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An effective technique to the design of dual band coaxial waveguide feed horn for reflector antenna excitation has been developed and tested. Matching the feed horn with free space in upper frequency range is achieved by dielectric insert disposed inside the circular waveguide fabricated in interior conductor of open ended coaxial waveguide. The presence of dielectric insert causes the high level of reflection in lower frequency range from the boundary between the open end of coaxial waveguide and free space. A novel technical solution of highly complicated matching problem has been proposed. The matching design was performed as the series of finite thickness diaphragm arranged on outer surface of interior conductor of coaxial waveguide. To obtain the rigorous solution of given electromagnetic problem for the complicated structure consisting of a great number of connected coaxial waveguide sections and dielectric insert, a new effective approach has been proposed. The distinctive feature of this approach consists in combination of rigorous procedure based on computation of generalized scattering matrices and another rigorous procedure based on the finite difference time domain method. To combine these essentially different approaches, the metallic discontinuity equivalent to overall investigated structure was employed. An optimization of equivalent matching structure has been performed by the variant of the evolution strategy method. The obtained design with metallic discontinuities was converted into real structure comprising the dielectric insert. The final calculations of real structure were performed by the finite difference time domain method. The evolutions of real structure were performed by the finite difference time domain method. The effectiveness of proposed technique was confirmed by the results of testing the developed feed horn in reflector antenna system.

### Introduction

In telecommunication wireless networks, the dual frequency microwave reflector antenna systems are widely used for transmitting and receiving of information in two frequency bands [1, 2]. For aims of dual frequency applications the reflector antennas are supplied by corresponding feeds [3, 4]. A configuration of dual frequency source of electromagnetic wave excitation is determined by appropriate designing a feed. Mostly, these microwave feeds are arranged on combination of metallic coaxial and circular waveguides which have considerably lower losses in comparison with other structures.

The coaxial waveguide is used in lower frequency band whereas the circular one is utilized for operation in the high frequency range. The simplest arrangement of the dual frequency feeds by using of smooth coaxial and circular waveguides does not allow archiving appropriate characteristics of all reflector antenna system. To obtain high-quality characteristics of such antenna structure, the coaxial and circular channels of the combined horn must be carefully matched.

In this work, the original device matching the combined horn with the feeding coaxial waveguide is proposed and extensively investigated. To improve the radiation characteristics of this horn, in output of circular waveguide, the dielectric insert from polystyrene is installed. The dielectric insert diameter was optimized to obtain necessary radiation characteristics in the upper frequency range. Numerical calculations show that the dielectric insert diameter is a few less then the diameter of circular waveguide which at the same time presents the inner conductor of coaxial waveguide used for excitation of the combined horn in lower frequency range. For mounting the matching dielectric insert inside the circular waveguide, the coaxial cylindrical bushing with significantly lower permittivity value is employed. Its outer surface is tight adjoined with the wall of circular waveguide whereas the inner surface holds the matching insert by small tension. As a result, we have the dual band device (Fig. 1) for excitation of corrugated circular horn.

#### Statement and solution of the problem

The purpose of this paper is to develop the effective technique for designing the complicated waveguide structure presented in Fig. 1. The originality of developed approach consists in replacement of combined coaxial waveguide connection with dielectric insert by equivalent metallic structure allowing the application of effective methods for its calculation. Based on constructive implementation of excitation unit, it can be noted that the development of matching the given structure setup is the most complicated problem since open ended circular waveguide together with symmetrically arranged dielectric insert has significant level of reflection.

The difficulty of technical solution choosing the named matching unit consists in impossibility of its designing based on known theories of stepped transformers and band-pass filters in wide frequency range. These theories can not be used for construction of matching device because they require using a great number of sequentially placed discontinuities. In this case, the first and the last discontinuities must have the small reflection coefficients that do not correspond to the values for real structure. The modulus of reflection coefficient from open ended coaxial waveguide with dielectric insert is significantly large, and besides, it increases with grow of frequency reaching a value 0.3.



Fig. 1. The schematic representation of initial configuration of matching unit.

To solve this problem important for operating the all antenna system, a several waveguide structures were considered. One of them, which most closely agree with the theory of longitudinally inhomogeneous waveguide structures, is a series connection of finite thickness diaphragms disposed on the inner conductor of coaxial waveguide. It was assumed that such technical solution allows obtaining the required level of matching. By using this connection as a transition from input coaxial waveguide of small transverse dimensions to output one of larger sizes, one can optimally employ such electromagnetic structure for decreasing the longitudinal dimensions of all coaxial feed horn. This connection in consecutive order is equivalent to physical model of longitudinally complicated waveguide structure. To obtain high-quality characteristics of such structure, all its elements must be carefully chosen based on a rigorous solution of corresponding electromagnetic problems [5, 6].

