Wiggler and Undulator Magnets

By Herman Winick, George Brown, Klaus Halbach and John Harris

Two new devices are being added to synchrotron radiation sources to extend the spectral range and increase brightness.

To scientists using vacuum ultraviolet and x rays the most important characteristics of an ideal radiation source would be a high intensity within a small solid angle and a high intensity within a small wavelength interval, both extending over a broad range of wavelengths. High spatial brightness (large flux within a small solid angle) permits the delivery of a large number of photons per second to a small sample. High spectral brightness (large flux within a narrow wavelength interval) is essential for high-resolution spectroscopy. A high-power tunable vuv and x-ray laser would be ideal, but unfortunately such a laser does not yet exist. Conventional vuv sources (such as gas-discharge lamps) and x-ray sources (such as electron-impact x-ray tubes) can produce a large flux of radiation, most of which is indeed within a narrow bandwidth at particular fluorescent lines. However, the flux is diffused over a large solid angle and the wavelength is fixed. The continuum radiation from these sources is less intense than the narrow fluorescence peaks by about four orders magnitude. Other sources exist, such as laser plasma sources and sliding-spark devices, that provide more intense continuum radiation but have low repetition rate, limited spectral range and attendant debris.

It is a remarkable fact that the normal synchrotron radiation emitted by relativistic electrons (traversing the bending magnets of storage rings) provides continuum electromagnetic radiation over a broad spectral range with spatial and spectral brightness many orders of magnitude higher than is available from other sources. Coupled with its other properties (such as natural collimation, high polarization and pulsed time structure-see the accompanying article by Ednor Rowe, page 28), these features of synchrotron radiation have made possible a great leap forward in research capability.

However, even though this radiation is far superior to other sources, close examination reveals that the use of storage-ring bending magnets to generate the radiation is, in many cases, not optimal.

To use bending-magnet radiation from machines that were built for high energy physics was, of course, a natural and effective beginning. With dedicated operation of these machines for synchrotron-radiation research and with the construction of new machines designed solely as radiation sources, even higher brightness is becoming available from ring bending magnets.

However, the primary function of storage-ring bending magnets is to maintain a circulating beam by bending electrons into a closed orbit. It is not surprising that one can design other magnetic structures that, when added to a storage ring, are more effective than the ring bending magnets in extracting radiation. This article describes wigglers and undulators, periodic magnetic devices that may be incorporated into a storage ring. They are designed to produce no net deflection or displacement of the circulating beam but to enable the relativistic electrons to produce synchrotron radiation with much higher brightness than can be obtained from the ring bending magnets and with a potential for more efficient use of radiated power by the experimenter. In addition, high-field wigglers significantly extend the spectral range available from the usually weaker ring bending magnets. This extension of the spectral range has important consequences for the design of new rings; making it possible, for example, for an intermediate-energy ring (about 1 GeV) to produce intense radiation in the important one angstrom wavelength region, whereas radiation from its bending magnets is restricted to longer wavelengths.

Because of the improvement that they provide in spectral range and brightness, wiggler and undulator magnets are being installed or seriously considered in synchrotron radiation laboratories throughout the world. The level and range of interest and activity can be judged from the proceedings of recent conferences.[1,2]

Both wiggler and undulator magnets deflect the electron beam in alternate directions, producing an angular excursion of the beam. However, the electrons leave the magnet in the same way they would if the magnet were turned off-that is, they produce no net deflection or displacement.

An undulator is a magnetic structure with many periods in which the angular excursion of the electron beam is less than or of the order of the natural emission angle of synchrotron radiation (given by $\gamma^{-1} = m_0c^2/E$, the ratio of the electron rest mass energy to its total energy). The undulator preserves the intrinsic high brightness of synchrotron radiation in the plane of deflection.

Unlike the smooth, continuous spectral distribution of radiation produced by ring bending magnets and wigglers, interference effects in undulator radiation result in peaks at one or a few wavelengths. At these tunable wavelengths undulator radiation has high spatial and spectral brightness.

A wiggler is a magnetic structure with one or a few periods in which the angular excursion of the electron beam is considerably greater than Υ^{-1} but less than the angular deviation produced by one ring bending magnet. A wiggler is usually designed so that the angular excursion of the radiation from each pole approximately fills the angular acceptance of the synchrotron light beam pipe.

Also, it is usually designed so that the peak magnetic field may be greater than the field in the bending magnets, resulting in a spectrum which may extend to higher photon energies. Thus, a wiggler magnet offers an extended spectral range and overall enhancement of the radiation brightness compared to the ring bending magnets.

Although the first consideration of wigglers and undulators goes back 25 or more years, their use as radiation sources for experiments is only recent. The first wiggler magnet to be used routinely as a radiation source began operation in March 1979 at the Stanford Synchrotron Radiation Laboratory. The substantial extension of spectral range and intensity provided by this magnet and the demonstration of its compatibility with storage-ring operation have had a profound effect on the development of the SSRL and other synchrotron radiation laboratories.

Two examples of this impact are:

- User interest in the first SSRL wiggler has been so great that plans to build additional beam lines from bending magnets were suspended and the emphasis at SSRL is now on building more wiggler lines.
- The design group planning the European Synchrotron Radiation Facility originally conceived of a 5-GeV, 565 mA machine to meet certain intensity and spectral specifications. The group is now considering an "all-wiggler machine"[3] in which virtually all of this radiation used by experimenters would be produced by wigglers (up to 40 of them). With wigglers, the ring would only have to operate at 3.5 GeV with 100 mA to meet the same intensity and spectral-range specifications as the bending-magnet beams on the higher energy, higher-current ring.

Undulators have not yet had a significant impact on synchrotron radiation research. Most of the work. date has concentrated on producing visible undulator radiation (see figure 1). It is only very recently that undulators designed to serve as radiation sources for experiments in the vuv and x-ray parts of the spectrum have been installed in storage rings (in Novosibirsk, USSR; Orsay, France; and at Stanford). Based on the brief experience with these devices plus the many studies done at longer wavelengths, it is our opinion that undulators will have an impact on research with synchrotron radiation comparable to or exceeding that of wigglers, particularly for difficult experiments requiring the highest brightness.

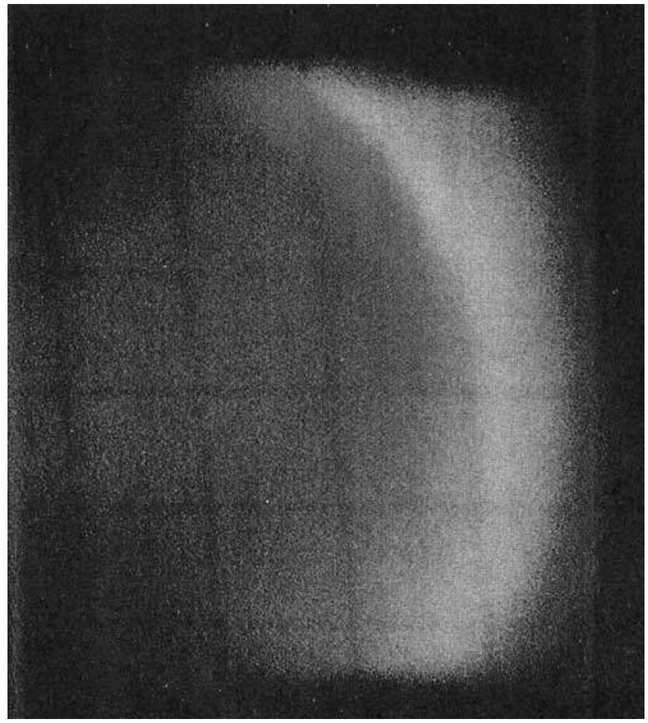


Figure 1 — Visible light from a superconducting undulator at the ACO storage ring operated at 240 MeV. The symmetric dark spots at the upper center and lower center of the photo correspond to emission angles of $\theta = \pm \Upsilon^{-1}$ The magnetic field is horizontal, so the undulation is in vertical plane. (Courtesy of Y. Farges and Y. Petroff, LURE, Orsay, France).

