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Why the proton spin is not due to quarks*

Marek Karliner

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

ABSTRACT

Recent EMC data on the spin-dependent proton structure function suggest that very little of the proton spin is due to the helicity of the quarks inside it. We argue that, at leading order in the $1/N_c$ expansion, none of the proton spin would be carried by quarks in the chiral limit where $m_q = 0$. This model-independent result is based on a physical picture of the nucleon as a soliton solution of the effective chiral Lagrangian of large-N_c QCD. The Skyrme model is then used to estimate quark contribution to the proton spin when chiral symmetry and flavor $SU(3)$ are broken: this contribution turns out to be small, as suggested by the EMC. Next, we discuss the other possible contributions to the proton helicity in the infinite-momentum frame – polarized gluons (ΔG), and orbital angular momentum (L_z). We argue on general grounds and by explicit example that $\Delta G = 0$ and that if the parameters of the chiral Lagrangian are adjusted so that gluons carry $\sim 50\%$ of the proton momentum, most of the orbital angular momentum L_z is carried by quarks. We mention several experiments to test the EMC results and their interpretation.

The EMC data[1] on polarized structure functions of the proton signals the need to re-examine our understanding of the various contributions to the proton spin. In the non-relativistic quark model (NQM) the proton is constructed as a bound state of three heavy quarks ($m_q \sim 300$ MeV) and its spin results from combining the spins of these objects. The structure of the proton as suggested by QCD

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and the deep inelastic scattering (DIS) experiments is very different. The proton contains an infinite number of partons, i.e. quarks and gluons, and the quarks are light. Both the quarks and the gluons can contribute to the proton angular momentum, either by combining their intrinsic spins or through their orbital angular momentum. This is reflected in the sum rule

$$\frac{1}{2}\sum_{q}\Delta q + \Delta G + \langle L_z\rangle = \frac{1}{2}. \tag{1}$$

where

$$\Delta q = \int_0^1 \Delta q(x) = \int_0^1 dx\left[q_\uparrow(x) + \bar{q}_\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)\right] \tag{2}$$

$$\Delta G = \int_0^1 \Delta G(x) = \int_0^1 dx\,\left[G_\uparrow(x) - G_\downarrow(x)\right] \tag{3}$$

The net quark helicities Δq are related to matrix elements of the various axial currents between proton states, *e.g.*

$$\langle p|\, A^0_\mu\, |p\rangle = \sqrt{2/3}\left(\Delta u + \Delta d + \Delta s\right)\cdot\Sigma_\mu(p) \tag{4}$$

where $\Sigma_\mu(p)$ is the proton spin.

What are the experimental sources of information about the axial form factors? Historically, the first piece of information comes from charged-current weak interactions. Because these currents are almost conserved, i.e. have soft divergence $\propto m_q$, they have no anomalous dimension. This allows us to relate, through the operator product expansion, their low-energy matrix elements to parton distributions observed in DIS. Thus from neutron decay we obtain

- $$\Delta u - \Delta d = g_A = 1.25 \tag{5}$$

Hyperon $\beta-$decay, combined with $SU(3)$ flavor symmetry yields[2]

- $$(\Delta u + \Delta d - 2\Delta s)/\sqrt{3} = 0.39 \tag{6}$$

So far we have two equations in three unknowns. The third equation can be obtained from DIS involving the electromagnetic current. Because of the vector nature of the electromagnetic interaction, information about axial form factors can only be obtained if both the proton and the photon are polarized and their spins are either parallel or anti-parallel. The difference A_1 between the anti-parallel ($\equiv \sigma_{1/2}$) and the parallel cross section ($\equiv \sigma_{3/2}$) is expressed in the parton model as

$$A_1 \equiv \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \xrightarrow[\text{Bjorken limit}]{} \frac{\sum_q e_q^2 \left[q_\uparrow(x) + \bar{q}_\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)\right]}{\sum_q e_q^2 \left[q_\uparrow(x) + \bar{q}_\uparrow(x) + q_\downarrow(x) + \bar{q}_\downarrow(x)\right]} \tag{7}$$