To achieve best results, physical parameters of such structures should be founded by using the optimization methods. The simple and relatively rapid optimization methods which require a small number of computations for finding the objective function lead as a rule to local minimum. In the local minimum of the objective function, it is impossible to achieve the broad band of operating frequencies. The outcome from this situation seems in the employment of optimization methods which essentially allow bypassing the problem of local minima.

Such optimization methods widely used for designing of filters, phase shifters, etc, are based on different modifications of approach called as the evolution strategy [7]. Application of the evolution strategy method guarantees finding a global minimum of the objective function. This evolution strategy method has been successfully applied to designing of a great number of complicated waveguide structures [6, 7].

As shown in [6-8], the computation of frequency characteristic of the parameter under optimization is carried out many times. The number of objective function calculations for complicated broadband devices can reach tens and even hundreds of thousands. Therefore, application of optimization methods based on the evolution strategy approach is associated with a lot of computer time even when high-performance computing facilities are used. Furthermore, the use of efficient optimization methods is usually followed by the calculation of objective function with high accuracy. Since expending the computer time at the expense of optimization itself can be considered as fixed, and they actually cannot be reduced, choosing the effective method of the objective function calculation that mostly reduces to designation of device frequency characteristics is of great importance.

Thus, when using optimization methods that ensure the guaranteed finding a global minimum of the objective function owing to a large number of iterations, it is desirable to have effective algorithms providing high calculation accuracy at low computing time expenses. These algorithms can be constructed on the basis of electromagnetic problems solution by the integral equations method. The examples of this approach application are substantially considered in [6]. We will apply some of these results in order to solve the problem stated above.

In considered matching device, there are three types of discontinuities such as diaphragm between two coaxial waveguides of different cross sections, symmetrical diaphragm in smooth coaxial waveguide and simple discontinuity in the form of coaxial waveguide junction. Solving the diffraction problem for the first and second discontinuities by using coupled integral equations can significantly reduce the computing time expenses as compared to the method based on combination of generalized scattering matrices of separate waveguide junctions. As applied to computing the objective function of overall matching device, the mathematical models obtained in [6] should be modified in accordance with the types of existing discontinuities.

Consider the mathematical model for diaphragm of finite thickness between two coaxial waveguides of different cross sections. Here, two scenarios should be investigated. First of them represents the case when coupling windows do not coincide with cross sections of coaxial waveguides under study. In this case, the internal boundary problem to find generalized scattering matrix of overall two junction waveguide structure without determining scattering matrices of separate discontinuities is reduced to following coupled systems of linear algebraic equations for the complex expansion coefficients  $C_{ul}^{(1)}$ ,  $C_{ul}^{(2)}$ 

$$\sum_{\mu} \sum_{l} C_{\mu l}^{(1)} [\sum_{\nu} \sum_{k} Y_{\nu k}^{(1)} \eta_{\nu k}^{(11\nu\nu)} \eta_{lk}^{(11\mu\nu)} + \\ + \sum_{\nu} \sum_{k} Y_{\nu k}^{(2)} \eta_{\nu k}^{(12\mu\nu)} \eta_{lk}^{(12\mu\nu)} \operatorname{coth} \gamma_{\nu k} t] - \\ - \sum_{\mu} \sum_{l} C_{\mu l}^{(2)} \sum_{\nu} \sum_{k} Y_{\nu k}^{(2)} \eta_{\nu k}^{(12\mu\nu)} \eta_{lk}^{(22\mu\nu)} / \operatorname{sinh} \gamma_{\nu k} t = \\ = 2\delta_{w1} Y_{pm}^{(1)} \eta_{\nu m}^{(11\mu p)};$$
  
$$u = 1, v = 1, 2, \dots, L_{t}^{(1)}; u = 2, v = 1, 2, \dots, L_{t}^{(1)};$$

$$w = 1, p = 1, m = 1, 2, ..., M_1; p = 2, m = 1, 2, ..., M_2;$$
(1)

$$-\sum_{\mu}\sum_{l}C_{\mu l}^{(1)}\sum_{\nu}\sum_{k}Y_{\nu k}^{(2)}\eta_{\nu k}^{(12u\nu)}\eta_{lk}^{(22\mu\nu)}/\sinh\gamma_{\nu k}t +$$

$$+\sum_{\mu}\sum_{l}C_{\mu l}^{(2)}[\sum_{\nu}\sum_{k}Y_{\nu k}^{(2)}\eta_{\nu k}^{(22u\nu)}\eta_{lk}^{(22\mu\nu)}\operatorname{coth}\gamma_{\nu k}t +$$

