

# Wideband Cognitive Radio Networks Based Compressed Spectrum Sensing: A Survey

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## Abstract

Spectrum sensing is a core function at cognitive radio systems to have spectrum awareness. This could be achieved by collecting samples from the frequency band under observation to make a conclusion whether the band is occupied, or it is a spectrum hole. The task of sensing is becoming more challenging especially at wideband spectrum scenario. The difficulty is due to conventional sampling rate theory which makes it infeasible to sample such very wide range of frequencies and the technical requirements are very costly. Recently, compressive sensing introduced itself as a pioneer solution that relaxed the wideband sampling rate requirements. It showed the ability to sample a signal below the Nyquist sampling rate and reconstructed it using very few measurements. In this paper, we discuss the approaches used for solving compressed spectrum sensing problem for wideband cognitive radio networks and how the problem is formulated and rendered to improve the detection performance.

## Keywords

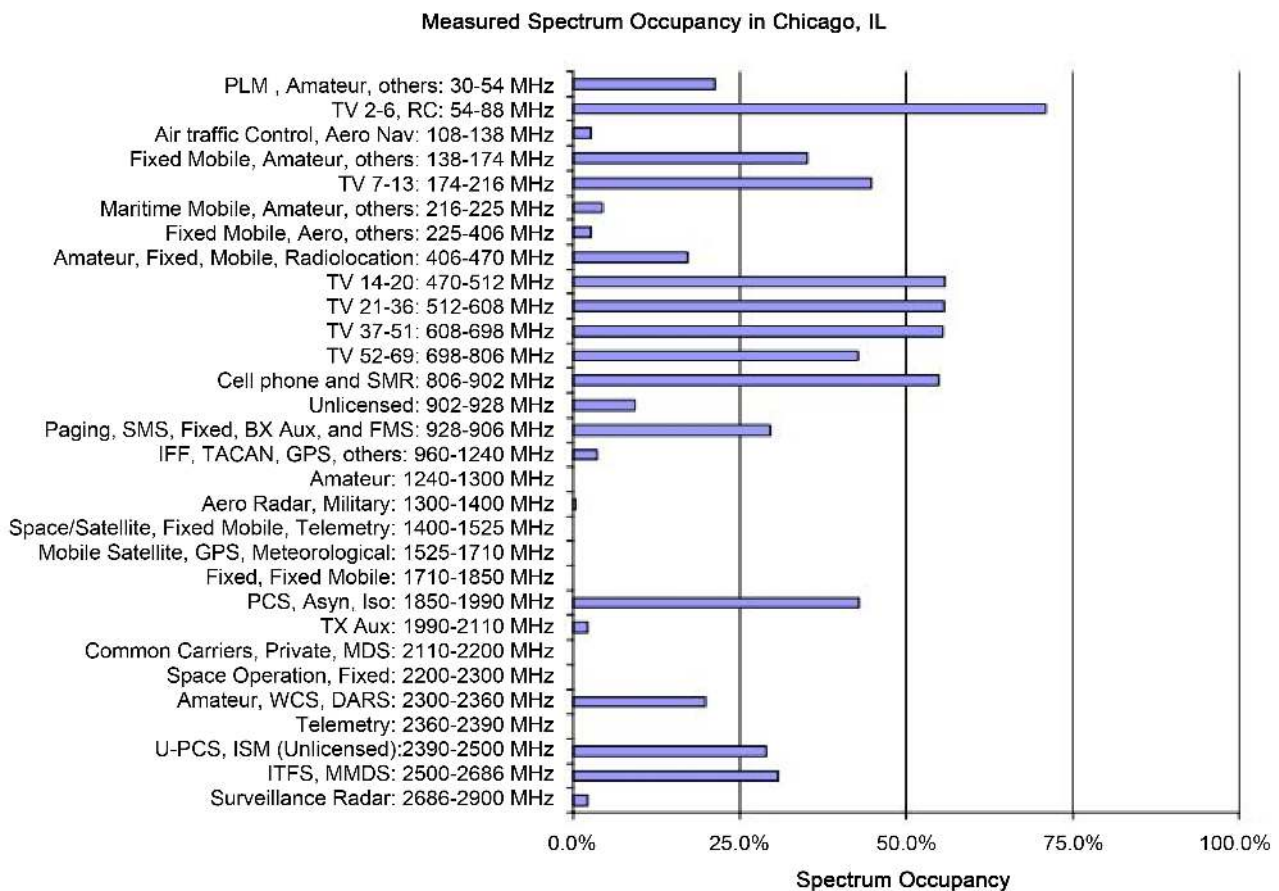
Cognitive Radio, Spectrum Sensing, Compressive Sensing, Compressed Measurements, Nyquist Sampling Rate

## 1. Introduction

The spectrum scarcity has created a false believe that there are no more useful

radio frequencies left. This false believe occurred due to the current spectrum allocation policy and made the spectrum underutilization, *i.e.*, nearly 70% of radio occupancy was not used most of the time [1] [2]. The problem of underutilization led to have many frequency bands not operative or Spectrum Holes (SHs). SHs are not but frequency bands assigned to Primary Users (PUs) at specific frequencies and geographic locations not used for period. So, to solve the spectrum issue, the Federal Communications Commission (FCC) suggested opening these frequency bands dedicated to the PUs to be shared with Secondary Users (SUs). Opening these frequency bands is known later as Dynamic Spectrum Access (DSA) [3] [4]. In the DSA, the SUs access these unoccupied bands whenever the PUs are absent and should terminate their transmission when PUs transmitted signals are detected. Also, the SUs should keep the inference background within acceptable limit that does no harm to the communication transmissions of both PUs and other SUs. **Figure 1** manifests that certain lot of the radio bands is sparsely used and substantial amount of these bands is unutilized [2].

However, a question may pose; how do SUs identify these hidden spectral opportunities or SHs? Simply without complication by searching the spectrum looking for empty slots/bands or detecting primary user transmitted signal on



**Figure 1.** Spectrum occupancy in each band measured in Chicago.

the band(s) under observation. For more clarity, spectrum sensing mechanism is nothing but collecting samples from the frequency band(s) of interest, then thresholding and draw up a conclusion about that band(s) statue [5]. Cognitive Radio (CR) as DSA technique enables SUs the spectrum awareness. This arises from that it has the ability to adapt its transmission parameters to meet spectral opportunities requirements such as transmission power, modulation schemes, code rates, etc. [4] [6] [7] [8].

Spectrum sensing is a mandatory component at IEEE 802.22 WRAN standardization to enable self-coexistence for CR and it could be achieved by two methods [9] [10]. In the first method, the radio itself becomes spectrum awareness by detecting and estimating the spectrum to identify which channels are occupied and which are unoccupied [11]. In the second method, geolocation and database or beacon are used for identifying SHs [9]. However, the second method requires the involvement of PUs cooperation which violates the condition of CR's independency on performing sensing task depending on its own ability without PU assistance. Moreover, spectrum sensing performance could be improved if cooperation between SUs is involved to mitigate hidden node problem, multipath fading and shadowing [12] [13]. And even further enhanced performance could be achieved by introducing Compressive Sensing (CS) technique. Most of researches focused on detecting primary transmitter signal rather than primary receiver detection due to some technical difficulties with the latter.

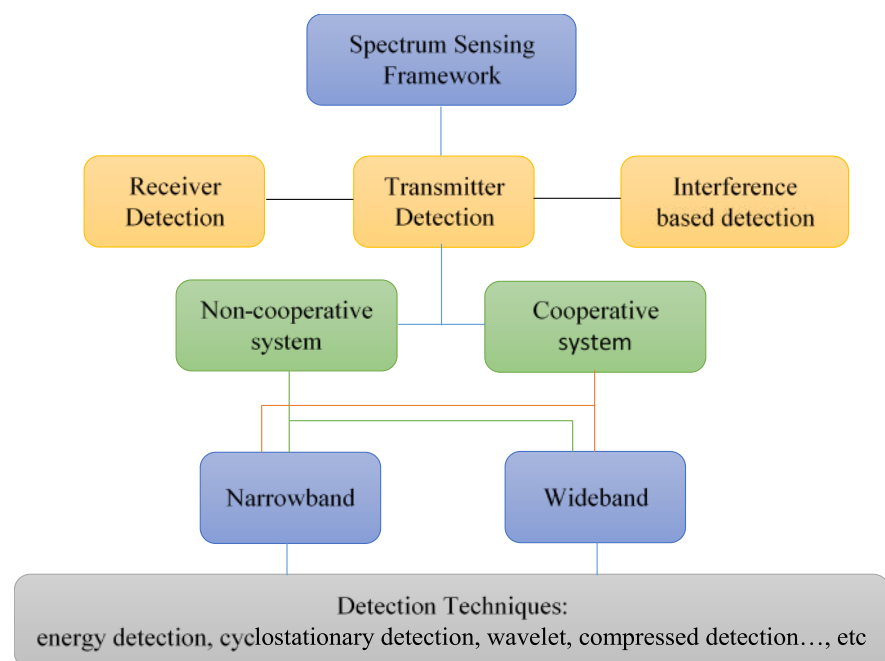
The sensing task becomes more challenging especially in wideband spectrum [14], e.g., a range up to 20 GHz. Conventional Digital Signal Processing (DSP) may take long time processing this wide range of frequencies and it's very expensive. To the best of our knowledge, the achievable sampling rate of the current Analog-to-digital convertor (ADC) is 3.6 Giga Sample per second or 3.6 GSps [15]. Filter bank technique could ease sampling requirements by turn wideband into narrowband [16] but we end with another problem which is the number of filters required. Nyquist sampling rate is very serious problem in wideband spectrum once it is rendered the rest is straight forward task. CS is a powerful tool that cracked the sampling rate problem [17]. It proved the possibility of sampling below Nyquist sampling rate, also known as sub-Nyquist sampling rate, and reconstructed the transmitted signal after sub-Nyquist sampling. However, it requires the signal to be sparse or compressible [18] [19]. If a signal is not sparse, it may transform to another domain at which sparsity condition is hold. Numerous research papers were published to identify spectrum holes-based CS through signal reconstruction algorithms. This kind of spectrum holes identification dominant since then. To the best of our knowledge there are few publications that concern with spectrum sensing problem as detection problem without going to reconstruction stage [20] [21] [22].