Using the measured values of the unpolarized structure function

$$F_2 = \sum_q e_q^2 x \left[q_\uparrow(x) + \bar{q}_\uparrow(x) + q_\downarrow(x) + \bar{q}_\downarrow(x)\right] \tag{8}$$

one can extract from A_1 the structure function

$$g_1(x) = \frac{1}{2}\sum_q e_q^2 \left[q_\uparrow(x) + \bar{q}_\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)\right] = \frac{1}{2}\sum_q e_q^2 \Delta q(x). \tag{9}$$

$g_1(x)$ was obtained in this way by the SLAC-Yale collaboration in the 1970's[3] and more recently, for a wider range of x, by the EMC collaboration.[1]

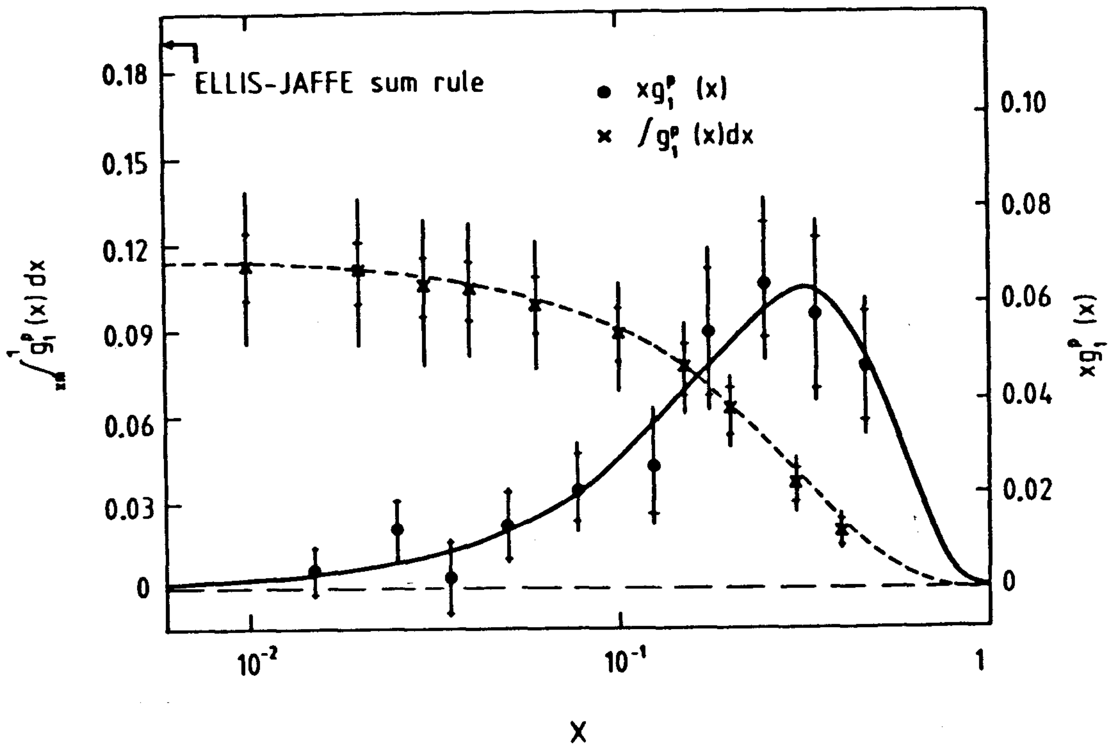

Fig. 1. EMC results for $xg_1^p(x)$ (Ref. 13)

Their combined[4] result is

$$\int_0^1 dx g_1^p(x) = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right) = 0.112 \pm 0.009 \pm 0.019 \tag{10}$$

At low x, one expects $g_1^p(x) \sim x^\alpha$ where[5] $\alpha \simeq 0$ is the intercept of the $a_1(1270)/f_1(1285)/f_1(1420)$ Regge trajectory. Since all meson Regge trajectories are expected to have equal slopes α', one expects the intercepts of the $a_1(1270)$ and $f_1(1285)$ trajectories to be almost equal, with the intercept of the $f_1(1420)$ trajectory slightly lower. Accordingly, we have fitted the data on $g_1^p(x)$ at low x with a single power of x: $g_1^p(x) \simeq B\, x^{-\alpha}$. We have made fits to the lowest 8, 7, 6 and 5 data points, as seen in Fig. 2. All the fits are of good quality and consistent with one another. For example, using the seven points in $x < 0.2$ one finds

$$\alpha = -0.07^{+0.42}_{-0.32} \, , \qquad B = 0.30^{+0.44}_{-0.17} \, . \tag{11}$$