$$+\sum_{\nu}\sum_{k}Y_{\nu k}^{(3)}\eta_{\nu k}^{(23u\nu)}\eta_{lk}^{(23\mu\nu)}] =$$

$$= 2\delta_{w2}Y_{qn}^{(3)}\eta_{\nu n}^{(23uq)};$$

$$u = 1, v = 1, 2, ..., L_{1}^{(2)}; u = 2, v = 1, 2, ..., L_{2}^{(2)};$$

$$w = 2, \ q = 1, \ n = 1, 2, ..., N_1; \ q = 2, \ n = 1, 2, ..., N_2;$$
$$\eta_{lk}^{(ij\mu\nu)} = \int \Phi_{\mu l}^{(i)} \Psi_{\nu k}^{(j)} ds .$$
(2)

where  $m = 1, 2, ..., M_p$ ;  $n = 1, 2, ..., N_q$ ;  $k = 1, 2, ..., K_v^{(j)}$ ;  $l = 1, 2, ..., L_{\mu}^{(i)}$ ; i = 1, 2; j = 1, 2, 3;  $\Phi_{\mu l}^{(i)}$  denotes the *k* th orthonormalized vector coordinate function of transverse-electric ( $\mu = 1$ ) or transverse-magnetic ( $\mu = 2$ ) types in *i* th coupling window;  $\Psi_{vk}^{(j)}$ ,  $Y_{vk}^{(j)}$  represent the orthonormalized vector eigenfunction and its admittance of *k* th mode of transverse-electric ( $\nu = 1$ ) or transverse-magnetic ( $\nu = 2$ ) types in *j* th waveguide;  $\gamma_{vk}$  is propagation coefficient of *k* th mode of transverse-electric (v=1) or transverse-magnetic (v=2) types in coupling waveguide;  $\delta_{w1}$ ,  $\delta_{w2}$  are Kronecker symbols; w=1, if the diffraction problem is considered for case of the incidence  $M_p$  transverse-electric (p=1) or transverse-magnetic (p=2) electromagnetic waves from the left side; w=2, if  $N_q$  electromagnetic waves of transverse-electric (q=1) or transverse-magnetic (q=2) types are incident from the right side;  $K_v^{(j)}$  is the number of modes of transverse-electric (v=1) or transverse-magnetic (v=2) types which are taken into account in *l* th waveguide;  $L_{\mu}^{(i)}$  is the number of terms of transverse-electric (v=1) or transverse-magnetic (v=2) types of transverse electric field  $\mathbf{E}^{(i)}$  expansion which are taken into account in *i* th coupling window;  $s_i$  is the area of *i* th coupling window; *t* is the diaphragm thickness.

As follows from (1), the obtained system has the same matrix of coefficients at unknowns and different right parts corresponding to electromagnetic problem solution under the alternative incidence on the inhomogeneous structure of eigenmodes of both the left and right waveguides. Such configuration of the system (11) simplifies finding its numerical solution.

To solve the coupled systems of linear algebraic equations (1), we must previously determine the coefficients (2) performing the relation between parameters of mathematical model and objective coaxial waveguide structures. Substituting the expressions for vector eigenfunctions of coaxial waveguide into (2) and estimating the integrals by formulas [9], we obtain the following relations for coupling coefficients:

$$\eta_{lk}^{(ij\mu\nu)} = \sum_{\lambda} (-1)^{\lambda} P_{\lambda} ;$$

$$P_{\lambda} = \pi / 2V_{\mu l}^{(i)} V_{\nu k}^{(j)} \sigma_{\nu l}^{(i)} \sigma_{\nu k}^{(j)} / (\xi_{\mu l}^{2} - \xi_{\nu k}^{2}) \times$$

$$\alpha Z_{1}(\alpha) [Z_{0}(\beta) - \zeta Z_{2}(\beta)] - \beta Z_{1}(\beta) [Z_{0}(\alpha) - \zeta Z_{2}(\alpha)] ;$$

$$\xi_{\mu l} = \sigma_{\mu l}^{(i)} ; \xi_{\nu k} = \sigma_{\nu k}^{(j)} ; \alpha = \sigma_{\mu l}^{(i)} r_{k}^{(i)} ; \beta = \sigma_{\nu k}^{(j)} r_{k}^{(i)} ,$$