In general, spectrum sensing problem can be discussed in several ways based on different criterions and different assumptions. For example, if the classification is based on what or how to sense, then discussion would classify the spec-

trum sensing to: 1) transmitter detection, 2) cooperative detection, and 3) interference-based detection [11]. Depending on the type of applications leads to highlight to the spectrum types, either narrow band sensing or wideband sensing. Proactive and reactive sensing are another aspect of classification. Mostly, in literature, spectrum sensing is based on transmitter detection due infeasibility to detect primary receiver signal such as the location of PU receiver [6] [11]. Different detection techniques, *i.e.*, energy detection, matched filter, cyclostationary detection, covariance-based detection wavelet, compressed detection..., etc, can be employed to detect the SHs. Spectrum sensing framework is depicted in **Figure 2**. Comparison of the main spectrum sensing is illustrated in **Table 1**.

Even though there are numerous research papers that discussed the problem of spectrum sensing for wideband cognitive radio based compressed sensing, nevertheless, the detection problem is formulated as detection based-signal reconstruction using compressed sensing which is the prevailing trend [5] [6] [18] [23]. In [23], the authors have focused on discussing technical implementation issues of Analog to Information Conversion (AIC). In this paper the compressed spectrum sensing problem at wideband CR networks is addressed. And the recent research achievements in solving the problem are discussed, *e.g.*, how the problem is formulated and rendered to improve the detection performance.

The rest of this paper is structured as follows. Section 2 highlighted the wideband CR spectrum sensing challenge. In Section 3, compressed spectrum sensing problem is formulated. Compressed wideband CR spectrum detection approaches are discussed in Section 4. The open research problems of the wideband CR networks based compressive spectrum sensing are introduced in Section 5. Finally, Section 6 presents the main conclusions.



**Figure 2.** Spectrum sensing framework.

**Table 1.** Comparison of main spectrum sensing techniques.

Detection technique	Performance metrics		
	Accuracy	Complexity	Robustness
Energy detection	<ul style="list-style-type: none"> <li>- at high SNRs, the performance is good</li> <li>- at low SNRs, the performance is unreliable</li> <li>- affected by noise uncertainty</li> </ul>	<ul style="list-style-type: none"> <li>- Low implementation complexity</li> <li>- convergence is reach by collecting higher number of samples</li> </ul>	<ul style="list-style-type: none"> <li>- orior information of PU signal isn't required</li> <li>- inappropriate for spread spectrum signals</li> <li>- cannot differentiate a PU signal from other signal sources</li> </ul>
Feature detection	<ul style="list-style-type: none"> <li>- the performance is good at all SNRs</li> </ul>	<ul style="list-style-type: none"> <li>- medium complexity</li> <li>- convergence requires small number of samples</li> </ul>	<ul style="list-style-type: none"> <li>- require partial knowledge of PU signal</li> <li>- robust against noise uncertainty and interference</li> <li>- differentiate PU signal among different types of signal source</li> </ul>
Matched filter and coherent detection	<ul style="list-style-type: none"> <li>- best performance at all SNRs</li> </ul>	<ul style="list-style-type: none"> <li>- high complexity</li> <li>- convergence requires fewest number of samples</li> </ul>	<ul style="list-style-type: none"> <li>- require precise prior information PU signals</li> </ul>
Covariance-based detection	<ul style="list-style-type: none"> <li>- detection accuracy is high</li> </ul>	<ul style="list-style-type: none"> <li>- low computational complexity</li> </ul>	<ul style="list-style-type: none"> <li>- uncorrelated PU signals degrade detection Performance.</li> <li>- blind detection</li> </ul>

## 2. Wideband CR Spectrum Sensing Challenge

In literature, there have been several signal processing challenges facing CR since certain tasks have become more complicated. Some of these were discussed in [6] [24]. In this section, we discuss the wideband spectrum sensing problem in the sense that the complication in this type of spectrum is very high. In fact, there is no signal processing approach yet exist that could provide the locations of all the hidden spectral opportunities and at what frequency band at one single shot. Several technical challenges are facing wideband sensing and limiting its operation. One of these challenges is the need for high data rate radio front end requirement to sense a wide frequency range. This adds additional constraint for CR system deployment especially on units with limited data rate processors. Conventional estimation methods based on Nyquist sampling are challenging task in wideband spectrum. Even though using filter banks may help to turn a wideband signal into narrowband to be easy sensed using conventional methods, but with the cost of using large number of narrowband filters. Even though the

above-mentioned problem is very challenging in wideband CR spectrum sensing, we will see how compressive sensing technique relaxes Nyquist sampling rate effectively alongside with reducing both complexity and power consumption requirements.

### 3. Compressive Spectrum Sensing Problem

The idea of using learning and sensing machines to probe the radio spectrum was envisioned several decades earlier. Two different frameworks regarding how to formulate spectrum sensing were presented. Signal detection and signal classification which encapsulated the spectrum sensing on coming up a decision whether the PU transmitted signal, e.g., deterministic or random, is present or not from the observed received signals [25]. Due to the importance of this topic, we will discuss the sensing or detection problem as entrance to compressive spectrum sensing problem template. Moreover, we will investigate how the detection based compressive sensing could improve the detection performance in CR systems.

#### 3.1. Signal Sensing Problem

Signal detection has been an important topic in wireless communication engineering and in CR. It is considered as a core function. In CR the radio frequency spectrum is sensed to come out with a decision whether the PU transmitted signal, mostly very weak PU signals, exists in a certain frequency band of interest or not. In other words, it enables the SUs identifying the SHs. So, we could formulate the spectrum sensing problem by the classical binary hypothesis test [5] [11] [26] as follows:

$$x(t) = \begin{cases} n(t) : \mathcal{H}_0, \text{PU is absent} \\ s(t) + n(t) : \mathcal{H}_1, \text{PU is present} \end{cases} \quad (1)$$

where,  $x(t)$  is the received signal at the SU receiver during observation window  $T$ .  $n(t)$  is the additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma^2$ , i.e.,  $n \sim \mathcal{N}(0, \sigma^2)$ .  $\sigma^2$  is also considered as a noise power.  $s(t)$  is the transmitted PU signal and it is independent and identical Gaussian random variable.  $\mathcal{H}_0$  and  $\mathcal{H}_1$  denote the absence and the presence of the PU respectively. The decision between the two hypotheses is based on the comparison of statistical test to a predetermined threshold  $\lambda$  as follows;

$$T \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \lambda \quad (2)$$

where,  $T$  is the statistical test generated to decide if the observed signal  $x(t)$  under hypothesis  $\mathcal{H}_0$  or  $\mathcal{H}_1$ . Most widely used detectors, e.g., energy detection, and cyclostationary detection, perform the statistical test to differentiate the PU transmitted signal from noise as depicted in **Figure 3**.

In the context of spectrum sensing, the performance of sensing is measured by the Receiver Operating Characteristics (ROC) curves. It is a tradeoff between

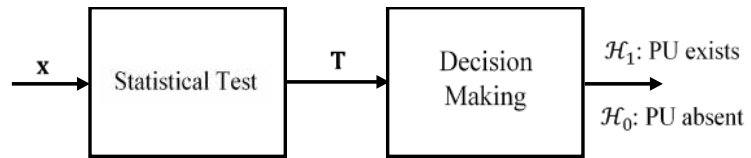


Figure 3. Primary user signal detection scenario.

probability of detection versus probability of false alarm or probability of miss-detection versus probability of false alarm [26]. Let us assume the received signal in (1) follows normal Gaussian distribution as depicted in Figure 4. Then then decision statistics under each hypothesis  $\mathcal{H}_0$  and  $\mathcal{H}_1$  could be denoted as  $f(\mathbf{x}|\mathcal{H}_0)$  and  $f(\mathbf{x}|\mathcal{H}_1)$ , respectively.