History of the devices

Undulators. The earliest eonsideration of undulators goes back to a theoretic paper by V. L. Ginzburg[4] in 1947. The first undulator was made by Hans Motz and coworkers[5] in 1953. It was a permanent magnet with a period of 40 mm and a gap of 4 mm; it achieved field strengths up to about 5 kG. Using beams from linear accelerators, Motz observed visible radiation when electron energies were in the range of 100 MeV and millimeter radiation at about 3 MeV.

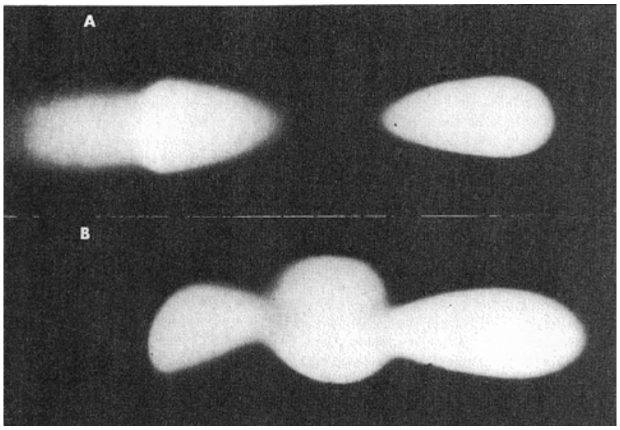


Figure 2 — First observation of undulator radiation (reference 6) from a circular electron machine-the Pachra 1.3-GeV synchrotron at the Lebedev Physics Institute, Moscow. (a) Undulator off: The two lobes are synchrotron radiation from the bending magnets adjacent to the undulator. (b) Undulator on: The central spot is due to the undulator. The electron energy is 175 MeV. (Courtesy V. V. Mikhailin, Moscow State University)

In the 1970s undulators were installed in two synchrotrons in the Soviet Union and systematic investigations of the properties of the radiation were carried out. One group (D. F. Alferov, Yu A. Bashmakov, E. G. Bessonov and ooworkers) used the Pachra 1.3-Ge V synchrotron of the Lebedev Physics Institute, Moscow. The other group (led by M. M. Nikitin) used Sirius, the 1.2-Ge V synchrotron at the Research Institute of Nuclear Physics of the Tomsk Polytechnic Institute.

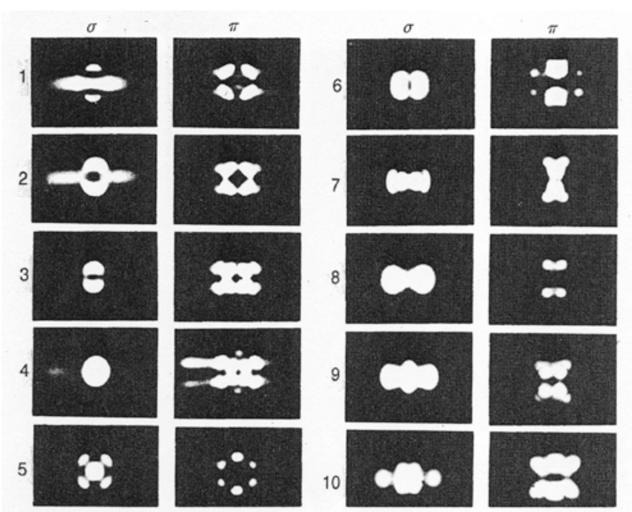


Figure 3 — Undulator radiation patterns for different polarization and varied electron energy. The angular distribution is shown at a wavelength of 500 nm with polarization parallel (τ) and perpendicular (π) to the electron acceleration vector at electron energies ranging from 473 MeV (1 σ,π) to 97.5 MeV (10 σ,π). The undulator had a period of 14 cm and was operated at 0.839 kG. (Reference 7).

In both cases relatively low-energy electrons (up to a few hundred MeV) were used, so that the undulator produced interference peaks in the visible part of the spectrum (see figures 2 and 3). Our present understanding of undulator radiation is due in large part to the excellent work of these groups. Their reviews[6,7] and the work of others[8,9,10,11,12,13,14] provide a comprehensive description of the properties (spectrum, intensity, angular distribution, polarization) of undulator radiation and extensive references to the rapidly growing literature on the subject.

Recently, efforts have begun to extend undulator radiation into the vuv and x-ray parts of the spectrum where, because of the absence of lasers, they are likely to be important sources of radiation.

A superconducting undulator[14] has been operated on the Orsay 540-MeV storage ring ACO, and permanent-magnet undulators have been operated on VEPP-3, the 2.2-Ge V storage ring at the Novosibirsk Physics Institute, and SPEAR,[15] the 4-GeV storage ring at SLAC, used by SSRL (see figure 4).

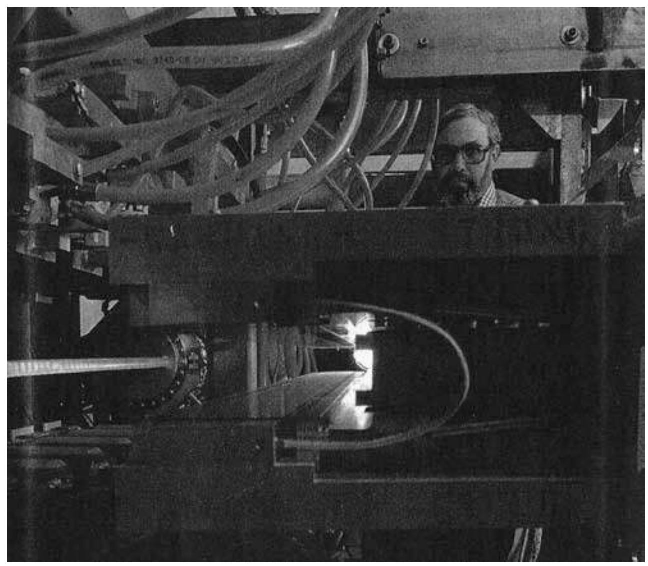


Figure 4 — Wiggler and undulator magnets may be alternately used in the same straight section of the SPEAR ring. The wiggler magnet is shown split and the top and bottom halves have been separated so that the undulator magnet (shown in the withdrawn position) could be inserted. The vacuum chamber, which is not disturbed in the changeover from wiggler to undulator, is midway between the wiggler halves. Photo by J. Faust.

Undulators are being included in the new dedicated synchrotron sources now being completed around the world. The reduced electron-beam size and angular divergence planned for some of these machines will enable undulators to produce even higher spectral and spatial brightness.

The devices built to date produce high-brightness undulator radiation at photon energies below about 2 keV. Higher photon energies can be reached by undulators with shorter periods and higher magnetic fields or by using higher-energy electrons. With electron energies of 3.5-4 GeV, such as are available with SPEAR, it should be possible to extend the higher-harmonic peaks from the undulator to about 8 keV.

We will describe the plans for such an undulator at SSRL later in this article. Even higher energies could attained if undulators were installed in recently completed, higher-energy colliding-beam storage rings in the Soviet Union and the US.