where  $\zeta = (-1)^{\mu+\nu}$ ;  $Z_{\tau}(\alpha) = J_{\tau}(\alpha) + D_{\mu}Y_{\tau}(\alpha)$ ;  $Z_{\tau}(\beta) = J_{\tau}(\beta) + D_{\nu}Y_{\tau}(\beta)$ ;  $D_{\mu}$ ,  $D_{\nu}$  denote the constants defined by boundary conditions on the walls of coaxial waveguide conductors;  $J_{\tau}(\alpha)$ ,  $J_{\tau}(\beta)$  are the first kind Bessel functions of  $\tau$  th order;  $Y_{\tau}(\alpha)$ ,  $Y_{\tau}(\beta)$  are the second kind Bessel functions of  $\tau$  th order;  $\sigma_{\mu l}^{(i)}$  define the critical mode numbers of coaxial waveguide forming *i* th coupling window;  $\sigma_{\nu k}^{(j)}$  represent the critical mode numbers of *j* th coaxial waveguide;  $r_{\lambda}^{(i)}$  are the inner  $(\lambda = 1)$  and outer  $(\lambda = 2)$  radii of *i* th coaxial coupling window;  $V_{\mu l}^{(i)}$ ,  $V_{\nu k}^{(j)}$  are the normalizing coefficients defined by the following relations:

$$V_{\mu l}^{(i)} = \left[ \int_{s_i} \Phi_{\mu l}^{(i)} \Phi_{\mu l}^{(i)} ds \right]^{-1/2};$$
  
$$V_{\nu k}^{(j)} = \left[ \int_{s_j} \Psi_{\nu k}^{(j)} \Psi_{\nu k}^{(j)} ds \right]^{-1/2}.$$

Using the expressions for vector eigenfunctions of coaxial waveguide and estimating the integrals by formulas [9], we obtain the following relations for the normalizing coefficients:

$$V_{\mu l}^{-2} = \frac{\pi}{4} \sum_{\lambda=1}^{2} (-1)^{\lambda} \alpha^{2} [Z_{0}^{2}(\alpha) +$$
  
+  $Z_{2}^{2}(\alpha) + 2Z_{1}^{2}(\alpha) - \frac{4}{\alpha} Z_{2}(\alpha) Z_{1}(\alpha)$   
 $V_{\nu k}^{-2} = \frac{\pi}{4} \sum_{\lambda=1}^{2} (-1)^{\lambda} \beta^{2} [Z_{0}^{2}(\beta) +$   
+  $Z_{2}^{2}(\beta) + 2Z_{1}^{2}(\beta) - \frac{4}{\beta} Z_{2}(\beta) Z_{1}(\beta).$ 

By solving the coupled systems of linear algebraic equations (1) with coefficients (2), one can find the distributions of tangential electric fields in first and second coupling windows. Based on these distributions, we can construct generalized scattering matrix in the form of separate blocks. By performing the corresponding transformation, we can obtain the relations for calculating electric field scattering matrix elements

$$S11_{vm}^{(up)} = \int_{s_{v}} \mathbf{E}^{(1)} \Psi_{pm}^{(1)} ds - \delta_{up} \delta_{vm} ; \qquad (3)$$

$$S21_{vm}^{(up)} = \int_{s_2} \mathbf{E}^{(2)} \Psi_{pm}^{(3)} ds ; \qquad (4)$$

$$S12_{\nu n}^{(uq)} = \int_{s_1} \mathbf{E}^{(1)} \Psi_{qn}^{(1)} ds ; \qquad (5)$$

$$S22_{vn}^{(uq)} = \int_{s_2} \mathbf{E}^{(2)} \Psi_{qn}^{(3)} ds - \delta_{uq} \delta_{vn} , \qquad (6)$$

where  $s_1$ ,  $s_2$  are the areas of first and second coupling windows.

Consider the case, when eigenmodes of the first waveguide are incident on the coaxial structure from the left side. Using relations (3), we find a block submatrix of the reflection coefficients. In this case, the relations (3) define own reflection coefficients which numerical values are disposed along the main diagonal of the block submatrix *S*11 and its mutual elements disposed symmetrically relative to main diagonal. These elements are transformation coefficients of incident modes into reflected modes. The relations (4) define transmission coefficients of modes with their transformations on the discontinuities of longitudinally inhomogeneous coaxial waveguide structure. As a result, the rectangular block submatrix S21 is received in contrast to the square submatrix S11 of reflection coefficients. These two submatrices form the left half of the generalized scattering matrix S of all waveguide structure. Considering the case when eigenmodes of the right waveguide are alternatively incident on the coaxial structure and using the relations (5) and (6), we also obtain two submatrices S12 and S22 forming the right half of the generalized scattering matrix S.