If the observed sample of  $x(t)$  is greater than the threshold, the detector is more likely to choose  $\mathcal{H}_1$  or  $f(\mathbf{x}|\mathcal{H}_1) > f(\mathbf{x}|\mathcal{H}_0)$ . The error in this scheme occurs as follows; if the detector decides  $\mathcal{H}_1$  when  $\mathcal{H}_0$  true and this type of error is known as false alarm. On the other hands, if the detector decides  $\mathcal{H}_0$  when  $\mathcal{H}_1$  true and this type of error is known as miss detection. These two errors are inevitable to some extent nevertheless may be traded off against each other to achieve high probability of detection  $P(\mathcal{H}_1|\mathcal{H}_1)$  to guarantee minimal interference with the PU [26].

At the same time, we need to have a low probability of false alarm  $P(\mathcal{H}_1|\mathcal{H}_0)$  to improve the SU throughput. This setup is called the Neyman-Pearson (NP) approach to spectrum sensing. The reason of using NP test is we do not know the prior probabilities for  $P(\mathcal{H}_0)$  and  $P(\mathcal{H}_1)$  which yields having a problem in minimizing the averaging probability of error. Also, we could not make risk minimization which requires having the prior probabilities of  $\mathcal{H}_0$  and  $\mathcal{H}_1$ . Maximum likelihood is an optimal detector but we have no way to evaluate its performance for the same problem of not having any knowledge of the prior probabilities [5]. The performance of the detector will be evaluated in term of these probabilities of detection, miss detection and false alarm, defined as;

*Probability of detection ( $P_D$ ):* it is the probability when SU detects PU transmitted signal correctly or declaring  $\mathcal{H}_1$  under  $\mathcal{H}_1$ ,  $P(\mathcal{H}_1|\mathcal{H}_1)$ .

$$P_D = P(\text{signal is detected} | \mathcal{H}_1) = P(X > \lambda | \mathcal{H}_1) \tag{3}$$

*Probability of false alarm ( $P_F$ ):* it is the probability when SU declares that PU is present, when it is not or declaring  $\mathcal{H}_1$  under  $\mathcal{H}_0$ ,  $P(\mathcal{H}_1|\mathcal{H}_0)$ .

$$P_F = P(\text{signal is detected} | \mathcal{H}_0) = P(X > \lambda | \mathcal{H}_0) = \int_{\lambda}^{\infty} f(\mathbf{x}|\mathcal{H}_0) dx \tag{4}$$

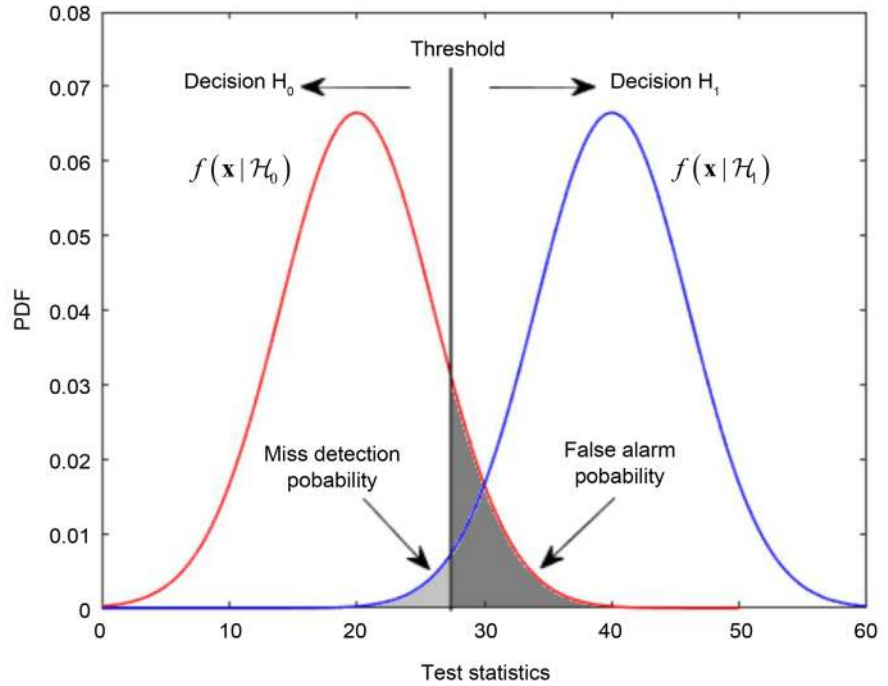
*Probability of miss-detection ( $P_M$ ):* it is probability of failing to detect the PU presence when it is there or declaring  $\mathcal{H}_0$  under  $\mathcal{H}_1$ ,  $P(\mathcal{H}_0|\mathcal{H}_1)$ .

$$P_M = P(\text{signal is not detected} | \mathcal{H}_1) = P(X < \lambda | \mathcal{H}_1) \tag{5}$$

### 3.1.1. Probability of Detection and False Alarm over AWGN Channels

The spectrum sensing problem could be formulated as a constrain optimization problem; maximize  $P_D$  where  $\alpha$  is a specified maximum tolerable false alarm





**Figure 4.** Trade off error by adjusting threshold.

probability [26]. In summary, ROC curves allow us to have insight understanding to that govern the relationship between the sensitivity ( $P_D$ ) and specificity ( $P_F$ ) of a spectrum sensing scheme for a variety of different algorithm parameters [27]. If the observed signal in (1) is deterministic, the test statistics could then be modeled as [28]

$$T \sim \begin{cases} N(0, \sigma^2 \|s\|^2) : \mathcal{H}_0 \\ N(\|s\|^2, \sigma^2 \|s\|^2) : \mathcal{H}_1 \end{cases} \quad (6)$$

And by solving (3) and (4), we have

$$P_F = \int_{\lambda}^{\infty} f(x | H_1) dx = \int_{\lambda}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\|s\|^2}{2\sigma^2}\right) dx = Q\left(-\frac{\lambda}{\sqrt{\sigma^2 \|s\|^2}}\right) \quad (7a)$$

$$P_D = \int_{\lambda}^{\infty} f(x | H_1) dx = \int_{\lambda}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x - \|s\|^2}{2\sigma^2}\right) dx = Q\left(-\frac{\lambda - \|s\|^2}{\sqrt{\sigma^2 \|s\|^2}}\right) \quad (7b)$$

where,  $Q(\cdot)$  is the complementary distribution function of a standard Gaussian. For indirect method, we could specify the amount of threshold required to maximize the  $P_D$  for certain  $P_F$  from (7a) as follows

$$\lambda = \sqrt{\sigma^2 \|s\|^2} Q^{-1}(P_F) \quad (8)$$

Substituting (8) into (7b), it yields

$$P_D = Q\left(Q^{-1}(P_F) - \|x\|/\sqrt{\sigma^2}\right) = Q\left(Q^{-1}(P_F) - \sqrt{\gamma}\right) \quad (9)$$



where,  $\gamma = \|s\|^2 / \sigma^2$ , i.e., the signal-to-noise ratio SNR. On the other hand, if PU transmitted signal  $s(t)$  assumed to be an unknown deterministic signal, the distribution of the decision statistics follows central chi-square  $\chi_N^2$  under  $\mathcal{H}_0$  and non-central chi-square  $\chi_N'^2$  with  $TW$  degrees of freedom under  $\mathcal{H}_1$  [29]. However, for the sake of simplicity, we exchange the time-bandwidth product by an integer number  $N$ . In literature, some researchers used instead of  $TW$ , they used  $2TW$  ( $N \rightarrow 2N$ ). In other words, we could use either even or odd degrees of freedom for the analytic processes relying on the type of applications.

$$\zeta \sim \begin{cases} \chi_N^2 : \mathcal{H}_0 \\ \chi_N'^2 : \mathcal{H}_1 \end{cases} \quad (10)$$