Wigglers. The earliest suggestion of a wiggler magnet to produce synchrotron radiation was made by K. W. Robinson in an unpublished report at the Cambridge Electron Accelerator (CEA-14) in 1956. The first wiggler was built CEA[16] in 1966, not as a source of synchrotron radiation, but to provide additional damping of betatron and synchrotron oscillations to convert the alternating gradient (and hence anti-damping) magnetic lattice of the CEA to a beam storage system. The PEP storage ring also contains wiggler magnets to control stored-beam size and polarization time constants.[17]

A wiggler magnet was first used as a synchrotron radiation source at SSRL in 1979.[18] This 7-pole, 1.2-m-long, 18 kG magnet provided the most powerful beam of x rays ever produced and also made possible an increase in the interaction rate (luminosity) of colliding beams on SPEAR. This success prompted the installation in 1980 of a pair of longer magnets (2 m, 18 kG) in SPEAR (see figure 5). With all eight coils powered in this magnet, the electron beam executes four full oscillations. The electrical circuit can be remotely changed to produce one, two or four oscillations to control the total power radiated. This ring is now operated for half the time as a single-beam synchrotron radiation source. A similar wiggler[2] is now operating in the Adone storage ring (1.5 GeV) at Frascati (see the photo on page 30) and a 35 kG, 21-pole superconducting wiggler has been installed in the VEPP-3 ring (2.2 GeV) in Novosibirsk.[19]

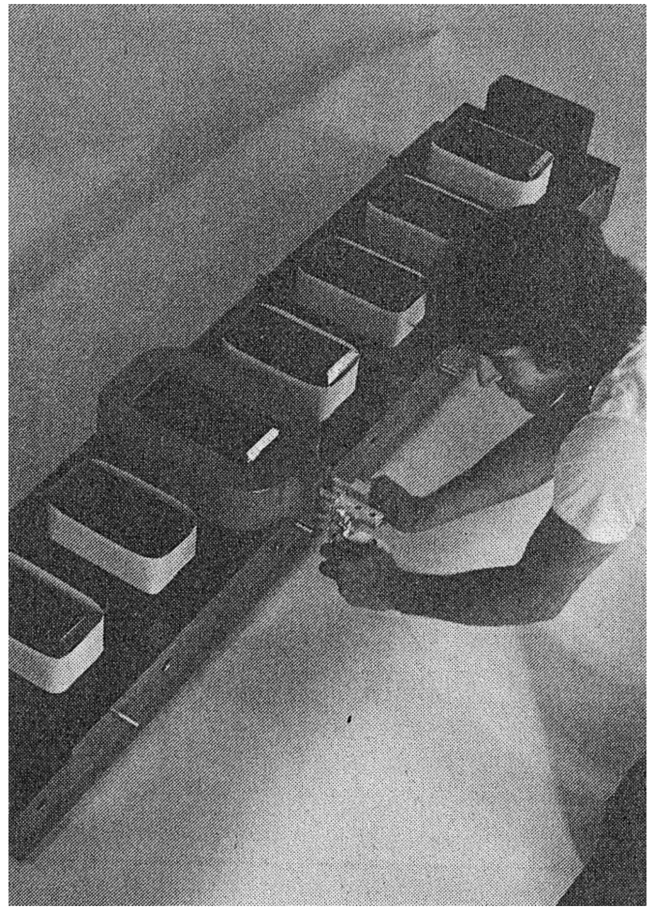


Figure 5 — Wiggler magnet for SPEAR (lower half) with seven full poles and two end half poles. Each pole is powered by a coil, only one of which is shown. (Photo by J. Faust, SLAG)

Wigglers are in construction or being planned for most of the new dedicated synchrotron sources. Spencer and Winick[12] review the characteristics of operating planned wigglers and undulators as of 1979.

Wiggler magnets

The simplest wiggler magnet consists of 3 poles and produces one oscillation of the electron beam. The total electron bending angle (and hence the angular divergence of the radiation), although small (about 1-5°), is larger than the opening angle of the radiation (≤ .05°). The fan of radiation produced by the wiggler is thus large enough that two or more simultaneous experiments can share the beam. The center pole is generally operated at a field higher than the bending magnets of the ring to extend the spectrum to higher photon energies. The spectral distribution is a smooth continuum as it is for the bending magnets. For wigglers and bending magnets the spectrum is characterized by the critical energy:

$$\varepsilon_c = \frac{3hc\gamma^3}{2\rho} = 0.06651BE^2 \quad (1)$$

where B is the magnetic field (in kG) of the wiggler or bending magnet and ρ is the radius of curvature; the critical energy is in keV, the beam energy is in GeV. The wiggler's effectiveness in extending the spectrum to higher photon energies is thus dependent on the ratio of the wiggler field to bending magnet field. Bending-magnet fields are typically in the range 5-15 kG. Conventional electromagnet wigglers have been used with fields up to about 20 kG and superconducting wigglers designed to reach 40-60 kG are in construction. It also appears that permanent-magnet wigglers with fields up to at least 16 kG should be possible. An example of the effectiveness of a wiggler in extending the spectrum available from the bending magnets is given in figure 6 for three facilities. All wiggler spectra are shown for one pole. The SPEAR wiggler has eight effective full poles so the wiggler spectrum should be increased by a factor of 7-8. The Daresbury wiggler has a strong central pole and weaker end poles so the overall intensity is enhanced by a factor that varies from 2 at low photon energy to 1 at the highest photon energy. The Hefei wiggler has not been designed yet. Multipole wigglers represent extended sources, which must be considered in the design of beam-line optics to maximize the flux available at the experiment. The flux is given per milliradian of bend integrated over all angles normal to the bending plane.

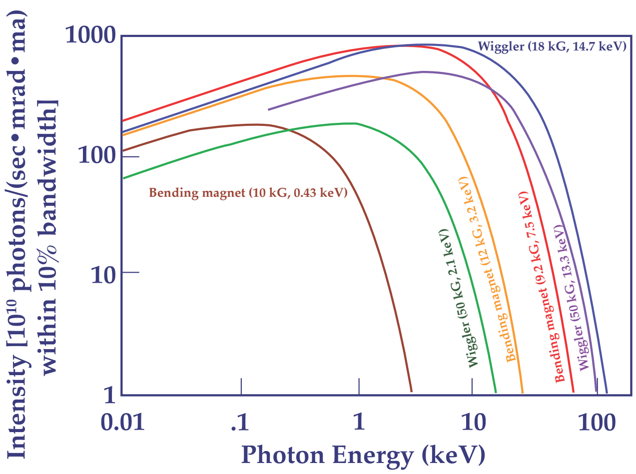


Figure 6 — Spectra compared for bending and wiggler magnets, calculated for the 800-MeV ring planned for Hefei, PRC (gray curves); the 2-GeV Synchrotron Radiation Source (SRS) recently completed at Daresbury, England (colored curves); the 4-GeV SPEAR operating at 3.5 GeV (black curves).

In most cases the magnetic field is vertical (that is, perpendicular to the ring plane) because the aperture required by a stored beam is generally less in that direction than in the other direction. This permits smaller gaps and stronger fields in the vertical direction. However, a horizontal-field device produces a vertical deflection and hence a vertically polarized synchrotron radiation beam. This is an advantage in certain diffraction experiments. Such a vertically deflecting wiggler, a 60-kG superconducting magnet, is under construction for the Photon Factory in Tsukuba, Japan.

To assure a net zero displacement of the beam, the wiggler field must have mirror symmetry about the transverse midplane of the magnet. Thus, although there are three physical poles in this simplest wiggler there are only two equivalent full poles: The two outside poles each produce half the deflection of the center pole, in the opposite direction. The outside poles may be operated at lower field than the, central full pole, contributing less high-energy radiation. Furthermore, the orbit displacement in the wiggler may be large compared to the transverse size of the beam; the radiation from the center then originates from a different! source point than radiation from the end poles, and only radiation from the central pole may be useful. The three-pole wiggler thus shifts the spectrum to higher photon energies but usually provides no overall intensity enhancement because only one pole is really used. Such a one-period wiggler is also called a wavelength shifter.

Adding more full poles produces more oscillations: one additional oscillation for each pair of additional poles. For example, a wiggler with three central full poles and two end half poles (equivalent to four full poles) produces two oscillations of the electron beam. If the amplitude of the oscillation is smaller than the transverse beam size (as it usually is), such a device would enhance the overall intensity (compared to the bending-magnet spectrum) by a factor of three to four (depending on the strength of the end poles) as well, as shift the spectrum to higher photon energies. Depending on the space available in the straight section of the ring and on the angular acceptance of the beam pipe for the radiation, the number of poles may be increased to eight or more equivalent full poles, providing an intensity enhancement of a factor of seven to eight or more, as well as a hardening of the spectrum. In cases where the radiation is focused, the full intensity enhancement may not be realized, because the source of the radiation is now extended.