The system of linear algebraic equations (1) is suitable for calculating of modal scattering matrices of longitudinally complicated waveguide structure of general form with two junctions. Depending upon the shapes and dimensions of connected waveguide sections some coupling coefficients can be equaled to unity or zero. Consider the single diaphragm in smooth coaxial waveguide when the coupling windows coincide with cross section of middle partial region.

As applied to this structure, the system of linear algebraic equations (1) is simplified because certain coefficients (2) are equaled to unity. Because for diaphragm of finite thickness  $\eta_{lk}^{(12\mu\nu)} = 1$ ,  $\eta_{\nu k}^{(12\mu\nu)} = 1$ ,  $\eta_{lk}^{(22\mu\nu)} = 1$ ,  $\eta_{\nu k}^{(22\mu\nu)} = 1$ , these coefficients are replaced by Kronecker symbols  $\delta_{\mu\mu}$ ,  $\delta_{\nu l}$ , and the system (1) is reduced to the form:

$$\begin{split} \sum_{\mu} \sum_{l} C_{\mu l}^{(1)} [\sum_{\nu} \sum_{k} Y_{\nu k}^{(1)} \eta_{\nu k}^{(11\mu\nu)} \eta_{lk}^{(11\mu\nu)} + \\ + Y_{\nu l}^{(2)} \delta_{\mu\mu} \delta_{\nu l} \operatorname{coth} \gamma_{\nu l} t] - \\ - \sum_{\mu} \sum_{l} C_{\mu l}^{(2)} Y_{\mu l}^{(2)} \delta_{\mu\mu} \delta_{\nu l} / \operatorname{sinh} \gamma_{\mu l} t = 2 \delta_{\nu l} Y_{pm}^{(1)} \eta_{\nu m}^{(11\mu p)}; \\ u = 1, v = 1, 2, \dots, L_{1}^{(1)}; u = 2, v = 1, 2, \dots, L_{2}^{(1)}; \\ w = 1, p = 1, m = 1, 2, \dots, M_{1}; p = 2, m = 1, 2, \dots, M_{2}; \\ - \sum_{\mu} \sum_{l} C_{\mu l}^{(1)} Y_{\mu l}^{(2)} \delta_{\mu\mu} \delta_{\nu l} / \operatorname{sinh} \gamma_{\mu l} t + \\ + \sum_{\mu} \sum_{l} C_{\mu l}^{(2)} [Y_{\nu l}^{(2)} \delta_{\mu\mu} \delta_{\nu l} \operatorname{coth} \gamma_{\nu l} t + \\ + \sum_{\nu} \sum_{k} Y_{\nu k}^{(3)} \eta_{\nu k}^{(23\mu\nu)} \eta_{l k}^{(23\mu\nu)}] = 2 \delta_{\nu 2} Y_{q n}^{(3)} \eta_{\nu n}^{(23\mu q)}; \\ u = 1, v = 1, 2, \dots, L_{1}^{(2)}; u = 2, v = 1, 2, \dots, L_{2}^{(2)}; \\ w = 2, q = 1, n = 1, 2, \dots, N_{1}; q = 2, n = 1, 2, \dots, N_{2}. \end{split}$$

It should be noted that another combinations of the diaphragm dimensions lead to the fulfillment of equalities  $\eta_{lk}^{(12\mu\nu)} = 1$ ,  $\eta_{\nu k}^{(12\mu\nu)} = 1$  or  $\eta_{lk}^{(22\mu\nu)} = 1$ ,  $\eta_{\nu k}^{(22\mu\nu)} = 1$  separately.

The advantage of the approach discussed above is particularly evident in the case when calculating generalized scattering matrices of coaxial waveguide diaphragm symmetrical with respect to a plane going through the middle of the structure perpendicularly to its longitudinal axes.

In this case, the tangential electric and magnetic fields on both sides of doubled discontinuity are identical. As a result, the coupled systems of linear algebraic equations (1) for the complex expansion coefficients  $C_{ud}^{(1)}$  are reduced to following form:

$$\sum_{\mu} \sum_{l} C_{\mu l}^{(1)} [\sum_{\nu} \sum_{k} Y_{\nu k}^{(1)} \eta_{\nu k}^{(11u\nu)} \eta_{l k}^{(11\mu\nu)} + \sum_{\nu} \sum_{k} Y_{\nu k}^{(2)} Q_{\xi} \eta_{\nu k}^{(12u\nu)} \eta_{l k}^{(12\mu\nu)}] =$$
(8)  
$$= 2Y_{pm}^{(1)} \eta_{\nu m}^{(11up)};$$
$$Q_{1} = \tanh(\gamma_{\nu k} t / 2); Q_{2} = \coth(\gamma_{\nu k} t / 2);$$
$$m = 1, 2, ..., M_{p}; k = 1, 2, ..., K_{\nu}; p = 1, 2; \nu = 1, 2;$$
$$u = 1, \nu = 1, 2, ..., L_{1}^{(2)}; u = 2, \nu = 1, 2, ..., L_{2}^{(2)}.$$

One can see that for every incident mode, the system of linear algebraic equations (8) have the same matrices of coefficients at unknowns. They differ only in the matrices of right parts which number equals a half of order of generalized scattering matrix of considered coaxial waveguide structure. Therefore, to solve (8), we use the subprogram of solution of linear algebraic equations with multiple right parts.