The probability density function of the decision statistics  $X_i$  is specified by

$$f_x(x) = \begin{cases} \frac{1}{\sigma^2 2^{N/2} \Gamma(N/2)} x^{(N/2-1)} \exp(-x/\sigma^2) : \mathcal{H}_0 \\ \frac{1}{2} \left(\frac{x}{\mu}\right)^{(N-2/4)} \exp(-\mu + x/2) I_{N/2-1}(\sqrt{\mu x}) : \mathcal{H}_1 \end{cases} \quad (11)$$

where,  $\mu$  is the noncentrality parameter ( $\mu \propto \|s\|^2 / \sigma^2$ ) and  $I_{\nu-1}(\cdot)$  is the  $\nu$ -th modified Bessel function of the first kind. The  $P_F$  and  $P_D$  over AWGN is given respectively by:

$$\begin{aligned} P_F &= \Pr(x > \lambda | \mathcal{H}_0) = \int_{\lambda}^{\infty} f_{x,\mathcal{H}_0}(x) dx \\ &= \int_{\lambda}^{\infty} \frac{1}{\sigma^2 2^{N/2} \Gamma(N/2)} x^{(N/2-1)} \exp(-x/\sigma^2) dx \end{aligned} \quad (12)$$

$$P_F = \Gamma(N/2, \lambda/2\sigma^2) / \Gamma(N/2) \quad (13)$$

Similarly,

$$\begin{aligned} P_D &= \Pr(x > \lambda | \mathcal{H}_1) = \int_{\lambda}^{\infty} f_{x,\mathcal{H}_1}(x) dx \\ &= \int_{\lambda}^{\infty} \frac{1}{2} \left(\frac{x}{\mu}\right)^{(N-2/4)} \exp(-\mu + x/2) I_{N/2-1}(\sqrt{\mu x}) dx \end{aligned} \quad (14)$$

$$P_D = Q_{\frac{N}{2}}\left(\sqrt{\mu/\sigma^2}, \sqrt{\lambda/\sigma^2}\right) \quad (15)$$

where,  $\Gamma(\cdot, \cdot)$ ,  $\Gamma(\cdot)$ , and  $Q_{\nu}(\cdot, \cdot)$  are complete and in complete gamma function and generalized Marcum Q-function, respectively. As the number of degrees of freedom increases, the  $\chi_N^2$  and  $\chi_N'^2$  probability density functions asymptotically turn out to be Gaussian by the central limit theorem [28].

$$PD = Q\left(Q^{-1}(PF) - \sqrt{N}\gamma\right) \quad (16)$$

A board published research has been conducted and discussed spectrum sensing in CR. To the best of our knowledge most of the researches are dedicated to improving the detection performance of the energy detector. Since it does not involve complicated signal processing and has short time response and low complexity cost [30]. It is called blind technique in that it does not require prior

information about the PU transmitted signal. Despite these merits made it preferable at wide variety of applications such as CR systems and ultra-wideband communications, it is subjected to noise uncertainty issue. On the other hand, cyclostationary detectors outperform all other detectors in term of spectrum sensing performance but with the expense of computational complexity. However, it requires long sensing time. **Table 2** includes some closed forms of spectrum sensing performance based on computations of  $P_D$ ,  $P_F$  and  $P_{MD}$  where,  $E$  is the received signal power.

### 3.1.2. Probability of Detection and False Alarm over Fading Channels

In the proceeding section, the spectrum sensing performance over AWGN channel and the derived detection probabilities pairs  $(P_D, P_F)$  have been discussed. To broad the picture for more than talking on AWGN, fading channel should not be ignored, e.g., Rayleigh, Nakagami-m, lognormal, ..., etc. The analytical expression  $P_D$  could be undergone of two popular methods; namely, the probability density function and the moment generating function (MGF) relying on the fading channel statistical characteristic to derive the performance analysis [31]. However, the MGF method is restricted to integer-valued fading descriptors. The conventional method for obtaining the average  $P_D$  over specific fading distribution is by averaging  $P_D$  of AWGN over  $f_\gamma(\gamma)$  of the output SNR. This method could be expressed as:

$$\bar{P}_D = \int_0^\infty P_D(\gamma) f_\gamma(\gamma) d\gamma \quad (17)$$

where  $f_\gamma(\gamma)$  is the fading channel distribution. Alternatively, it could be expressed by:

$$\bar{P}_D = \int_0^\infty Q_{\frac{N}{2}}\left(\sqrt{\frac{\mu}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right) f_\gamma(\lambda) d\gamma \quad (18)$$

with  $\mu \propto \gamma$  ( $\mu = \alpha\gamma$ ) and  $\alpha$  is a positive value. Many channel models have been suggested to define the amplitude and phase of multipath fading signals statistics. Among them Nakagami-m distribution is a generalized distribution to model different fading environments [32]. It has greater flexibility and accuracy

**Table 2.** Probabilities of detection and false alarm of well-known detectors over AWGN.

Detector type	Probability of detection	Probability of false alarm	Probability of miss-detection
Matched filtering	$Q\left(\sqrt{\frac{2E}{\sigma^2}}, \sqrt{\frac{2\lambda^2}{E\sigma^2}}\right)$	$\exp\left(-\frac{\lambda^2}{E\sigma^2}\right)$	$1 - Q\left(\sqrt{\frac{2E}{\sigma^2}}, \sqrt{\frac{2\lambda^2}{E\sigma^2}}\right)$
Energy	$Q_{\frac{N}{2}}\left(\sqrt{\frac{\mu}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right)$	$\frac{\Gamma\left(\frac{N}{2}, \frac{\lambda}{2\sigma^2}\right)}{\Gamma\left(\frac{N}{2}\right)}$	$1 - Q_{\frac{N}{2}}\left(\sqrt{\frac{\mu}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right)$
cyclostationary	$Q\left(\frac{\sqrt{2\gamma}}{\sigma}, \frac{(2N+1)\lambda}{\sigma^2}\right)$	$\exp\left(-\frac{(2N+1)\lambda^2}{E\sigma^2}\right)$	$1 - Q\left(\frac{\sqrt{2\gamma}}{\sigma}, \frac{(2N+1)\lambda}{\sigma^2}\right)$

in matching some experimental data than the Rayleigh, lognormal or Rice distributions [31] [33]. The letter  $-m$  in the word “Nakagami- $m$ ” is called Nakagami shape factor or the fading order. For example,  $m = 1$  is Rayleigh distribution which is sufficient to model amplitude in urban areas. While for  $m > 1$  Rician distribution suits better in sub-urban area where LOS components exist. Both Rayleigh and Rician distributions are special cases of Nakagami- $m$  distribution. The problem of spectrum sensing over fading channels is addressed in [34]. And for sake of simplicity, we tabulate the closed forms obtained in Table 3 (after mathematical manipulations see [34]).

In the following section, the sensing problem in case of compressive sensing and its formulation will be discussed.

### 3.2. Compressed Signal Detection Problem

Compressed sensing proved the conventional sampling theory was not completely true [17] [35] [36]. In the sampling theory, the transmitted signal requires to be sampled, at least, twice the maximum frequency component in the signal to be able to reconstruct it successfully at the receiver side. In wideband spectrum scenario, the signal has 3 GHz bandwidth and after sampling we get 6 GHz bandwidth which infeasible for implementation and signal processing. Even though current advance analog-to-digital conversion, to the best of our knowledge, achieve sampling rate at speed to 3.6 GSps [15] but the cost of this signal processing is very high. CS showed that the possibility to sample below Nyquist sampling rate as long the signal is sparse or compressible [36]. We say that a certain signal is sparse if it contains few coefficients are nonzero and the rest are zeros. And the signal is compressible it has few large coefficients and the remainders are small or zero. Also, signal sparsity could be maintained by transformation, e.g., Fourier, DCT, wavelet, ..., etc. Signals with this type of representation yield to underdetermined system problem because we are seeking to reconstruct the transmitted signal with  $N$  dimensions ( $x \in \mathbb{R}^{N \times 1}$ ) from the received signal with  $m$  dimensions ( $y \in \mathbb{R}^{m \times 1}$ ) where,  $m \ll N$ . Mathematically

**Table 3.** Probability of detection over different types of fading channels.

Type of fading channel	Closed form expression of $\overline{P}_d$
Rayleigh	$\exp(-\lambda/2\sigma^2)[1+\eta]^{(N/2-1)} \left[ \exp(-\lambda/2\sigma^2(1+\eta^2)) - \sum_{k=0}^{N/2-2} \frac{1}{k!} (\lambda/2\sigma^2(1+\eta^2))^k \right]$ $+ \exp(-\lambda/2\sigma^2) \sum_{k=0}^{N/2-2} \frac{1}{k!} (\lambda/2\sigma^2)^k$
Rician	$Q(\sqrt{2K\overline{\gamma}}/K + 1 + \overline{\gamma}, \sqrt{\lambda(1+K)/K + 1 + \overline{\gamma}})$
Nakagami- $m$	$1 - (2m/2m + \overline{\gamma}) \{ \overline{\gamma} (\lambda/2\sigma^2, N/2) / \Gamma(N/2) \}$

where  $K$  is Rician factor,  $\overline{\gamma} = h^2 \mu / \sigma^2$  and  $h$  is the channel amplitude gain, which is supposed to be constant during  $N$  observed samples.

speaking, in case of discrete received signal the compressed sensing problem could be formulated as [17] [21] [35] [37]:

$$y = \Phi x = \Phi \Psi s = A s \quad (19)$$

where  $y$  is an  $m \times 1$  measurement vector,  $\Phi$  is an  $m \times N$  measurement or sensing matrix,  $\Psi$  is an  $N \times N$  sparsity basis matrix, and  $s$  is an  $N \times 1$  weighting vector. The measurement matrix design is an important issue in CS problem. We seek for stable  $\Phi$  to reduce the dimension of  $x \in \mathbb{R}^N$  to  $y \in \mathbb{R}^m$  without losing signal information. Also,  $\Phi$  requires to be non-adaptive and could not be sparse representation to the columns of  $\Psi$  and vice versa. It should be noted that since  $m \ll N$ , there are infinitely many solutions that satisfy (19) and among all of them we seek for the sparsest solution only. Several reconstruction algorithms have been existed in literature to obtain the optimum solution. Example of these algorithms are Basis Pursuit (BP), Matching Pursuit (MP), Orthogonal Matching Pursuit (OMP), Gradient Pursuit, Stage-wise Orthogonal Matching Pursuit (StOMP), Regularized Orthogonal Matching Pursuit (ROMP), Compressive Sampling Matching Pursuit (CoSaMP), ..., etc., (see [18] [37] and the references therein). The number of measurements required for the signal reconstruction is in the order of  $O(m \log N)$ . On the other hand, if the signals are analog, then we use analog to information converter (AIC) to extract the information of the signal. Due to that the information rate of a sparse signal is less than its sampling rate. AIC be implemented in different ways, e.g., Wideband Modulation Converter (WMC), multi-coset sampling, multi-rate sampling, and random demodulator [23]. The advantages and disadvantages of the wideband sensing AICs are summarized in **Table 4**.