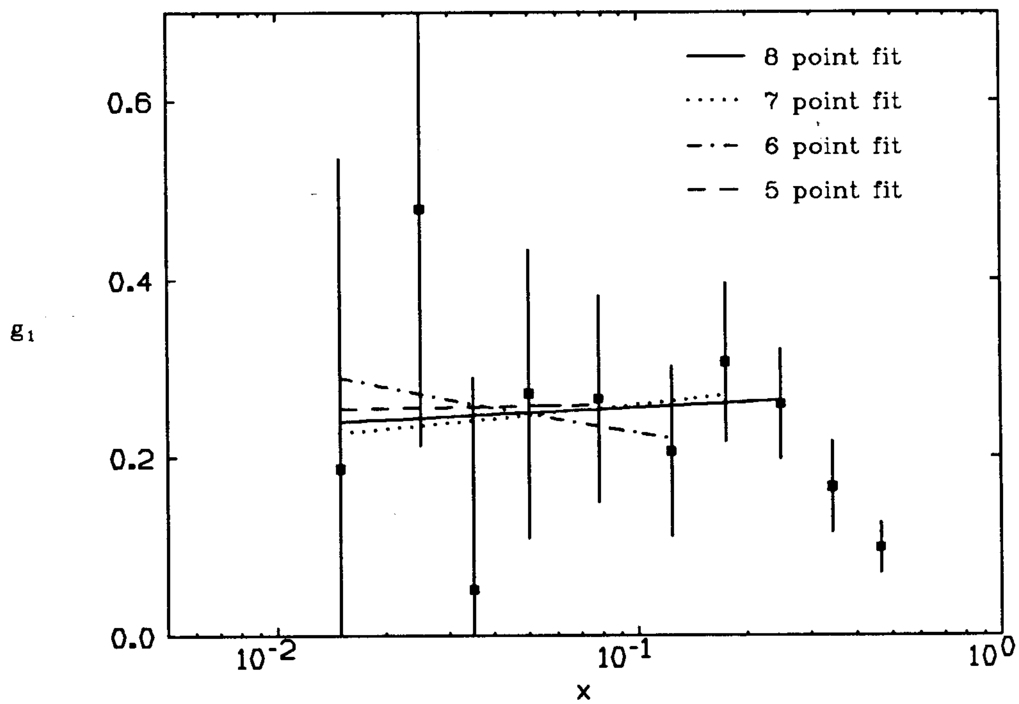

Fig. 2. Fits to the EMC data[13] on $g_1^p(x)$ of the form $Bx^{-\alpha}$. The data points at the 8,7,6 and 5 lowest values of x are used. (Ref. 12)

The result (11) gives us confidence that the EMC data at low x can be trusted. Let us then see what are the implications of (10).

In 1974, using the experimental values of the charged current matrix elements, taken together with $SU(3)$ flavor symmetry and the assumption $\Delta s = 0$, Ellis and

Jaffe[6] wrote down a sum rule

$$\int_0^1 dx g_1^p(x) = 0.19 \tag{12}$$

which is violated by the EMC result. Now that we have three equations denoted by •, its failure can be traced back[7,8,9] to the assumption $\Delta s = 0$, as the solution of the equations is

$$\left.\begin{aligned}\Delta u &= 0.73 \pm 0.07\\ \Delta d &= -0.52 \pm 0.07\\ \Delta s &= -0.24 \pm 0.07\end{aligned}\right\} \quad \Delta u + \Delta d + \Delta s = -0.01 \pm 0.21 \tag{13}$$

The first surprise is the large value of Δs. But perhaps we shouldn't have been so surprised. The large value of the σ-term in πN scattering has for some years been known to indicate[10] rather large strange sea in the proton, $\langle p|\, \bar{s}s\, |p\rangle$.