Solving the system (8) at  $\xi = 1, 2$ , we find the sums  $\mathbf{F}_1$  and differences  $\mathbf{F}_2$  of tangential electric fields in first and second coupling windows of doubled discontinuity. Using complex coefficients  $C_{\mu l}^{(1)}$  found by solving the system (8) and expressions for  $\mathbf{F}_1$ ,  $\mathbf{F}_2$ , we define the distributions of tangential components of electric fields in coupling windows

$$\mathbf{E}_1 = (\mathbf{F}_1 + \mathbf{F}_2) / 2; \ \mathbf{E}_2 = (\mathbf{F}_1 - \mathbf{F}_2) / 2.$$
 (9)

Then, the elements of generalized scattering matrix of doubled coaxial discontinuity can be obtained according to relations (3)—(6).

Considering the coaxial waveguide structure describing by (7) for symmetric scenario, we derive the following two systems ( $\xi = 1, 2$ ) of linear algebraic equations

$$\sum_{\mu} \sum_{l} C_{\mu l}^{(1)} [\sum_{\nu} \sum_{k} Y_{\nu k}^{(1)} \eta_{\nu k}^{(1 \, l \nu \nu)} \eta_{l k}^{(1 \, l \mu \nu)} + Y_{\nu l}^{(2)} Q_{\xi} \delta_{u \mu} \delta_{\nu l}] = 2Y_{pm}^{(1)} \eta_{\nu m}^{(1 \, l \mu p)}; \qquad (10)$$

$$Q_1 = \tanh(\gamma_{\nu k} t / 2); \ Q_2 = \coth(\gamma_{\nu k} t / 2);$$

$$\begin{split} m = 1, 2, ..., M_p; \ k = 1, 2, ..., K_v; \ p = 1, 2; \ v = 1, 2; \\ u = 1, \ v = 1, 2, ..., L_1^{(2)}; \ u = 2, \ v = 1, 2, ..., L_2^{(2)}. \end{split}$$

The elements of generalized scattering matrix of symmetrical doubled coaxial discontinuity can be found by using the solution of the system (10) and relations (9) as well as formulas (3)—(6).

The proposed technical solution of matching device suggests the employment of single junctions of coaxial waveguides with different cross sections. The implementation of these structures requires the computations of their generalized scattering matrices. As in previous consideration, two variants of coaxial waveguide junctions can be occurred. The first one corresponds to the junction in which the coupling window does not coincide with cross sections of connected waveguides. The required solution of electromagnetic problem for single junction can be obtained as a special case of the half of coaxial doubled discontinuity describing by relations (1). Carrying out the corresponding transformations, we obtain the following system of linear algebraic equations for the unknown complex expansion coefficients of tangential electric field in coupling window by the series of its orthonormalized vector coordinate functions.

$$\sum_{\mu} \sum_{l} C_{\mu l}^{(1)} [Y_{\mu l}^{(1)} \eta_{\nu k}^{(11\mu\nu)} \eta_{lk}^{(11\mu\nu)} + \sum_{\nu} \sum_{k} Y_{\nu k}^{(2)} \eta_{\nu k}^{(12\mu\nu)} \eta_{lk}^{(12\mu\nu)}] =$$
(11)  
$$= 2\delta_{w l} Y_{pm}^{(1)} \delta_{up} \delta_{\nu m} + 2\delta_{w 2} Y_{qn}^{(2)} \eta_{\nu n}^{(12\mu q)};$$
  
$$u = 1, v = 1, 2, ..., L_{1}^{(1)}; u = 2, v = 1, 2, ..., L_{2}^{(1)};$$
  
$$v = 1, p = 1, m = 1, 2, ..., M_{1}; p = 2, m = 1, 2, ..., M_{2};$$
  
$$w = 2, q = 1, n = 1, 2, ..., N_{1}; q = 2, n = 1, 2, ..., N_{2}.$$

w

The second scenario describing by the relations (7) corresponds to the junction in which the coupling window coincides with cross sections of one connected waveguide. In this case, the system of linear algebraic equations for the unknown complex expansion coefficients of tangential electric field in coupling window by the series of orthonormalized vector eigenfunctions of coaxial waveguide of smaller cross section can be obtained in the form:

$$\sum_{\mu} \sum_{l} C_{\mu l}^{(1)} [Y_{\mu l}^{(1)} \delta_{u\mu} \delta_{vl} + \sum_{\nu} \sum_{k} Y_{\nu k}^{(2)} \eta_{\nu k}^{(12u\nu)} \eta_{lk}^{(12\mu\nu)}] =$$
(12)  
=  $2\delta_{wl} Y_{pm}^{(1)} \delta_{up} \delta_{\nu m} + 2\delta_{w2} Y_{qn}^{(2)} \eta_{\nu n}^{(12uq)};$ 

$$\begin{split} &u=1, \ v=1,2,...,L_1^{(1)}; \ u=2, \ v=1,2,...,L_2^{(1)}; \\ &w=1, \ p=1, \ m=1,2,...,M_1; \ p=2, \ m=1,2,...,M_2; \\ &w=2, \ q=1, \ n=1,2,...,N_1; \ q=2, \ n=1,2,...,N_2. \end{split}$$

The presence of frequency-independent coupling coefficients in the systems of linear algebraic equations is the specificity of the obtained solution. For invariable dimension of structure, these coefficients may be determined one time and be stored in the computer memory. The critical mode numbers of waveguides and coupling coefficients of eigenmodes were also computed separately and stored in the computer memory. This largely accelerates the computation of frequency characteristics of longitudinally inhomogeneous waveguide structure because the main computer time expenses are connected with computation of scattering matrices.

All systems of linear algebraic equations obtained have the same matrices of coefficients under unknowns and differ only in right-side parts. Therefore, the subprogram of solution of the systems of linear algebraic equations with several right-side parts is used to numerically realize the obtained mathematical models. When calculating the coefficients of these systems for propagating modes, the hyperbolic functions are replaced by trigonometric ones.

The mathematical models obtained are realized as a complex of FORTRAN programs. Calculations of a great number of coaxial longitudinally inhomogeneous waveguide structures were performed by using this program. The results of these calculations confirm the efficiency of developed approach.

A further solution of electromagnetic problem consists in choice of coaxial waveguide configuration with metallic conductors, equivalent to real structure comprising the dielectric insert, as shown in Fig. 1. To achieve an equivalence of real and virtual coaxial structures, it is necessary to get the approximate equality of its coefficients reflection modules. Such equivalence can be obtained by fulfillment of the following procedure. Initially, we calculate the reflection coefficient from the structure with dielectric insert by finite difference time domain method which allows the computation of electromagnetic parameters of waveguide connections with partial dielectric filling.

To achieve the high accuracy of the procedure for obtaining the equivalence, we print the calculated coefficient reflection module in large scale. These calculated results are depicted in Fig. 2 by solid curve. Further, we count values of the coefficient reflection module in discrete points uniformly distributed in lower frequency range. By using the scattering matrix method described above, we find the configuration of equivalent waveguide structure with metallic discontinuities without dielectric filling. For this, we solve the optimization problem by the evolution strategy method using fifty thousand iterations.



Fig. 2. The calculated coefficient reflection module of equivalent metallic structure.

As the result, we can obtain several variants of the equivalent waveguide structure. The coaxial waveguide combination which allows achieving the desired result is shown in Fig. 3. It represents the finite thickness diaphragm arranged on inner conductor of coaxial waveguide. This sample not comprising the dielectric insert is disposed on the boundary between metallic coaxial waveguide and radiating circular waveguide used as an aperture of feed horn.



Fig. 3. The schematic representation of equivalent metallic waveguide structure.

We form the objective function for finding this structure dimensions under which the maximal conformity between frequency characteristics of real and virtual structures can be attained. As the objective function, we take the sum of squares of coefficient reflection modules differences of real and equivalent structures. The frequency characteristic of equivalent virtual coaxial structure is calculated by generalized scattering matrix approach considered above. For this objective function, we again solve the optimization problem by the evolution strategy method using fifty thousand iterations. The calculated coefficient reflection module of equivalent virtual structure is depicted in Fig. 2 by points. It can be seen that the discrepancies between calculated results for real and virtual structures do not exceed 0.8 percent. Therefore, this virtual model can adequate reflect the physical processes in real coaxial waveguide structure comprising the dielectric insert and can be applied to solving the optimization problem for finding the configuration of matching device.