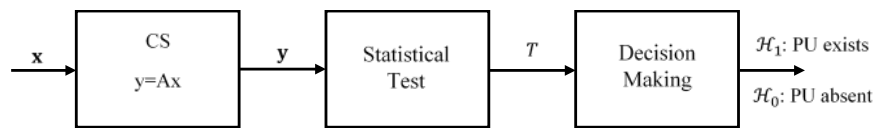
Current CS literature is generally centralized on signal reconstruction or approximation while in most communication processes and radars, signals are processed for making detection or classification decision [20] [21]. In detection, we do not ever reconstruct the signal rather than extract sufficient statistics from a small number of random projections or compressive measurements from the frequency band under observation as depicted in **Figure 5**.

In **Figure 5**, the PU transmitted signal is detected from the compressed measurements without any reconstruction stage. This generic structure outperforms the conventional structure of detection in two folds: 1) by reducing the computational complexity and 2) guaranteeing better performance with remarkably reduction in the number of measurements due to that the information of PU is preserved in the compressed measurements. Also, there is no obligation to precise reconstruction of  $s$  coefficients in (19). However, the detection decision is accurate because the compressed measurements still contain sufficient data for detection.

By considering the same detection problem as in Section (3.1), however, in compressive detection problem the observing  $x$  is changed to  $y$  as in (19) where  $\Phi \in \mathbb{R}^{m \times N}$ ,  $m \leq N$  and  $\Psi = I_{N \times N}$  is an identity matrix. Hence, the compressive spectrum detection could be formulated as [21]:

**Table 4.** Advantages and disadvantages of different wideband sensing AIC approaches.

AIC Method	advantages	disadvantages
Random Modulation	- signal sparsity is reconstructed using partial measurements	- affected by design imperfection
modulated wideband converter (MWC)	- robustness against the noise and model mismatches. - reduction in the measurement matrix dimensionality	- requires large number of low pass filters
Multi-coset sampler	- the sampling rate in each channel is lower than the Nyquist rate - less measurements	- the channel synchronization should be met to satisfy a specific sampling pattern for robust spectral reconstruction - requires many sampling channels
Multi-rate Sampling	- Low sampling rate - Less sampling channels	- stringent implementation



**Figure 5.** Compressed measurements-based spectrum sensing framework.

$$y = \begin{cases} \Phi n & \mathcal{H}_0 \\ \Phi(x + n) & \mathcal{H}_1 \end{cases} \quad (20)$$

where  $x \in \mathbb{R}^{N \times 1}$  is the PU transmitted signal and  $n \sim \mathcal{N}(0, \sigma^2 I_N)$  is independent and identical distribution (i.i.d.) Gaussian noise. The optimal decision of NP detector is:

$$T = \frac{f(y > \lambda | \mathcal{H}_1)}{f(y > \lambda | \mathcal{H}_0)} \underset{\mathcal{H}_0}{\underset{\mathcal{H}_1}{\geq}} \lambda \quad (21)$$

where  $\lambda$  is a threshold and the distribution of compressed measurements under  $\mathcal{H}_0$  and  $\mathcal{H}_1$  is:

$$f_x(x) = \begin{cases} \frac{1}{\sqrt{\sigma^2 \Phi \Phi^T} (2\pi)^N} \exp(-yy^T / 2\sigma^2 \Phi \Phi^T) & \mathcal{H}_0 \\ \frac{1}{\sqrt{\sigma^2 \Phi \Phi^T} (2\pi)^N} \exp(-(y - \Phi x)^T (y - \Phi x) / 2\sigma^2 \Phi \Phi^T) & \mathcal{H}_1 \end{cases} \quad (22)$$

Using the same method described in section (3.1) to obtain  $P_F$  and  $P_D$  (see (7a) and (7b)) we get:

$$P_F = P(y > \lambda | \mathcal{H}_0) = Q \left( -\frac{\lambda}{\sqrt{x^T \Phi^T (\Phi \Phi^T)^{-1} \Phi x}} \right) \quad (23a)$$

$$P_D = \mathbb{P}(y > \lambda | \mathcal{H}_1) = \mathcal{Q} \left( \frac{\lambda - x^T \Phi^T (\Phi \Phi^T)^{-1} \Phi x}{\sqrt{x^T \Phi^T (\Phi \Phi^T)^{-1} \Phi x}} \right) \quad (23b)$$

From (23a), the threshold  $\lambda$  required to maximize the  $P_D$  for specific  $P_F$  could be computed as:

$$\lambda = \sqrt{x^T \Phi^T (\Phi \Phi^T)^{-1} \Phi x} \mathcal{Q}^{-1}(P_F) \quad (24)$$

Let  $\Phi \Phi^T = I_m$  and substituting (24) for (23-b), we get  $\max_{(P_F, \lambda)} P_D$ :

$$\begin{aligned} P_D &= \mathcal{Q} \left( \mathcal{Q}^{-1}(P_F) - \sqrt{x^T \Phi^T (\Phi \Phi^T)^{-1} \Phi x} / \sqrt{\sigma^2} \right) \\ &= \mathcal{Q} \left( \mathcal{Q}^{-1}(P_F) - \sqrt{\|\Phi x\|_2} / \sqrt{\sigma^2} \right) \end{aligned} \quad (25)$$

As  $m \ll N$ , the matrix  $\Phi$  is ill-conditioned. For a small controllable perturbation error  $\epsilon > 0$ ,  $\Phi$  is enabled to preserve Euclidean length between  $x$ -sparse vectors. Therefore,  $\Phi$  needs to satisfy the restricted isometry property (RIP) [37]. Thus,

$$(1 - \epsilon) \sqrt{m/N} \|x\|_2 \leq \|\Phi x\|_2 \leq (1 + \epsilon) \sqrt{m/N} \|x\|_2 \quad (26)$$

Hence, the compressed detector performance is bounded by

$$P_D \approx \mathcal{Q} \left( \mathcal{Q}^{-1}(P_F) - \sqrt{m/N} \sqrt{\gamma} \right) \quad (27)$$

Next, approaches adopted so far for identifying the spectrum holes under wideband spectrum using CS will be discussed in the following sections.

## 4. Compressed Wideband CR Spectrum Detection Approaches

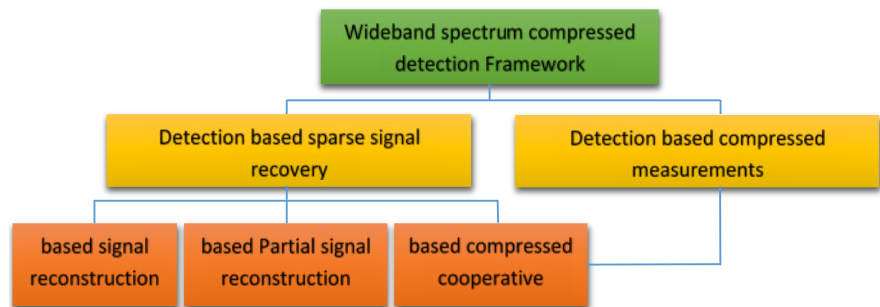
For years, detecting PU transmitted signal has been acquired from compressed measurements by signal reconstruction [21]. In this section compressed wideband detection approaches will be highlighted. Our discussion will be categorized into two main categories;

1) Sparse recovery for detection which could be divided into three subcategories: detection-based signal reconstruction, detection based partial signal reconstruction and detection based compressive cooperative.

2) detection based compressed measurements where compressed measurements are used directly to decide whether the PU transmitted signal exists or not on the observed frequency band of interest. Further, compressed detection based cooperative approaches could also comprise the second category as depicted in **Figure 6**.