A more striking conclusion is that the total contribution of quark helicities to proton helicity is zero. Loosely speaking, the contribution of valence quarks is cancelled out by the sea quarks. As noted, a crucial ingredient is the relatively large and negative Δs. An independent corroboration of the above estimate of Δs can be obtained[11,12] from weak neutral current, elastic $\nu p \to \nu p$ and $\bar{\nu} p \to \bar{\nu} p$ scattering:[13] since Z^0 couples to $(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d - \bar{s}\gamma_\mu\gamma_5 s)$ the deviation of the axial form factor $G_1(q^2 = 0)$ from $g_A = \Delta u - \Delta d$ provides an estimate $\Delta s = -0.15 \pm 0.09$.[12] The neutral current result in itself would not be sufficient to establish that $\Delta s < 0$, but is very important as independent verification of the EMC result.

It has recently been observed[14,15] that the Δu, Δd, Δs appearing in the parton model expression for $\int_0^1 dx g_1^p(x)$ and elsewhere acquire QCD radiative corrections and should be replaced[15] by $\widetilde{\Delta u} = \Delta u - (\alpha_s/2\pi)\Delta G$, etc... It has been suggested[15] that perhaps $\Delta s = 0$ and the discrepancy between the EMC result for $\int_0^1 dx g_1^p(x)$ and the previously expected value of 0.19[6] might be entirely due to ΔG. This would require $\Delta G \simeq 8 \pm 2$ at $Q^2 \simeq 10\,\mathrm{GeV}^2$, where $\alpha_s \simeq 0.2$, and $L_z \simeq -8$, surprisingly large values. We will in fact argue in the following that $\Delta G \simeq 0$.

The result $\Delta u + \Delta d + \Delta s \simeq 0$ can be rephrased as a statement about the matrix element of the ninth axial current,

$$J^0_{\mu 5} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s; \qquad \langle p|\, J^0_{\mu 5}\, |p\rangle = -0.01 \pm 0.21 \tag{14}$$

It should be stressed here that u, d and s in $\Delta u + \Delta d + \Delta s \simeq 0$ are current, not constituent, quarks. Because of the many successes of NQM we sometimes forget the difference between the two and tend to apply our NQM intuition to DIS phenomena and that is part of the reason why the result (13) is so surprising. In fact, it turns out that $\langle p|\, J^0_{\mu 5}\, |p\rangle = 0$ occurs naturally in large-N_c QCD in the chiral limit, i.e. with current masses of quarks taken to be zero.[9] Given that we are interested in the matrix element of an axial current at zero momentum transfer, it is natural to calculate it in an effective Lagrangian. Since the early sixties it has been known that chiral Lagrangians provide a very successful description of soft pion physics.[16] One approximates the QCD Lagrangian with an effective Lagrangian describing low energy dynamics of a chiral field U:

$$\mathcal{L}_{QCD}(q,g) \longrightarrow \mathcal{L}_{eff}(U); \quad U = \exp\left(2i\pi_a\tau_a/f_\pi\right) \tag{15}$$

More recently it has been realized that in large-N_c QCD the chiral Lagrangians describe baryon, as well as pion physics, provided only that the momentum transfer is small compared to the QCD scale.[17,18] Baryons appear as solitons of the chiral Lagrangian - "Skyrmions". Baryon number is identified with topologically conserved winding number. The solitons, when quantized, have precisely the same spin and flavor quantum numbers as lowest lying baryons - J=1/2 isodoublet for $SU(2)$ flavor and J=1/2 octet together with J=3/2 decuplet for $SU(3)$ flavor.[19] All the qualitative counting rules of large-N_c QCD are correctly reproduced, including N_c dependence of baryon masses, radii and hadronic cross sections.[19]

On a more quantitative level, N_c independent ratios of experimental quantities as well as pion-nucleon partial-wave amplitudes are reproduced rather well.[20,21] Thus, to the extent that the real world with $N_c = 3$ is well described by large-N_c QCD, that description is also present in the chiral Lagrangian language. A useful analogy is the Thomas-Fermi model of the atom[22] where one replaces the electron wave function by the average of its bilinear. Similarly, in this framework one replaces the quark field operator by the average of its chiral bilinear:

$$\text{atom: } \rho(r) \sim \psi^*\psi\,; \Leftrightarrow \text{ proton}\cdot\ U \sim \bar{q}_L q_R.$$

The full effective Lagrangian contains a very large number of couplings and fields. The Skyrme model is only a rough approximation to the full $\mathcal{L}_{eff}$. It does, however, have all the right symmetries and can be used to illustrate model independent results which are valid in any chiral Lagrangian in which the nucleon corresponds to a hedgehog soliton (see below). The result (14) is precisely of this kind.[9] To see this, consider a "generic" Lagrangian of the form

$$\mathcal{L} = \frac{f_\phi^2}{16}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \ldots \quad ; U(x) = \exp\left[\left(\frac{2i}{f_\phi}\right)\sum_{i=0}^{8}\lambda_i\phi_i(x)\right] \qquad (16)$$

where $\{\phi_i \equiv \eta_0,\ \underset{\sim}{\pi},\ K_a,\ \eta_8\}$.