Wigglers have proven immensely useful at SSRL. In particular, during the 50% of the time that SPEAR operates primarily for colliding beam experiments, the energy is usually kept at, only 2 GeV and the maximum current: is limited to only 10 mA by beam-beam, instabilities. Under these conditions the photon intensity above about 7 keV from the bending magnets (which operate at 5.3 kG and produce a spectrum; with a critical energy of 1.4 keV) is too low for most experiments. The wiggler operating at 18 kG with up to eight full poles produces a spectrum with a critical energy of 4.8 keV and provides sufficient intensity for experiments requiring photons up to about 20 keV. In effect, for many experiments the wiggler beam at 2 GeV, 10 mA is equivlent to a bending magnet beam at 3 GeV and about 70 mA.

During the 50% of the time that SPEAR operates as a dedicated synchrotron radiation source, currents of 50-100 mA are stored at energies from 3 to 3.5 GeV. The user on the wiggler line can select magnetic fields up to 18 kG, providing critical energies up to 11 keV at 3 GeV and 15 keV at 3.5 GeV. Often the harmonic content of monochromatized beam (that is, the λ/2, λ/ 3, etc. components that can pass a crystal monochromator set to transmit radiation by Bragg diffraction at a wavelength λ) is higher than desired when the critical energy is so high. In these cases the wiggler field is reduced, frequently to a level comparable to the bending-magnet field (8 kG at 3 GeV) to obtain a spectrum with the desired energy (about 5 keV). Thus the user on the wiggler line has control over the spectrum as well as the important benefit of up to eight times higher intensity due to the eight wiggler poles.

The total power radiated by a wiggler is given numerically in kilowatts by

$$P = 1.267x10^{-2} E^2 \left\langle B^2 \right\rangle IL \quad (2)$$

where $<B>^2$ is the average value of the square of the magnetic field (in kG) over the length L (in m); the current I is in amps and the energy E is, again, in GeV. This equation applies to any distribution of magnetic field and can also be used for undulators and constant-field bending magnets. For an N-pole wiggler, the peak power along the beam axis (when the amplitude of oscillation is less than the beam size) has the numerical value in watts per milliradian

$$P = 0.422 B_{\max} E^3 IN \quad (3)$$

where the power is given per milliradian of wiggler bend angle integrated over the full range of vertical angles (typically less than 1 mrad). The maximum field is in kG and the current is, again, in amps.

For example, the SPEAR wigglers (18 kG, 1.8 m, 8 poles) each produce a total power of about 3.5 kW and a maximum power on axis of about 150 W/mrad with SPEAR operating at 3 GeV and 100 mA. Before the system operates at these high levels of radiated power, some of its components had to be rebuilt with improved cooling to deal with the energy deposited by the radiation. Some limitations still remain (such as heating of beryllium windows and monochromator crystals) and it is sometimes necessary to reduce the radiated power. One way to do this is to lower the wiggler field but, of course, this reduces the critical energy as well. It is often preferable, therefore, to reduce the number of active poles by changes in the electrical circuit of the wiggler.

This approach has been used in the SPEAR wigglers, which can be operated so as to produce one, two or four oscillations of the electron beam. The higher power available from wigglers is pacing the development of beam lines and experimental equipment that can handle the power (for example, detectors able to count at higher rates and crystal monochromators with cooling and feedback systems to compensate for thermal detuning of Bragg peaks).

Effects on storage ring operations

As several operating wigglers have demonstrated, their effects on the operation of the ring can be minimal. The net deflection of the electron beam can be made negligibly small by making the field integral along the beam path vanish and the net displacement can also be made small by designing the magnet to be mirror symmetric across the transverse midplane. The principal observable effects of a wiggler are on the betatron oscillation frequency, energy spread and emittance of the electron beam. We discuss these briefly here; the interested reader is referred to more detailed treatments elsewhere.[12,18,20,21]

The magnetic field of the wiggler causes some focusing of the beam in the direction parallel to the wiggler field lines, thereby increasing the betatron oscillation frequency (the so-called v value). However, this effect is usually small enough that one can either ignore it or compensate for it simply by adjusting the strength of the ring quadrupole magnets.

The energy spread and emittance of a stored electron beam under equilibrium conditions are determined by the balance between the damping effects of synchrotron radiation and the excitation of betatron and synchrotron oscillations due to the quantum nature of this emission. Detailed considerations require the so-called synchrotron radiation integrals,[12] which we will not discuss. The energy spread of the stored beam in a storage ring is usually so small ($\Delta E/E \approx 0.1\%$) that the small changes (usually increases) caused by wigglers are of no importance to users of synchrotron radiation. With one exception, the energy spread is also unimportant to colliding beam users. It becomes relevant when the center-of-mass energy is equal to the equivalent mass of a narrow resonance such as the ψ/J particle. The natural width of these resonances can be comparable to the energy spread of the stored beam, in which case an increase in energy spread can result in a decrease in counting rate.

In a storage ring the position of an electron and the angle of its trajectory (both measured relative to the equilibrium orbit) are correlated. Phasespace plots of angle in the median plane versus radial electron position summarize this correlation in a “horizontal electron emittance ellipse.” A similar plot of vertical angle versus vertical position gives the “vertical electron emittance ellipse.” (The electron beam emittance is the area of these ellipses divided by π.) The precise shape and orientation of these phase space ellipses change along the orbit but the area is invariant-an example of Liouville’s theorem. The vertical emittance is usually a small fraction (about 1%) of the horizontal emittance and is determined largely by coupling between vertical and horizontal betatron oscillations-due, for example, to imperfections for alignment errors of the quadrupole magnets. Smaller emittance means a higher source-point brightness of the synchrotron radiation (because of smaller beam dimensions or divergence angles or both).

As mentioned earlier, the precise value of the emittance is determined by the balance between the damping and the excitation effects of synchrotron radiation on betatron oscillations. Because wigglers contribute additional synchrotron radiation they also affect the emittance, but the effect is usually significant only at low electron energies, where the radiation effects of the wiggler on the electron beam are significant compared to the effects of bending magnets. In colliding-beam operation, an increase in emittance at low energy can lead to a higher interaction rate (luminosity), because larger currents can be allowed to collide before the beam-beam instability begins. The first wiggler installed in SPEAR in 1978 as an improved source of synchrotron radiation was eagerly used by high-energy physicists to increase the luminosity of the stored beam even before the team line for the radiation was completed. By appropriately designing the magnet lattice and by placing the wiggler in a particular location in the lattice where the momentum dispersion function is zero or very small, one can make the effects of a wiggler on the electron-beam emittance negligibly small at all electron energies. Such locations are now routinely designed into new dedicated synchrotron radiation sources (see Rowe’s article, page 28).

Undulators

Undulator radiation differs from the radiation produced by wigglers and bending magnets in two important respects:

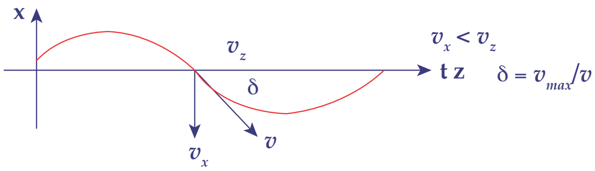
- The radiation has the same intrinsic brightness as the synchrotron radiation because the angular excursions of the beam in the undulator are no larger than the natural emission angle of the synchrotron radiation (which is on the order of $1/\gamma$)
- The radiation is the result of the interference effects that can occur when an electron radiates in a periodic field.

Because undulator radiation is highly collimated, it is not feasible split the beam to serve more than one experiment simultaneously, as one usually does with the more divergent beams from wigglers or bending magnets.