### 4.1. Detection Based Sparse Signal Recovery

Prior to the CS introduction as a solution to alleviate sampling rates bottleneck which represents a main challenge for wideband spectrum sensing, it was performed using band-by-band approach. In this case each band has been modulated



**Figure 6.** Compressed wideband spectrum detection framework.

to know the locations of these vacant frequency bands. However, this approach of sensing introduces substantially large amount of latency. Later, to beat the latency, filter banks approach was used where each filter is dedicated to scan a specific frequency band. Although filter banks helped on relaxing latency issue introduced by band-by-band method, the number of band pass filters required to achieve sensing purpose was the major problem. On the other hand, wavelet detectors may be suitable solution to overcome filter banks drawbacks, but it may end up with power issue. The nature of the wideband spectrum exhibits sparse structure in open wireless environment in the sense that the number of operative frequency bands is less in comparison to the total number of frequency bands offered by vendors. The sparsity property eases applying CS.

#### 4.1.1. Detection Based on Signal Reconstruction

Detection of spectrum holes based on the signal reconstruction has been adopted as the major method used for years before a decision took place. In this method, CR required to build up the spectrum occupancy or PUs transmitted signals using one of the signal reconstruction algorithms [18]. To the best of our knowledge, Tian and Giannakis [38] were the first who built a model to acquire measurement vector depending on Power Spectral Density (PSD) level. It was estimated from the autocorrelation sample sequence, such as high, medium or low to identify spectrum hole. They exploited coarse sensing on analog wideband signal after converted to discrete time sequence using Nyquist sampling rate and then applied CS on the discrete sequence under noiseless channel. PSD gained a great attention where the estimate is acquired from the diagonal matrix of weakly or Wide Sense Stationary (WSS) signals representation in the covariance matrix [39] [40]. Hence, spectrum holes could be identified from the estimated autocorrelation coefficients. Then wavelet edge detector took place to identify edges between adjacent bands to locate the empty bands. In [39] Polo et. al. skip the converting analog to discrete stage and suggested obtaining autocorrelation samples directly from the analog wideband signal unlike [38]. The results in [38] [39] were based on single CR. In [41], a more generalized approach has been introduced based on the autocorrelation vectors obtained from multiple CRs distributing among a network used joint sparse model. However, due to the fact that compressive measurements matrices are not all truly stationary,



the authors of [42] assumed non WSS signal. The developed algorithm depended on slow channel variation due to small change between two close time instants of the spectrum. In addition, it is focused on monitoring these quite small changing in borders of bands where sparsity found. In this case, the autocorrelation coefficients were estimated from several compressed measurements using random demodulator, however it requires memory support. Eventually, Fourier transform took place to estimate the PSDs.

Other researchers suggested to use the available known information of VHF/UHF TV channels for spectral shape estimate [43] [44] [45], e.g., modulation parameters, channelization, and PU activity. IEEE802.22 WRAN suggested TV channels for CR systems [9]. Bayesian based iterative scheme to estimate a set of occupied bands and their spectral powers simultaneously in wideband spectrum from compressed measurements was proposed in [45]. As well in [43], detecting PU activity was suggested because compressed autocorrelation function computation is conserved. So, PSD of the compressed received signal wouldn't be accessed likens the preset spectral profile. In [45], the sensing task could be further enhanced based on geodesic distance, minimum eigenvalue and by conserving positive semi definite symbol. For this purpose, the difference between the reference spectral profile and auto-correlation matrices sample without reconstructing the signal completely has been used. Even in case of low leakage interference come from other SU, it may increase both the detection performance as in [44]. The signal support is obtained from the over complete dictionary to attain sparsity to elude band by band sweeping. Of course, spectral shape-based detection reduced the sensing time and mitigate interference, *i.e.*, TV bands are sparse in frequency domain. Nonetheless, it requires prior knowledge of interesting signal for comparison with the estimated signal. In other words, memory issue is posed. Cyclostationary feature detection could relax the previous memory with some acceptable complexity as in [46]. Compressive Spectrum Estimator (CSE) was proposed that not only could reconstruct PSD of PU signals from sparse structure by reducing two-dimension cyclic feature. However, it could also have extended to the non-sparse signal to estimate occupancy across a wideband spectrum in CR network. CSE showed robustness against noise uncertainty and interference. Further, the linear relationship between the preferable cyclic feature and the compressed measurements of time-varying cross-correlations used to reconstruct power spectrum was derived in [47]. It rendered the problem to be formulated as convex  $\ell_1$  minimization solution. Despite these advantages of CSE, the adaptive block size affects the sensing time performance causing the loss of available spectral opportunities.

Undoubtedly, better PSD estimate was achieved exploiting Kronecker matrix as sparsifying basis compared to traditional approaches through substantially reduction of Mean Square Error (MSE) [48]. On the other hand, if sparsity constraints on the power spectrum were relaxed using Least Squares (LSs) and after some rank conditions were hold, blind sampling of power spectrum was pro-

posed in [49]. Estimating PSDs of the wideband spectrum could be obtained using the cross-correlations between the different outcomes of the sampler. As an extension to the work presented in [46], minimum sparse ruler based multi-coset sampling was used to acquire compressed measurements [49]. It guaranteed unique LS solution when full column rank is met with at least single digit at each column of autocorrelation matrix. In early cooperative approach, Simultaneous Orthogonal Matching Pursuit (SOMP) took place for signal recovery at fusion center [38] [39] [50]. When there is no constraints on sparsity, Simultaneous Sparsity Adaptive Matching Pursuit (SSAMP) was used [51] for more stable reconstruction. When deterioration of spectrum recovery is occurred due to mismatching in standard  $\ell_2/\ell_1$  optimization, the problem could be rendered by dividing the spectrum into blocks with different lengths [52]. Occupancy could be reconstructed using, mixed  $\ell_2/\ell_1$  denoising operator. First,  $\ell_1$  minimization to each cluster of PSD vector was performed; then  $\ell_2$  minimization was applied to the aggregation of the minimized cluster by  $\ell_2$  norm. While Distributed Online Scheme (DOS) uses online distributed least-absolute shrinkage, selection operator Least Absolute Shrinkage and Selection Operator (D-LASSO) algorithm to track the variations on PU's PSDs in ad hoc networks [53]. If the measurements contain complex value, DFT was used as sparsifying basis, Tree based Orthogonal Matching Pursuit (TOMP) would be expedient algorithm [38]. Early mentioned sensing performance is characterized by receiver operating characteristics (ROC) as described in section 3.1 which is a tradeoff between false and detection probabilities, modified blindly optimized compressive sensing (MBOCS) was used in [54]. It used optimized sparsifying basis rather than being fixed. It showed better reconstruction accuracy when compared to simultaneous orthogonal matching pursuit (SOMP) under the same number of measurements. Moreover, when low complexity computations with fast convergence are posed, an iterative technique known as weighted orthogonal matching pursuit (WOMP) [44] was introduced due to its ability rebuild sparsity representation very fast. Following that the optimal sparsifying basis attained by Karhunen-Loève Transform (KLT) [55] yielded to accuracy, reduction in number of measurements and estimation improvement as [54] due to its coefficients redundancy avoidance in signal representation. Also, it could detect the signal correlation structure and directly update the basis from compressed received signal. In [56], the sensing performance using partially knowledge of the spectrum occupancy has been proposed. The partially knowledge could be provided by remote database, collaborative sensors or any auxiliary entity may deliver that kind of data where only subbands location were known but not their varying PSDs. When signal support is not available, the performance of weighted  $\ell_1$  minimization is comparable to Modified Basis Pursuit Denoising (MBPDN). It assigned small weights to occupied bands while assigned large weights to the unoccupied bands. It showed some robustness to error when partially knowledge is available.

It follows that reconstruction algorithm requires having termination point to reduce the sensing period to maximize data throughput when certain satisfaction criterion is met using  $\ell_1$ -norm validation to stop data acquisition as in [56]. Besides, detection performance and computational complexity will be more enhanced and reduced respectively as stated in [57]. The algorithm searches for highly sparse frequency band in the spectrum without a prior knowledge of PU signal by dividing the received wideband spectrum to several frequency band via several bandpass filters followed by AIC. Energy vector is acquired by comparing and averaging these measurements to estimate spectrum occupancy. Also, the choice of sensing matrices, e.g., Discrete Walsh-Hadamard Transform (WHT) and DCT, introduced even far improvement to sensing performance as in [58]. WHT is a matrix that its columns and rows are orthogonal to each other and comprise  $\pm 1$  values only. Decision of compressed detector is thresholding based on centralized chi squared distribution which requires less processing time to unveil spectrum holes. Moreover, WHT and DCT have comparable probability of detection. DCT uses real arithmetic computation due to its lack of complex coefficients and concentrates the energy on the first low order coefficients which make it sparser [59].

The benefits of sparsity led to Sub-Nyquist sampling rate, spectral leakage of spatial, frequency reduction and enhance detection sensitivity. In [53] an arbitrary spatial map of estimating PSDs to identify SHs, but with the expense of battery exhausting of CR device due to the online monitoring has been created. According to the domain where the signal is sparse, [60] proposed an estimator to just acquire the PSD without reconstruction via solving least squares problem rather than  $\ell_1$  norm that is used for signal reconstruction. Also by feeding the wideband received signal to a number of filters less in number than the available spectrum bands will reduce the latency caused via band-by-band sensing approach [38] [61]. However exploiting the number of BPF in [61] adds another problem which is how many filters are required. According to the simulation results of [62], it is stated that this number is 12, which may not be valid in all circumstances.