$\mathcal{L}$ is invariant under $SU(3)_L \times SU(3)_R$: $U \to VUW^\dagger$. The corresponding Noether currents can be written explicitly in terms of U. Since U has non-zero expectation value, $SU(3)_L \times SU(3)_R$ is spontaneously broken down to vector $SU(3)$ and the remaining axial $SU(3)$ is realized in Goldstone mode. The vacuum corresponds to $\langle U\rangle = 1$, while in the sector with baryon number $= 1$ the classical ground state is given by a "hedgehog" soliton $U_0 = \exp[iF(r)\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau}]$. This ground state has a large degeneracy, $U_0 \to VU_0V^\dagger$ where V is any constant $SU(3)$ matrix. This degeneracy is removed when V-s are treated as collective coordinates and the corresponding Hamiltonian is diagonalized. Baryon wavefunctions $B(V)$ are the eigenstates of the collective coordinate hamiltonian. Matrix elements of the currents can now be evaluated explicitly.[19] For example, for the axial isovector current

$$\langle B|\, J^a_{i5}\, |B\rangle \propto \langle B(V)|\, \mathrm{Tr}(\lambda_i V \lambda_a V)\, |B(V)\rangle \tag{17}$$

where a is the isospin index and i is a spacelike Lorentz index. For the isoscalar current, $\lambda_a \to \lambda_0 = \sqrt{\frac{2}{3}}\mathbf{1}$ and therefore[9]

$$\langle B|\, J^0_{i5}\, |B\rangle \propto \langle B(V)|\, \mathrm{Tr}(\lambda_i V \lambda_0 V)\, |B(V)\rangle = 0 \tag{18}$$

which is equivalent to $\Delta u + \Delta d + \Delta s = 0$, (*cf.* (13) and (14).) Let me stress again that this is a model independent result, relying only on the particular symmetry of the U_0. The vanishing of $\langle p|\, J^0_5\, |p\rangle$ can also be understood by considering the soliton topology. The soliton exists because the mappings from the real space to internal $SU(2)$ or $SU(3)$ flavor group space fall into distinct classes which cannot be continuously deformed into each other: $\Pi_3(SU(2)) = \mathbb{Z}$. But the same is not true for when the internal target space is $U(1)$, for $\Pi_3(U(1)) = 0$. That means that the soliton has no tail in the isoscalar direction and that the corresponding current decouples.

We have just obtained the matrix element of J^0_5 at $Q^2 = 0$. Unlike the flavor non-singlet currents however, J^0_5 has a "hard" divergence, due to the triangle anomaly[23]. Because of that, it also has non-zero anomalous dimension[24] and its matrix elements have some Q^2 dependence. We should therefore proceed with caution[25] when attempting to relate the $Q^2 = 0$ result to DIS data. Fortunately, the renormalization in this case is multiplicative, so if $\langle p|\, J^0_5\, |p\rangle = 0$ at some Q^2, it will remain so at all Q^2 so that (18) which is derived in low Q^2 effective Lagrangian remains valid in the kinematic region explored by the EMC[9]

We thus see that in the double limit $N_c \to \infty$ and $m_q = 0$ the result (13) occurs naturally. We do not know at present how to compute the $1/N_c$ corrections, but we can estimate corrections of $\mathcal{O}(m_q/\Lambda)$. This is done by adding to $\mathcal{L}$ a mass term