Other examples of radiation by electrons subjected to periodic fields include channeling radiation, coherent bremstrahlung from crystals and Compton backscattering. The basic physics is very similar in all these cases. The free-electron laser consists of an electron beam traversing an undulator magnet in a resonant cavity, so that the initial spontaneous undulator radiation gives rise to extremely high intensity stimulated radiation with even narrower bandwidth. Several undulators are in use or under construction primarily as free-electron lasers or the related optical klystrons.

Electron Motion in an Undulator

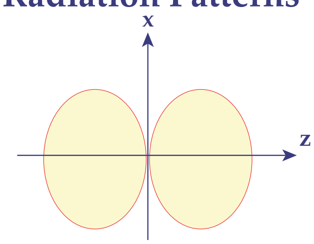
Consider the motion of a single electron for one period of its path through an undulator.



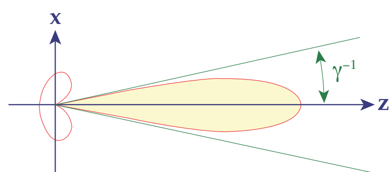
Laboratory Frame

For a weak undulator ($\delta << \gamma^{-1}$, $K = \gamma\delta << 1$) in the electron rest frame, the electron exhibits purely transverse simple harmonic motion. The resulting radiation pattern is the usual dipole pattern and contains only the frequency of oscillation of the electron. The radiation pattern in the laboratory frame is peaked forward and contains only one frequency at each viewing angle.

Radiation Patterns



Rest Frame



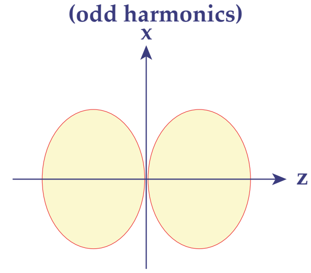
Laboratory Frame

For a strong undulator ($\delta > \gamma^{-1}$, $K = \gamma\delta > 1$) average motion in electron rest frame resembles a figure 8 having both transverse and longitudianal components.

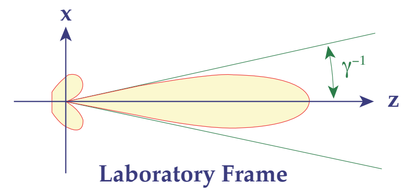
Transverse motion contains only odd harmonics and has a peak intensity in the forward direction ($\theta = 0$) in the laboratory system. Longitudinal motion contains even harmonics and is peaked at $\theta \sim \gamma^{-1}$ in laboratory system.

Radiation Patterns

Transverse motion (odd harmonics)

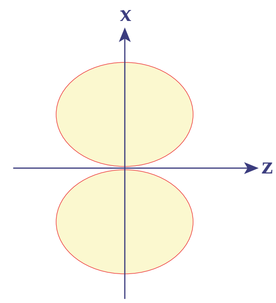


Rest Frame



Laboratory Frame

Longitudinal motion (even harmonics)



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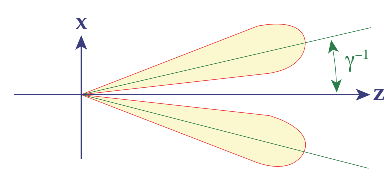

Laboratory Frame

The weak undulator

The basic characteristics of undulator radiation can be readily understood by first considering the motion of a single electron in a weak undulator — one in which the maximum deflection angle from the average direction is much less than Υ^{-1} (see the box on page 58). For a general treatment, see reference 10. In each half period the electron is deflected through this small angle. Thus, a weak undulator with *N* periods can be thought of as a succession of 2N short magnets alternating in polarity.

For a qualitative treatment of the weak undulator we can regard the longitudinal velocity as constant and consider the electron motion from a reference frame moving with this velocity. In this frame the electron exhibits purely transverse, simple harmonic motion (see the box). The resulting radiation pattern is the usual dipole pattern and, for an infinitely long undulator, this pattern contains only the frequency of oscillation of the electron. In the laboratory frame the radiation is shifted to higher energy and the pattern is folded forward so that most of the radiation is within a cone with an opening angle of $1/\Upsilon$. Only one frequency is observed at each observation point in the laboratory system. For an undulator of finite length and for a beam with finite transverse size and divergence, the radiation has a spread of frequencies centered on the shifted oscillation frequency (see figure 7).

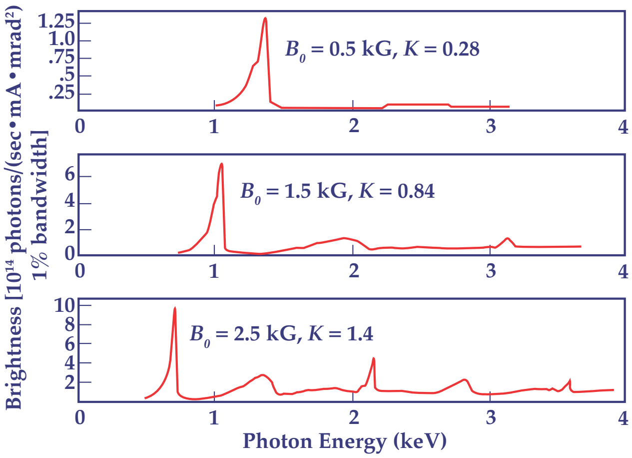

Figure 7 — Spectra from undulator magnet at LI3L-SSRL calculated for 3.0 GeV and different values of the peak magnetic field. The effects of. finite electron beam size and divergence have been included. The detector is assumed to be 25 m away from the undulator. (Program ΣyY. Sambre, Stanford.)

The frequency of the photons observed in the laboratory system can be simply calculated by considering the interference between the radiation emitted by the electron at two successive crests of its sinusoidal path. In the weak undulator approximation we ignore the extra path length travelled by the electron and consider its time to move a distant λ_u (the undulator period) as $\lambda_u/\beta c$. The photon, of course, takes only λ_u/c to go this distance. When the difference between these times is equal to one oscillation period of the light (λ/c) we have constructive interference. Thus the interference condition is

$$\frac{\lambda}{c} = \frac{\lambda_u}{\beta c} - \frac{\lambda_u}{c} \quad (3b)$$

For values of β near unity, $1/\beta - 1$ is very nearly equal to $1/2\Upsilon^2$, so the condition for constructive interference is

$$\lambda = \frac{\lambda_u}{2\gamma^2} \quad (4)$$

Harmonics of this wavelength also satisfy the interference condition but their amplitude is negligible for the weak undulator. For radiation emitted at a small angle ($\theta<<1$) to the undulator axis it is easily shown that equation (4) becomes

$$\lambda = \left(\frac{\lambda_u}{2\gamma^2}\right)\left(1+\gamma^2\theta^2\right) \quad (5)$$

The strong undulator

For a strong undulator (one in which the maximum deflection angle is on the order of Υ^{-1}) the transverse velocity of the electrons is larger and, because the total velocity must be constant in a magnetic undulator, the transverse oscillatory motion is accompanied by longitudinal velocity variations that can no longer be ignored. We can define an average rest frame which moves at a constant velocity equal to the average longitudinal velocity of the electron. In this frame the electron executes a figure 8 (see box) made up of transverse and longitudinal oscillations. The transverse motion is composed of odd harmonics and the longitudinal motion has even harmonics. For a single electron only odd harmonics are present on-axis. However, for a real electron beam with finite transverse dimensions and divergence angles, some electrons are always viewed off-axis so the spectrum also contains even harmonics (see figures 7 and 9).

The electron beam size and divergence also affect the polarization of the radiation. For a single electron the radiation emitted in the plane of the undulator is 100% linearly polarized. Off this plane the polarization varies with polar and azimuthal angles in a fairly complex manner.[22]

For the strong undulator it can readily be shown that the interference condition becomes

$$\lambda = \left(\frac{\lambda_u}{2\gamma^2}\right)\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right) \quad (6)$$

where

$$K = \gamma\delta = \frac{eB_0\lambda_u}{2\pi m_0 c} = 0.934\,B_0\lambda_u \quad (6b)$$

B_o is the peak value of the magnetic field (measured in teslas), the wavelengths λ and λ_u are measured in cm, and δ is the maximum angle of deflection with respect to the average beam direction. The angle δ is also given approximately by v_{xmax}/v (see box).