Although beamforming was not included within IEEE802.22 WRAN framework according to antenna structures [9], CS Minimum Variance Distortion less Response (CSMVDR) algorithm introduced significant reduction in computation complexity at created the reconstruction process. Due to varying the threshold, the probability of detection was improved [62]. Furthermore, spatial beamforming is not only avoiding unnecessary interference to PUs to provide higher throughput for SUs, but it also alleviates the main challenge in traditional methods in estimating the angle of arrival of the signal received by many transmitters. Steven and Sachin [63] designed MAC Spatial Compressed Detection (MAC-SCD) that exploits a spatial degree of freedom as low power wideband detector to estimate spectrum holes efficiently and accurately in CR networks. Also, they used CS to recover angles of arrival by fewer spatial samples obtained

from less number of antennas than required by traditional one. Ones should keep in mind when the signal of interest of wideband issue is sampled at non-uniform sub-Nyquist sampling rate, approaches such as energy and cyclostationary is no longer valid. So, Aziz *et al.* [64] formulated the problem of spectrum sensing as missing data problem and proposed Lomb-Scargle method for PSD estimate.

We summarize our discussion in **Table 5** to show the advantages and disadvantages of each method.

**Table 5.** Detection based signal reconstruction methods dis/advantages.

Article Ref.	Advantages	Disadvantages
[44]	<ul style="list-style-type: none"> <li>- elude band by band sweeping</li> <li>- reduced the sensing time</li> <li>- mitigate interference</li> </ul>	<ul style="list-style-type: none"> <li>- Computational complexity</li> <li>- requires prior knowledge</li> <li>- memory issue</li> </ul>
[46]	<ul style="list-style-type: none"> <li>- reducing two-dimension cyclic feature</li> <li>- can be applied on sparse and non-sparse signals.</li> <li>- robustness against noise uncertainty and interference</li> </ul>	<ul style="list-style-type: none"> <li>- Computational complexity</li> <li>- block size affects the sensing time performance</li> </ul>
[38] [39] [50]	<ul style="list-style-type: none"> <li>- fast algorithm</li> <li>- easy to implement</li> </ul>	<ul style="list-style-type: none"> <li>- requires more measurements to have perfect reconstruction</li> <li>- lacks provable quality of reconstruction</li> </ul>
[50]	<ul style="list-style-type: none"> <li>- sparsity constraints due to the use of overcomplete method</li> </ul>	<ul style="list-style-type: none"> <li>- the unique solution is conditioned by complex Gaussian matrix <math>M^2 \geq (2N - 1)^1</math> and the selection of M is subject to the auto- and cross correlation between the row of sensing matrix used</li> </ul>
[51]	<ul style="list-style-type: none"> <li>- prior sparse level is not required</li> <li>- sequential measurements are used to fasten detection process</li> </ul>	<ul style="list-style-type: none"> <li>- quality of reconstruction performance is subject to complexity</li> </ul>
[52]	<ul style="list-style-type: none"> <li>- using mixed l1/l2 norm denoising operator with LASSO algorithm increase detection performance.</li> </ul>	<ul style="list-style-type: none"> <li>- require information of the spectrum boundaries between different PU as a priori information</li> </ul>
[53]	<ul style="list-style-type: none"> <li>- localized transmitting CRs</li> <li>- sensitive to changes in the system</li> <li>- single hop with low overhead message between neighbors</li> </ul>	<ul style="list-style-type: none"> <li>- battery issue</li> </ul>
[56]	<ul style="list-style-type: none"> <li>- Robust to error</li> <li>- comparable detection performance compare to MBPDN algorithm</li> </ul>	<ul style="list-style-type: none"> <li>- Priori knowledge of subband locations</li> </ul>
[57]	<ul style="list-style-type: none"> <li>- complexity reduction</li> <li>- improved detection accuracy</li> </ul>	<ul style="list-style-type: none"> <li>- Priori knowledge is required</li> </ul>

<sup>1</sup>M and N denote the number rows and columns in the measurement matrix respectively.

#### 4.1.2. Detection Based Partial Signal Reconstruction

Before continuing our discussion of partial signal reconstruction, we need to define what does partial reconstruction-based detection approach mean? It could be defined as an intermediate stage prior to what we discussed in Section 4.1.1 where final decision is declared after signal reconstruction nor does exploiting fewer measurements directly as highlighted in Section 3.2. For this reason, we call it partially signal reconstruction. A multi-resolution Bayesian CS algorithm (BCS) [65] tunes with this concept with fine computational complexity and sensing time reduction whereas the CS problem is inversely solved by Bayesian coefficients obtained by Relevance Vector Machine (VRM). The accuracy of recovered signal is obtained from error bars which it is an indicator for up/down sampling rate according to the varying nature of spectrum occupancy [66]. The error bars strategy is even included in [67].

#### 4.2. Detection Based Compressed Measurements

Notwithstanding spectrum occupancy estimate from power spectrum provides more insight on spectrum holes' distribution or spectrum grid. However, in many cases of signal detection problem, we are interested in determining whether the presence of PU transmitted signal on the frequency band under observation or not without ever reconstructing the signal. Thus, compressed measurements-based spectrum detection approach improves sensing into three folds; 1) minimum sensing duration depending on sensing matrices used, 2) increasing CR throughput, and 3) reducing computation cost and implementation complexity. In other words, traditional CS is applied on the received signal firstly. Then reconstruction process took place and after that a decision about existence PU is made as in [38] [53]. In fact, this outcome is obtained with high computation cost and complexity even though the reduction of sampling rate is below the Nyquist rate. The difference between detection based compressed measurements and -signal reconstruction is summarized in **Table 6**.

Compressed detection or compressed measurements-based detection aims to detect PU transmitted signal without ever reconstruction. Therefore, in [22] direct detection from CS measurements rather than went through reconstruction stage has been proposed. This will increase the CR user data transmission due to the fast detection and the improvement of the sensing time. In [76] the number of measurements required for compressed detection has been specified. Also, as

**Table 6.** Comparison between two methods of detection based compressed measurements and signal reconstruction.

detection based compressed measurements	detection based signal reconstruction
<ul style="list-style-type: none"> <li>- minimum sensing duration depending on sensing matrices used,</li> <li>- increasing CR throughput, and</li> <li>- reducing computation cost and implementation complexity</li> </ul>	<ul style="list-style-type: none"> <li>- high computation cost and</li> <li>- high system complexity</li> <li>- sensing decision is subject to the used reconstruction algorithms speed</li> </ul>

mentioned earlier, the wireless communication environment exhibits sparsity property by nature. Hence the spectral opportunities that viewed as Markov hidden chain motivated [68] to propose optimum myopic strategy to improve detection performance. The authors of [68] combined CS with online learning of multi-branches bandit, where these branches denote the rows and columns of the sensing matrix, to create an adaptive compressive sensing known as optimum myopic strategy. Nevertheless, the approach of [68] was derived according to an idealistic state of noise free channel with signal support is guaranteed definitely.

Moreover, the optimality of spectrum occupancy decision needs to take into consideration all the previous states of channels occupancy both observation and decision. However, it requires long term of channels activities history database on how it is being taken and left. To enhance the detection performance, in [69] the Algebraic Detection (AD) for compressed wideband spectrum sensing was designed. It exploited the distribution discontinuities method to save the linearity of PU sparse signal by using measurement matrix consists of Dirac function. On the other hand, by relaxing the convex optimization problem used to solve the underdetermined system in estimate compressed wideband spectrum, [70] suggested to design a Nonlinear Least Square Estimator (NLLSE). It is used to unconcealed the SHs directly from few collected samples of multiband signal either correlated or not in time domain. Furthermore, since the noise does not bear any sparsifying basis unlike interference, it has sparsifying basis in some dictionaries. The sparse coefficients positions were used as detector in [67] by measuring the coherence between the sparsity coefficients position on received signal that is free or even corrupted with noise.

### 4.3. Detection Based Compressive Cooperative Schemes

Cooperative compressed spectrum sensing approaches either centralized or distributive CR networks will have a great effect in alleviating challenges related to high sampling rate requirements for wideband spectrum, computation, complexity and power constraint reduction. In addition, noise, and interference issues such as fading, shadowing, hidden node/terminal problem, and network overheads transmission might devastatingly deteriorate spectrum detection performance.