$\sim$ $\mathrm{Tr}[m_q(U+U^\dagger-2)]$ and an extra kinetic term,

$$\Delta\mathcal{L}_K = \epsilon\frac{f_\phi^2}{16}\,\mathrm{Tr}\left[\frac{\lambda_8}{2}\left(U^\dagger U_{\mu L}U^{\mu L} + U^\dagger U_{\mu R}U^{\mu R}\right)\right] \tag{19}$$

which have the effect of introducing $\eta-\eta\prime$ mixing and $f_\pi \neq f_K$. When these effects are taken into account, we obtain

$$\frac{\langle p|\,J_5^0\,|p\rangle}{\langle p|\,J_5^8\,|p\rangle} = -0.38 \qquad \text{(to be compared with } \sqrt{2} \text{ in NQM)}, \tag{20}$$

leading to a corrected estimate[9] $\Delta u+\Delta d+\Delta s = -0.18$ (*vs.* exp. value -0.01 ± 0.21). Please keep in mind, though, that this does not take into account possible $1/N_c$ or higher order m_s/Λ_{QCD} corrections.

Given the sum rule (1) and the result $\sum_q \Delta q \simeq 0$, we would like to find out where the proton spin does come from. In the chiral soliton approach the proton angular momentum is purely orbital. To see this explicitly and to make sure that the glue does not contribute to L_z, we will make $\mathcal{L}(U)$ scale invariant, as expected[26] of $\mathcal{L}_{eff}$ for QCD. To that effect we introduce a scalar gluonium field χ.[27,28] The modified kinetic term in $\mathcal{L}$ reads

$$\mathcal{L} = \frac{f_\phi^2}{16}\mathrm{Tr}\left(\partial_\mu U\partial^\mu U^\dagger\right)\chi^2 + \ldots \tag{21}$$

The classical solution of (21) is given in terms of U_0 and the glue "profile" $\chi(r)$. U and χ indicate the relative contribution of quarks and gluons, respectively, to the energy-momentum tensor $\theta_{\mu\nu}$, and through it to the various observables. For example, soliton mass M_0 is given by

$$M_0 = \int d^3\boldsymbol{r}\,\theta_{00}(\boldsymbol{r}) \tag{22}$$

With this in mind, we first compute $\theta_{++} = \theta_{00} + \theta_{33}$ and require that half of proton's linear momentum in the infinite momentum frame be carried by gluons.

The adjustable parameters in $\mathcal{L}$ are thereby fixed. Next, we consider the angular momentum. In the collective coordinate approach the spin of the proton is due to rotation of the soliton as a whole, i.e. $\langle \Delta G \rangle = 0$, $J_z = L_z = \frac{1}{2}$. In other words, $J_z = \omega I$, where $I \sim N_c$ is the moment of inertia and $\omega \sim 1/N_c$ is the (slow) angular frequency of rotation. The slow rotation justifies the semi-classical treatment of the problem. The moment of inertia I is given by