Harmonics of this wavelength are important for the strong undulator. Their intensity depends on the value of K, the undulator parameter, as explained in the next section. For values of K near 1 the power radiated into the fundamental is a maximum. For values of K between 1 and 2, harmonics become quite strong. As K gets even larger many more harmonics appear and their spacing decreases, so that the spectrum approaches a smooth continuum-the spectrum normally associated with a bending magnet or wiggler. The value of K is the real distinction between a wiggler and undulator, because a multi-period wiggler could also be operated as an undulator by reducing the field until K becomes small enough for the interference peaks to become prominent. Normally a wiggler is operated at high magnetic field to extend the spectrum. For example, the Frascati wiggler[2] has a peak field of 19 kG and a period of 65.4 cm for a K value of 116. It has also been operated at 0.16 kG to produce a K value of about 1. Undulator radiation was observed at visible wavelengths with 0.69 GeV electrons under these conditions.

Undulator brightness

The brightness function, that is, the number of photons per second per square milliradian in a 1% bandwidth, is given by[11,14]

$$\frac{\frac{dn}{dt}}{d\Omega\frac{d\nu}{\nu}} = 8.7x10^{14}\, IE^2\, \frac{N}{n\Delta_0} F_n\left(K\right) \quad (7)$$

where N is the number of undulator periods, n is the harmonic number, Δ_0 is the fractional bandwidth of the peak and $F_n(K)$ is a dimensionless function with a peak value of about 0.45. Tables and graphs of $F_n(K)$ are given by several authors.[11,13] For K less than 1, $F_n(K)$ decreases rapidly for higher harmonics. For larger values of K the amplitude of harmonics stays quite strong. The radiation is contained within an opening angle of about $1/\gamma$, which is usually much less than 1 mrad.

Several factors determine the width of the peak. For a detector with high spatial resolution, the most important contributions are due to the finite number of periods (which contributes $\Delta\lambda/\lambda_n \approx 1/nN$) and the size and divergence angle in both transverse planes of the electron-beam source. The finite beam size and divergence make it possible for a point detector situated on the average beam axis to receive photons emitted off axis by some electrons. Photons emitted at an angle θ relative to the electron direction are shifted to longer wavelengths according to equation 5, producing a peak which is sharper on the high-energy side. The properties of the electron beam contribute substantially to the widths of the peaks shown in figures 7 and 9.

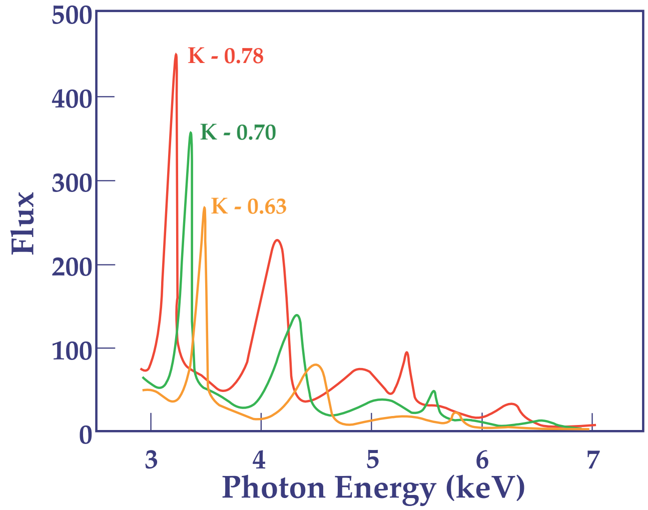

Figure 9 — Higher-harmonic spectrum produced by 3-GeV electrons in the LBL-SSRL 3D-period permanent-magnet undulator. Slits were used to define a very small angular acceptance (18 x 10^{-6} radians horizontal, 8.8 x 10^{-6} radians vertical)

To achieve minimum width, and hence maximum brightness, one needs low-emittance electron beams. This is a primary motivation for reducing the emittance of existing rings and for designing new rings with low emittance. For the peak width to be determined primarily by the number of undulator periods, the transverse beam size should be less than $\gamma^{-1}N^{-1/2}$, where L is the distance from the undulator to the detector; the electron beam divergence angles should also be less than $\gamma^{-1}N^{-1/2}$.

When the electron beam size and divergence satisfy these conditions the fractional bandwidth is a minimum, given by $\Delta\lambda/\lambda_n \approx 1/nN$. The brightness function of equation 7 is then proportional to N^2. A normal ring bending magnet is approximately an undulator with $N = 1$; the undulator brightness is thus about N^2 times the brightness of a bending magnet with a magnetic field equal to the peak undulator field and with the same electron energy and current. Thus, with N in the range 30-100 we would have a brightness enhancement by three to four orders of magnitude. In the other extreme, if no interference peaks were present the undulator would be an N-period wiggler with a brightness $2N$ times larger than the bending magnet at the same field. Thus, the brightness enhancement is in the range $2N$ to N^2 depending on the emittance of the electron beam.

The concentration of undulator radiation into one or a few peaks and the reduction in the experimentally unused continuum radiation means that heating of beamline components is minimized. In other words, an undulator produces less heating per useful photon than do wigglers or bending magnets. This feature is likely to become increasingly important as thermal effects become more severe on bending-magnet and wiggler lines with increasing stored current and more powerful wigglers. In some cases the undulator peaks are narrow enough to be used directly in an experiment without the need for a monochromator. This could result in an increase in usable flux of about a factor of 100 beyond the increase due to the higher intensity produced by the undulator. For example, vuv monochromators often have a transmission of about 1%. Although higher transmission is available in x-ray monochromators, their bandwidth is often very small ($\Delta\lambda/\lambda$ around 10^{-4}). Thus for vuv and x-ray experiments that can utilize the full width of undulator peaks (1-10%) a large increase in usable flux is available. For example, the study of Auger emission does not require highly monochromatic radiation. A band of radiation several percent wide situated above the threshold for the process is ideal for such work. Similarly x-ray microscopy with zone plates can be performed with a bandwidth of about 5%.

Effects on storage-ring operation

In principle all of the effects discussed earlier for wigglers are also present for undulators. However, because undulator magnetic fields are weaker and the deflection angles are smaller, the net effect on the beam is expected to be even less than it is for wigglers. For undulators in the beam vacuum chamber, one must take care to minimize the interaction of the stored beam with the parts of the undulator, which are no longer separated from the beam by the vacuum chamber walls. Such interactions (called parasitic or higher-mode losses) can cause severe heating of the undulator and also beam instabilities.

Undulator technology

Undulators have been built using electromagnets, superconducting magnets and permanent magnets. Most produce alternating transverse fields. Some produce helical fields[8] — that is a constant transverse field with a direction that rotates with the period of the magnet. For the helical geometry, $K^2/2$ in equation 6 must be replaced by K^2. Although the helical undulator has elegant symmetry properties, and indeed produces more power than a transverse undulator at the same peak field (because $<B^2>$ in equation 2 is larger), its use in a storage ring poses difficulties because its circular aperture is a poor match to the different vertical and horizontal beam requirements of most storage rings. A helical undulator produces circularly polarized radiation.

Most undulators are designed so that K can reach a value of at least unity. This maximizes the radiated power in the fundamental. Higher values of K increase the strength of harmonics, as discussed above. Frequently it is desirable to reach certain photon energies with the undulator in a particular storage ring, that is, when the maximum electron energy is fixed. This may be accomplished in two ways:

- Keeping K near 1, one can choose a value of λ_u such that the fundamental has the desired wavelength according to equation 6.
- One can choose higher values of K (for example, K between 1 and 2). Although this decreases the photon energy at which the fundamental occurs, higher photon energy may be reached using higher harmonics.