Proceeding in Section 4.2 it has been mentioned that spectrum sensing could be achieved cooperatively either in centralized or distributed scheme. In centralized scheme, the FC collects the sensory information from CR users to determine the decision about spectrum occupancy and broadcasts that result among CR users to be shared opportunistically. While in distributed scheme, a global decision on SHs converge iteratively and jointly through communications performed among CRs. Fading is one of the major challenges in any wireless network which aggravates the detection performance. To tackle this issue, multiple CR terminals cooperate with each other during sensing assignment performing

spectral estimate locally using data fusion to bring global convergence without the aid of FC to proliferate decision accuracy. Node failure avoidance is obtained by distributive implementation where each SU communicates with its neighboring SUs only using single hop to consume less power throughout sensing mechanism [71]. Another serious issue is the hidden node problem, when an obstacle blocking the CR user from seeing PU signal which lead unintentionally to raise the interference. Albeit the combination of CS with an algebraic detector at FC will provide better detection performance than that obtained with energy detector [69]. However, processing each income from each CR node separately may cause latency unlike if all collected compressed measurements process together at once. A general case of multiple CR users in centralized scheme, rather than the one in [38], is performed in [41] using joint sparsity matrix to improve decision making.

The centralized detection performance [72] could be further improved by formulating spectrum detection problem as support recovery. This improvement could be achieved using several fusion schemes such as decision, quantized data and data fusion in FC with no a prior knowledge of channel state condition and noise statistics. Applying this to the sequential spectrum sensing leads to an increase in the detection sensitivity against noise uncertainty at low SNR using minimum number of measurements. Because sequential detection task is distributed among CR users in the network. On the other hand, Farrag *et al.* [54] proposed algorithm based on distributed detection matrix targets to increase detection performance of PU with low sensing node complexity among clusters of sensing nodes rather than individuals. In contrast to [72] where each node sends its measurements as a sign vector only to FC. In [54], each sensing node uses its own local detection matrix to sense the wideband spectrum. decision report is generated and sent to FC. In this case, global spectrum occupancy was determined from the aggregation of all received measured signals.

Moreover, sensing node hardware complexity was avoided using upper bound value while maintaining acceptable SNR. Alternatively, in [73] the authors stated that detection could even far improve if sensing problem merged with localization using a grid of available spectrum holes to be easily accessed by SU was created. In other words, the joint framework proposed in [73] might be imagined as a 3D map or grid. It shows the PSD of occupied bands at PU location in the network from sparse observation. For more clarity, just imagine  $(x, y, z)$  space where the  $xy$ -plane is used for PU positioning and  $z$ -axis represents the occupied bands. The PSD is considered as point in that space or 3D map. As the location of the PU is known to the CR users, the effect hidden node problem could be minimized. However, it requires large storage space to record data or having frequently updated external database. Also, power consumption issue is presented by CR users due to the continuous update. The modified Kronecker joint sparsity matrix in cooperative approach was recommended in [48]. It reduces the mean square error of estimated PSD and maintains the lower sampling



rate under low compression ratio. In [54] an optimized sparsifying basis has been adopted. The beauty of modified Kronecker comes from the  $n$ -th dimension, it has. It could represent frequency, time, space and more. This is due to the fact that each dimension exhibits different sparsity representation basis rather than Distributed CS (DCS) which is one-dimension joint sparse matrix. Similarly in [74], Bayesian compressed sensing is used jointly with localization to enhance reconstruction. The PSD map could be estimated using online iterative process. It also yields uncovering the other nearby CR user to minimize the interference caused by CR users themselves [53].

In [40], Ariananda and Leus considered PSD estimation by multiple CR sensors through collected measurements deliver to FC along with Channel State Information (CSI) and sensing matrices to make the final decision about spectrum occupancy. This is for sake of lowering sampling rate per CR sensors cooperated in centralized network in estimate the spectrum state search for spectral opportunities. Later on Romero *et al.* [75] exploited the model of [40] however, without CSI and instead using cross spectra between the different collected data. In this case, the sensors are grouped, and each group uses different CS structure and sweep different portion of the spectrum; where PSD estimated the correlation of collected measurements. Further reduction on overhead transmission and reports are sent from CR sensor to FC using matrix completion and joint sparsity recovery [76]. Unfortunately, each sensor is equipped with frequency selective filter to sense multiple bands collecting sensory data will encounter latency due to band by band detection. On the other hand, reduction of hardware complexity and sensing time in [77] is a new direction worth-noting. Instead of assigning each CR device with  $m$  channels to observe, the  $m$  channels will be divided among  $m$  CR devices. So each CR device is assigned with only one channel unlike traditional MWC [75]. In this case,  $m$  channels in each CR device are adopted to sweep the whole spectrum search for SHs which reduced the sampling rate to be the same as bandwidth of one subband.

## 5. Open Research Problems

### 5.1. Compressed Detection of PU Signal at Wideband CR Networks over Fading Channels

Although the magnificent advantages being gained by introducing CS as a key success for relaxing sampling rate burden at wideband CR networks, the dominant use of CS has been focused almost on signal reconstruction. In literature, there have been several handy reconstruction algorithms used, e.g., basis pursue, greedy algorithm (matching pursue and its extensions),..., etc. In solving detection or sensing problem, we are pivotal interested on deciding if the PU transmitted signal on a frequency band under observation exists or not without ever reconstructing. To the best of our knowledge, this topic of compressed detection or sensing problem isn't attained enough attention yet. Despite the very few published articles to cover this important topic, they ended discussing recon-

structuring the spectrum grid by building the PSD of spectrum occupancy to uncover hidden spectral opportunities location. However, some of these frequency bands may highly likely be occupied by the time for final decision. CS is useful for wide range of statistical inference tasks. Recent published papers dealt compressed detection over AWGN channel, and to the best of our knowledge the topic over several types of fading channels hasn't investigated yet.

### 5.2. Minimizing Sensing Time via Wideband Compressed Cooperation CR Networks

The collaborative approaches have their enormous impact in minimizing sampling rate requirements and computational and hardware complexities per single CR node and/or to overall CR network performance. In collaborative approach CR users, nodes or sensors agree to form a coalition to detect the hidden WS at wideband spectrum. This coalition could be categorized as centralized or distributed. In distributed mode, spectrum occupancy status on the observed frequency band is converged via a global decision. On the other hand, the final decision on spectrum occupancy status is made by FC as in centralized mode. Furthermore, in both modes, individually or cooperatively, CR users involve sweeping wide range of frequencies sensing WS. The process is performed by dividing the wideband spectrum into  $N$  non-overlapping subbands. The process is time consuming which may end to lose the opportunity to use free frequency bands by CR users. However, it is worth noting that the wideband spectrum is sparse in open environment and stressing on this point sensing scenario could be achieved as:

- The spectrum may be divided into sparsity blocks. Let's say  $J$  and each CR users or CR group work together by adopting their reception parameters to specific block of interest. And they deliver their sensing compressed measurements to FC for final decision to build the spectrum occupancy.
- Or two different groups of CR may sense the same block to minimize the errors due to noise and interference and send their compressed measurements to FC.

The highlighted point requires an investigation on how this technique will minimizing sensing time and complexity per node and reduce miss detection and false alarm and maximize throughput and detection performance.

### 5.3. Optimal Compressed Decision for CR Networks

As already stated before, spectrum sensing involves the detection of the hidden unused spectral opportunities. However, specifying an optimal threshold for detection is one of the most noteworthy contests. Under optimal requirements, it is required to maintain the probability of false alarm as low as possible and the probability of detection as high as possible as regulated by IEEE802.22 WRAN work group. Minimum false alarm probability improves the utilization of the spectrum, while maximum detection probability reduces the chances of interfe-

rence due to miss-detection. Miss-detection probability reduction bids protection to the PU against potential CR transmissions. These two metrics is also known as ROC curves where the statistics decision must be adjusted adaptively to satisfy these two opposing requirements for various channel conditions. In most of the published research works, statistics decision is formulated to follow the central chi square distribution with  $L$  degrees of freedom under hypothesis  $\mathcal{H}_0$  and noncentral chi square distribution under hypothesis  $\mathcal{H}_1$  with  $\mu$  is the non-centrality parameter. Since for large number of observation the chi-square distribution could be approximated as Gaussian distribution, the Neyman-Pearson rule may take place to find the optimal threshold for specific false alarm probabilities. A compressed sensing form of threshold is already derived as in [21] at form of Gaussian distribution. An effort to derive an optimal traditional energy detection has been investigated in [78]. However, analytical closed form has not been reached yet under chi-square distribution and more research are required to obtain the closed form optimal threshold for detection compressively as well traditionally. Any breakthrough in this research point will happen in minimizing the sensing time required for detection.

## 6. Conclusion

Spectrum sensing is a mandatory feature to enable self-coexistence for CR in IEEE 802.22 WRAN standardization. Sensing becomes very challenging at wideband CR network due to the cost of signal processing requirements, *i.e.*, Nyquist sampling rates. Thanks to compressive sensing technique that eases the sampling rates crisis by proving sub-Nyquist sampling and reconstructing the signal using few random projections. It also reduces cost computations, hardware complexity and energy issue. Further, it improved sensing performance over AWGN and fading channels. In this paper compressed spectrum sensing problem at wideband CR networks is addressed and the recent research achievements in solving the problem are discussed.

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