$$I = \int d^3\mathbf{r}\theta_{00}(\mathbf{r})r^2 \tag{23}$$

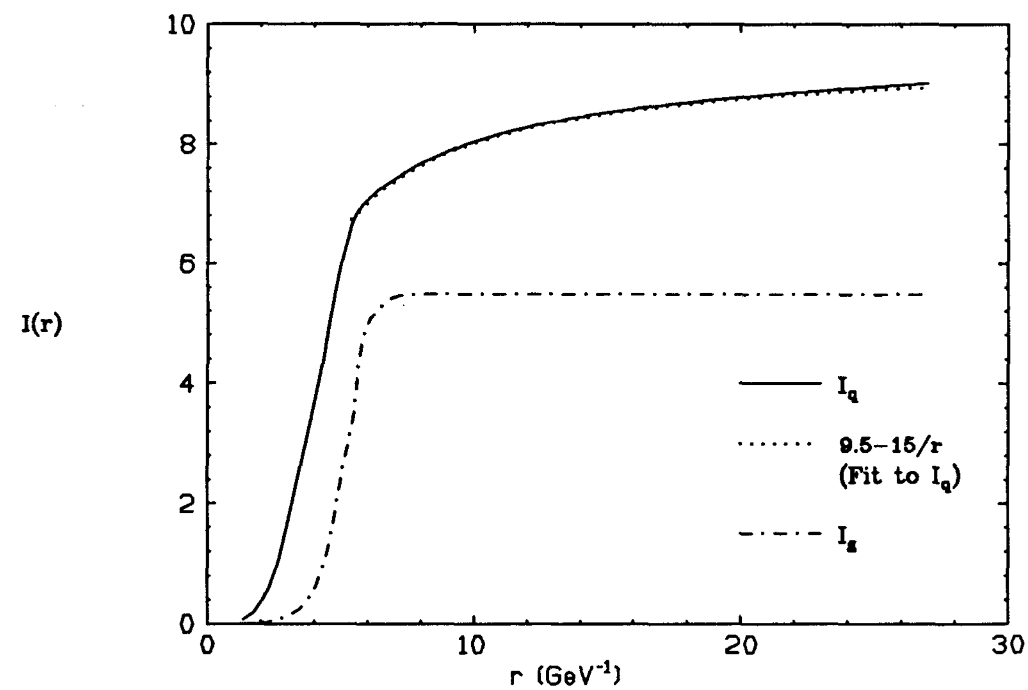

Fig. 3. Quark and gluon contributions, I_q and I_g, to the moment of inertia I (Ref. 12)

The relative contribution of quarks and gluons to the spin is determined by their relative contributions to I. In the chiral limit these turn out to be 36% and 64%, respectively.[12] The glue contributes less, because its energy density is concentrated in a small region of space ~ 1 fm, as suggested by the bag model, while the chiral field extends farther away.

The physical picture of the proton spin as suggested by this work has several rather interesting experimental consequences.

Clearly it is of great importance to confirm the EMC result (10) and to measure also $\int_0^1 dx g_1^n(x)$ using polarized neutrons, so as to check the Bjorken sum rule.[29] The theoretical interest in new experiments to measure these quantities is enhanced by the fundamental information about chiral symmetry and its breaking that they provide. We also remind the reader of the relevance of $\langle p| A_\mu^0 |p\rangle$ to dark matter searches[7,8] and to axion couplings.[30] Assuming that the EMC measurement (10) is essentially correct, the next priority is to determine the origin of the bulk of the proton spin, which must be carried by gluons and/or orbital angular momentum: $\frac{1}{2}(\Delta u + \Delta d + \Delta s) + \Delta G + \langle L_z \rangle = \frac{1}{2}$. There are various possibilities for measuring ΔG, including the following.*

(*a*) Measurement of J/ψ production and decay properties in deep inelastic muon scattering off polarized targets;[32]

(*b*) Measurement of $\chi_2(3555)$ production and decay properties in hadronic collisions[33]

(*c*) Measurements of charm distributions in deep inelastic scattering off a polarized target using dimuon events from $c(\bar{c}) \to \mu^+(\mu^-) + X$ decays;

(*d*) Hadronic jet asymmetries in polarized pp collisions;[34]

(*e*) Direct photon production at large p_T by polarized protons;[34]

(*f*) Hyperon production at large p_T in polarized pp collisions;[35]†

⋆ For a review and other references on spin physics at short distances, see Ref. 31.

† The fact that $\Delta s < 0$ suggests that there may be significant spin anticorrelation for hyperons produced by polarized protons, even at low p_T.

(*g*) Higher order effects in polarized *ep* collisions;[36]

(*h*) Drell-Yan l^+l^- production with polarized beams;[37]

(*i*) Large p_T hadron production in photoproduction off polarized targets.[38]

We have been discussing the polarized structure function of the proton and its physical interpretation. The physical picture I have described here is based on large-N_c QCD and on spontaneous breaking of chiral symmetry. It is in agreement with the data and has some interesting predictions. There have been several attempts to understand the EMC data by other means. Some of those have been mentioned in some detail in the text, together with their drawbacks, as we see them[9,12]:

- " $\langle p|\, J^0_{\mu 5}\, |p\rangle$ *varies rapidly as function of renormalization scale* Q^2"[25]

- " *Isospin breaking effects:* $m_u \neq m_d$ *are important* "[40]

- " *A crisis in the parton model*"[39]

- " $\int dx g_1^p(x)$ *gets a large contribution from glue* "[14,15]

- " $\int\limits_0^\infty \frac{d\nu}{\nu} G_1^p(\nu, Q^2)$ *not yet asymptotic at* $Q^2 = 10$ *GeV, due to higher twists.*"[41]

- " *Perturbative QCD is wrong*"[42]

- " *EMC is wrong*"[43]

- " *The naive interpretation of quark model is wrong*"[9]

The above list summarizes the various suggestions that have been made. I hope it will serve as a catalyst for further research into proton spin structure, both experimental and theoretical.

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