The first approach is generally preferable because less power is radiated at unwanted photon energies. However, it may not be possible to reach desired wavelength in this way. The second approach usually yields higher photon energies. In both cases higher photon energy is reached with smaller values of λ_u. Hence, in general, one prefers undulators having a short period. Also, the shorter the period of undulator, the more periods can be fit into the usually fixed available length resulting in sharper and higher peaks in the spectrum. However, a shorter period reduces the maximum field, B_o and hence also K, that can be obtained with a fixed magnet gap. The minimum magnet gap is determined by the beam aperture requirements. Thus the starting point in designing an undulator is the beam aperture requirement.

In an undulator based on an electromagnet, the required current density in the coils increases as the period of the undulator decreases. A limiting current density is reached beyond which normal conducting coils cannot be cooled. Higher current densities can be reached with superconducting coils, which thus permit shorter undulator periods. A short-period superconducting undulator[12,14] has been operated at Orsay, France, with ACO, the 540- MeV storage ring there. The magnet has 23 periods each 4 cm long, a 2.3 cm pole gap and a horizontal magnetic field with a peak value of 5 kG on the orbit (K_{max} = 1.9).

The magnet coils and pole pieces of the Orsay undulator are outside a vacuum chamber that has an inside aperture of only 1.2 cm. Since this aperture is insufficient for injection of the beam, the vacuum chamber is in the shape of an inverted "T", with a larger horizontal aperture on the bottom. This larger-aperture part is used during injection, after which the entire assembly is lowered and the stored beam then occupies the part with the 1.2 cm aperture. The peak value of the magnetic field at the pole tips is about 8 kG. This is only a modest field, yet it is unattainable with conventional electromagnets because the short period leaves insufficient space for conventional coils.

When the ACO ring is operated at about 240 Me V this undulator produces a beautiful spectrum of visible light, as shown in figure 1.

Permanent magnets can also produce the required fields with the desired short periods. In particular, rare-earth-cobalt materials (such as $SmCo_5$), have the required strength as well as other unusual properties that make it ideally suited to undulators.[15] The materials are highly anisotropic with an "easy" axis, along which the *BH* curve is a straight line in the first and second and part of the third quadrants, with a slope close to unity, and a remanent field (the intersection of that line with the *B* axis) of 8 to 11 kG, depending on the material. In the direction perpendicular to the easy axis the *B-H* curve is a straight line through the origin with a slope that is also close to unity.

As a result of these properties the material behaves magnetically very nearly like a vacuum with an impressed current. This makes it straightforward to predict analytically the field that will result from almost any configuration of blocks. Some ferrite materials have similar properties, but with a lower remanent field. Other materials, such as the various Alnicos, have high remanent field, but a very nonlinear *B-H* curve with a very low value of the coercive field, H_c, which makes these materials unsuitable for undulators.

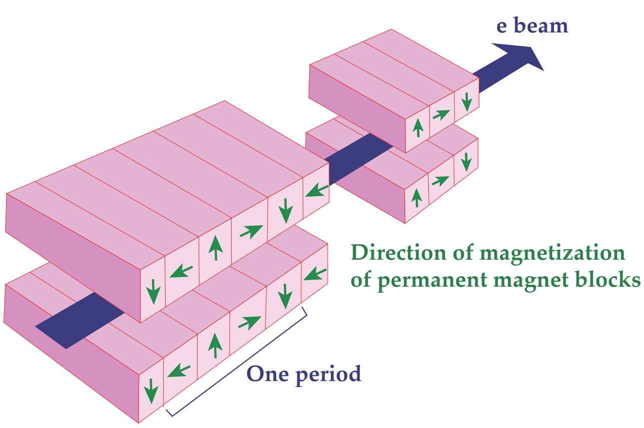

Figure 8 — Permanent-magnet undulator with vertical field made from blocks of rare earth-cobalt ($SmCo_5$) material. The beam is shown traveling in a straight line because the amplitude of oscillation is very small (typically 0.02 nm) compared to the beam width.

LBL-SSRL undulator. A 30-period $SmCo_5$ permanent-magnet undulator[15] has been built at LBL for use at SSRL. The concept, originally proposed by Klaus Halbach of LBL, is shown in figure 8. The field on the axis of the undulator is very close to a sinusoid with wavelength λ_u and amplitude B_o given by[15]

$$B_0 = eB_r e^{\frac{-\pi g}{\lambda_u}} \frac{\sin\left(\frac{\pi}{M}\right)}{\left(\frac{\pi}{M}\right)} \left[1 - e^{\frac{-2\pi h}{\lambda_u}}\right] \quad (8)$$

where B_r is the remanent field, M is the number of blocks per undulator period λ_u, h is the height of the blocks and g is the full gap height of the magnet. For the LBL-SSRL undulator B_r = 8 kG, M = 4, λ_u = 6.1 cm, $h = \lambda_u/4$ and g is remotely variable from 2.7-6 cm. The peak field on the orbit varies from about 2.8 to 0.5 kG and a value of K that varies from 1.4 to 0.3. The minimum gap is set by the outside dimension of the vacuum chamber which is 2.9 cm. Calculated spectra up to 4 keV for this undulator are shown in figure 7. Measurements of the much weaker but more easily observed higher harmonics from 3-7 keV.

The undulator shares a location in SPEAR that is also used by a wiggler magnet; it also shares the wiggler beam line. This approach considerably reduces the time and expense necessary to utilize the undulator. Anticipating this shared use, the wiggler magnet was designed to permit it to be readily split along the median plane and the two halves separated to allow insertion of the C-shaped undulator. Figure 4 is a photograph of the wiggler magnet (shown split and the halves separated) and the undulator (shown ready to be pushed into place).

The first measurements of the radiation produced by this undulator were made in December 1980. An example of the higher harmonic part of the spectrum is shown in figure 9. The measurements were made with a 12-cm long air ionization chamber and a high resolution ($\Delta\lambda/\lambda \approx 10^{-4}$) two-crystal monochromator operating in a He system separated from the storage-ring high vacuum by 533 microns of beryllium. Corrections have been made for attenuation in the beryllium and other absorbing material in the line. The attenuation of the Be is very large below 3 keV so the more intense first and second harmonics were observed by removing the Be windows and measuring the energy distribution of photoelectrons from a polycrystalline graphite sample in a high-vacuum system.

Depending on the undulator gap and the energy of SPEAR, the fundamental peak can be tuned from a few hundred eV to about 2 keV. At the smallest gap setting, the brightness of this peak exceeds that of the the bending magnets by about two order of magnitude. The first few harmonics are also quite strong and should prove useful.

Future developments

Wigglers. The operations flexibility and added capabilities that wiggler magnets have provided at SSRL can also be used to great advantage in new storage rings, particularly when they are designed from the start with wigglers in mind. The "all-wiggler machine"[3] being developed for the European Synchrotron Radiation Facility has already been mentioned. Another example is the use of wigglers to extend the spectral range and intensity in smaller machines such as the Hefei Synchrotron Radiation Facility now being designed in the Peoples Republic of China. The machine being considered would have an electron energy of 800 MeV and a bending-magnet field of 10 kG, resulting in a critical energy of 0.43 keV (see equation 1). This ring could supply many bending-magnet beam lines with high flux up to photon energies of about 2 keV. This makes it a good source for a large number of experimental studies requiring photons in that spectra range, such as surface studies using vuv photoemission and soft x-ray absorption by surfaces (SEXAFS), soft x-ray microscopy and lithography, and so on. (See the article by Dean Eastman and Franz Himpsel on page 64). A 50 kG super; conducting wiggler (which could be added in the future) would produce a beam with a critical energy of 2 keV and high flux up to about 10 keV (see figure 6). This opens up the important x-ray diffraction applications and extends x-ray absorption studies to include the important *K* edges up to copper (see the article by Cully Sparks on page 40). Thus, it is possible for a country embarking on its first synchrotron radiation project to obtain an impressive spectral range and broad scientific capability with a single cost-effective ring.