Let us continue the computation of overall matching device with employment of chosen technical solution. It represents a sequential connection of finite thickness diaphragm situated on inner conductor of coaxial waveguide. To increase the matching quality in lower boundary of considered frequency range, the single step transition from outer diameter 50 mm to outer one 70 mm is envisaged in device construction. This real coaxial structure is combined with calculated equivalent unit accounting the influence of dielectric insert.

Similarly to previous consideration of virtual unit optimization, we form the objective function for finding the parameters of chosen configuration of matching device. As the optimization parameters, the radii of diaphragms and distances between them were used without changing the equivalent unit. For computation of frequency characteristics of matching device, the algorithms based on developed mathematical models were employed. To avoid the problem of local minima, one hundred thousand iterations were used in optimization process.

The results of fulfilled computations showed that small values of reflection coefficient in such structure can not be obtained because of considerable level of reflection from junction of two coaxial waveguides having the outer diameters 50 and 70 mm, respectively. It was decided to replace one junction of coaxial waveguides by two additional junctions. The radii and lengths of resulting two steps between these three junctions were included to the variety of optimization parameters. And this technical solution allows obtaining the desired result.

By the optimization including the outer radii of sections constituted by three adjacent junctions of coaxial waveguides and their lengths, the optimal configuration of matching device with the metallic equivalent was found.

At the last designing step, the metallic equivalent was replaced by real structure with dielectric insert. As a result, the real matching device of coaxial feed horn looks as shown in Fig. 4. The final calculation of this structure was performed by finite difference time domain method. It is reduced to the selection of distance between the first diaphragm and open end of circular waveguide manufactured inside the inner conductor of coaxial waveguide. This procedure was necessary since the reflection coefficients phases for real and virtual structures does not taken into account in previous calculations. The theoretical frequency dependence of VSWR for the matched design of coaxial horn is shown in Fig. 5.



Fig. 4. The actual unit of matching device of dual frequency coaxial feed horn to be applied as a radiator of reflector antenna.

It is seen that the maximal value of VSWR in all matching frequencies band do not exceed 1.16. The designed coaxial horn was applied for the irradiation of reflector antenna. The high technical parameters of developed coaxial horn allow obtaining the excellent performance of the reflector antenna in both frequency bands.

### Conclusion

An effective technique to the design of dual band coaxial waveguide feed horn for reflector antenna excitation has been developed. A novel technical solution of highly complicated matching problem has been proposed.

The dielectric insert disposed inside the circular waveguide fabricated in interior conductor of open ended coaxial waveguide is applied to match the feed horn with free space in upper frequency range. The matching design of the feed in lower frequency range performed as the series of finite thickness diaphragm arranged on outer surface of interior conductor of coaxial waveguide was investigated.

Originality of developed approach consists in replacement of combined coaxial waveguide connection with dielectric insert by equivalent metallic structure allowing the application of effective methods for its calculation. The optimal configuration of the metallic equivalent replacing the real structure with dielectric insert was found by optimization procedure based on by the variant of the evolution strategy method. The distinctive feature of the developed approach consists in combination of rigorous procedure based on computation of generalized scattering matrices and another rigorous procedure based on the finite difference time domain method.



Fig. 5. The feature of developed matching device of dual frequency coaxial feed horn in lower frequency range.

The new effective approach to obtain the rigorous solution of given electromagnetic problem for the complicated structure consisting of a great number of connected coaxial waveguide sections and dielectric insert has been proposed. The rigorous procedure based on computation of generalized scattering matrices was combined with another broad procedure based on the finite difference time domain method. The employment of metallic discontinuity equivalent to overall investigated structure was proposed in order to combine these essentially different approaches.

To optimize the dimensions of equivalent unit and overall matching device, the expressions for calculating generalized scattering matrices of complicated coaxial structures consisting of several waveguide sections are derived. The formulas for description of symmetrical coaxial waveguide diaphragm scattering matrix in the form of two independent systems of linear algebraic equations are obtained.

An optimization of equivalent matching structure has been performed by the variant of the evolution strategy method. The obtained design with metallic discontinuities was converted into real structure comprising the dielectric insert. The final calculations of real structure were performed by the finite difference time domain method. As a result of performed investigations, the excellent final VSWR characteristic of proposed matching device was obtained. The maximal value of VSWR in all matching frequencies band does not exceed 1.16. The designed coaxial horn was applied for the irradiation of reflector antenna. The high technical parameters of developed coaxial horn allow obtaining the excellent performance of the reflector antenna in both frequency bands. Thus, the effectiveness of proposed technique was confirmed by the results of testing the developed feed horn in reflector antenna system.

The technique proposed can be applied for the optimization of complicated coaxial waveguide structures containing the dielectric inserts, where the multiple calculations of frequency characteristics at finding of objective functions are required.

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