Undulators. Extension of the energy range of undulator radiation beyond the 2 keV now used will have a large scientific payoff for many types of experimental studies that require higher photon energy and the high brightness of undulators. An immediate goal is to reach 8-10 keV. Such a radiation source would enhance the prospects fora wide variety of x-ray experiments such as high-resolution inelastic x-ray scattering, time resolved small-angle scattering and nuclear Bragg diffraction. One way to accomplish this is with stronger undulators having shorter periods and smaller gaps. For example, a permanent-magnet undulator is being planned for SPEAR with a period of about 3 cm, a gap of about 1 cm and a peak magnetic field of about 6 kG (from equation 8), corresponding to a *K* value of 1. 7. With an electron energy of 3.5 GeV the fundamental would occur at 1.6 keV (equation 6) and there would be a strong fifth harmonic at 8 keV. To obtain the smallest gap, the undulator would be placed inside a vacuum tank. The gap would be opened to provide the larger aperture required for injection, after which it would be closed down to the smaller aperture permissible for the stored beam.

Further extension to higher proton energies is most readily accomplished with the higher electron energies available now at several other storage rings such as those in Hamburg, Germany (DORIS at 5 GeV, PETRA at 18 GeV), Novosibirsk USSR (VEPP-4 at 4-7 GeV), Cornell (CESR at 6-8 Ge V) and Stanford (PEP at 15-18 GeV). Some of these machines have flexible magnet optics which permits them to operate with lower emittance, enhancing the potential brightness of an undulator.

At an electron energy of 18 GeV, a narrow-gap, short-period undulator could produce a fundamental peak at 45 keV with strong harmonics well above 100 keV. A. Hofmann[23] has considered undulators on LEP, the 80-130 GeV colliding-beam ring planned at CERN. At these electron energies an undulator could produce photons in the 10 MeV region.

Technology of the future

While it is unlikely that the design of conventionally powered wigglers will change significantly in the future, we will probably see some changes in the other two wiggler technologies. We expect improvements in superconducting wigglers that will give, by today's standards, spectacular performance. And we also expect to see some permanent magnet wigglers (probably also made from steel) that will give fields around 15 kG, and which will be easy and inexpensive to build and operate.

Because undulators should have a short spatial period (combined with $K \approx 1$), conventionally powered undulators are not likely to be used frequently in the future. Changes in the design of permanent-magnet undulators will probably improve their performance and reduce their cost. Since they are so easy to operate and so much less expensive to build than superconducting-undulators, we expect that superconducting undulators will be used only under special circumstances.

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References

[1] Herman Winick, T. Knight; editors, Wiggler Magnets, SSRL Report 77/05 (From SSRL, SLAC Bin 69, Stanford, Cal. 94305).

[2] A. Luccio, A. Reales, S, Stipcich; editors, Proceedings Wiggler Conference June 1978. From Laboratori Nazionale de Frascati casella Postale 13, I-00044 Frascati (Roma)

[3] D. Thompson, R. Coisson, J. Le Duff, F. Dupont, M. Erickson, Albert Hofmann, D. Husmann, G. Mulhaupt, M. Poole, M. Renard, M. Sommer, V. Suller, S. Tazzari, F. Wang; "The 'all Wiggler' Synchrotron Radiation Source," IEEE Transactions on Nuclear Science, Volume 28, Issue 3, June 1981, doi: 10.1109/TNS.1981.4332037

[4] V. L. Ginzberg; Izv. Akad. Nauk. SSSR, Ser. Fiz., 11, 165 (1947).

[5] H. Motz, W. Thon, R. N. Whitehurst; "Experiments on Radiation by Fast Electron Beams," Journal Applied Physics 24, Issue 7, pp. 826 (1953), doi: 10.1063/1.1721389

[6] D. F. Alferov, Yu. A. Bashmakov, K. A. Belovintsev, E. G. Bessonov, P. A. Cerenkov; Particle Accelerators 9, 223 (1979).

[7] A. N. Didenko, A. V. Kozhevnikov, A. F. Medvedev, M. M. Nikitin; V. Ya. Epp. Zh. Eksp. Teor. Fiz., Volume 76, pp. 1919 (1979); or Soviet Physics — Journal of Experimental and Theoretical Physics, Volume 49, pp. 973 (1979).

[8] Brian M. Kincaid; "A Short-Period Helical Wiggler as an Improved Source of Synchrotron Radiation," Journal of Applied Physics, Volume 48, Issue 7, pp. 2684 (1977), doi: 10.1063/1.324138

[9] Albert Hofmann; "Quasi-monochromatic synchrotron radiation from undulators," Nuclear Instrument Methods, Volume 152, Issue 1, pp. 17 (1978), doi: 10.1016/0029-554X(78)90231-8

[10] R. Coisson; "Angular Spectral Distribution and Polarization of Synchrotron Radiation From a Short Magnet," Physical Review A, Volume 20, Issue 2, pp. 524-528 (1979), doi: 10.1103/PhysRevA.20.524

[11] D. J. Thompson, M. W. Poole; editors, European Synchrotron Radiation Facility Supplement II; pages 52, also page 17; European Science Foundation, Strasbourg, (1979).

[12] J. Spencer, Herman Winick; in Synchrotron Radiation Research, Herman Winick, S. Doniach; editors, Plenum, New York 1980, page 663.

[13] S. Krinsky; "An undulator for the 700 MeV VUV-ring of the National Synchrotron Light Source," Nuclear Instrument Methods, Volume 172, Issue 1-2, pp. 73 (1980), doi: 10.1016/0029-554X(80)90611-4

[14] Y. Farge; "Emission of Photons by Undulators," Applied Optics, Volume 19, Issue 23, 4021-4026 (1980), doi: 10.1364/AO.19.004021

[15] Klaus Halbach, J. Chin, E. Hoyer, Herman Winick, R. Cronin, J. Yang, Y. Zambre; "A Permanent Magnet Undulator for SPEAR," IEEE Transactions on Nuclear Science, Volume 28, Issue 3, June 1981, pp. 3136-3138, doi: 10.1109/TNS.1981.4332031; see also Klaus Halbach, Nuclear Instrument Methods (to be published).

[16] Albert Hofman, R. Little, J. M. Paterson, K. W. Robinson, G. A. Voss, Herman Winick; Proceedings 6th International Conference on High Energy Accelerators CEAL-2000 September 1967, pp. 123.

[17] J. R. Rees; "Positron-Electron Project PEP," IEEE Transactions on Nuclear Science, Volume 24, Issue 3, 1836-1841 (1977), doi: 10.1109/TNS.1977.4329104

[18] M. Berndt, W. Brunk, R. Cronin, D. Jenson, A. King, J. Spencer, T. Taylor, Herman Winick; "Initial Operation of SSRL Wiggler in SPEAR," IEEE Transactions Nuclear Science, Volume 26, Issue 3, 3812-3815 (1979); also Herman Winick, J. Spencer; "Wiggler magnets at SSRL-present experience and future plans," Nuclear Instrument Methods 172, Issue 1-2, pp. 45 (1980), doi: 10.1016/0029-554X(80)90606-0

[19] L. M. Barkov, V. B. Baryshev, G. N. Kulipanov, N. A. Mezentsev, V. G. Pindyurin, A. N. Skrinsky, V. M. Khorev; "A proposal to install a superconducting wiggler magnet on the storage ring VEPP-3 for generation of the synchrotron radiation," Nuclear Instrument Methods, Volume 152, Issue 1, pp. 23 (1978), doi: 10.1016/0029-554X(78)90232-X

[20] Herman Winick, R. Helm; "Standard wiggler magnet," Nuclear Instrument Methods 152, Issue 1, pp. 9 (1978), doi: 10.1016/0029-554X(78)90230-6

[21] V. Suller; "The interaction of wigglers and undulators with stored electron beams," Nuclear Instrument Methods, Volume 172, Issue 1-2, pp. 39 (1980), doi: 10.1016/0029-554X(80)90605-9

[22] H. Kitamura; Japan Journal Applied Physics 19, L185 (1980).

[23] Albert Hofmann; Physics Reports 64, 253 (1980).