

Wilson Renormalization Group Equations for the Critical Dynamics of Chiral Symmetry

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The critical dynamics of the chiral symmetry breaking induced by a gauge interaction is examined in the Wilson renormalization group framework in comparison with the Schwinger-Dyson approach. We derive the beta functions for the four-fermi couplings in the sharp cutoff renormalization group scheme, from which the critical couplings and the anomalous dimensions of the fermion composite operators near criticality are immediately obtained. It is also shown that the beta functions lead to the same critical behavior found by solving the so-called ladder Schwinger-Dyson equation, if we restrict the radiative corrections to a certain limited type.

§1. Introduction

Chiral symmetry breaking phenomena represent one of the key issues in understanding the non-perturbative dynamics of gauge theories. The analytical study of this problem has been initiated by the Nambu-Jona-Lasinio (NJL) model,¹⁾ which was introduced as the effective theory with four-fermi interactions. For gauge theories, in particular the Schwinger-Dyson (SD) equations in the ladder approximation with Landau gauge^{2), 3)} have been intensively studied and applied not only to QCD but also to the various models of dynamical electroweak symmetry breaking.^{4), 5)} In QCD, the ladder SD equation with the running gauge coupling constant, the improved ladder,⁶⁾ was found to give good results even quantitatively.⁷⁾ However, the ladder SD equations are known to suffer from some serious problems, in particular the strong gauge dependence⁸⁾ and the difficulty to proceed beyond the ladder approximation.⁹⁾

The Wilson renormalization group (RG)¹⁰⁾ has been known to offer a powerful method to analyze critical phenomena and has been applied to various dynamical problems, mainly in statistical mechanics. The so-called exact RG equations,^{10) - 12)} which are the concrete formulation of the Wilson RG in momentum space, have recently been applied to numerical study of non-perturbative dynamics in field theories. Application to the QCD dynamics has also been considered in this framework.¹³⁾ The advantageous features of this method, compared with the SD approach, are that the critical behavior is analyzed directly from the RG equations, and that it pro-

duces a systematic improvement of the approximation obtained from the derivative expansion and truncation of the Wilsonian effective action.^{14),15)} Interestingly, very recently the RG method has been applied to the fermi liquid theory of superconductivity,¹⁶⁾ which the NJL model was considered in analogy with. Also, it should be noted that the fermi liquid theory of high density QCD was studied with the RG analyses.¹⁷⁾

In this paper we examine the chiral critical dynamics in gauge theories by using the exact RG equations, especially focusing on the comparison with the SD approach. There are several formulations of exact RG. Here we simply employ the Wegner-Houghton RG equations,¹¹⁾ which are derived with a sharp momentum cutoff, in the so-called local potential approximation for our present purpose. Analyses with the exact RG equations with smooth cutoff may be performed as well.¹⁸⁾ It is found that the critical behavior is determined from the beta functions of the effective four-fermi couplings induced by gauge interactions with a remarkably simple calculation. The phase boundary and also the anomalous dimensions of the composite operators of fermions near the criticality will be evaluated. Our approximation scheme adopted here is even better than the ladder approximation used with the SD equations for critical behavior. Actually, as seen below, if we make further approximations so as to pick up only a few types of the radiative corrections, then the critical behavior is reduced to be identical to that obtained by solving the ladder SD equation.

§2. Scheme of the RG equations

The ladder SD equation for the fermion mass function is given in the form of an integral equation, where the momentum integration is carried out with a sharp cutoff. In order to see the direct relation between the critical dynamics obtained with the two methods; the SD equation and the RG equation, we consider the exact RG equation defined with a sharp cutoff in this paper. There are several formalisms of exact RG.^{10),12)} Here we adopt the so-called Wegner-Houghton RGE¹¹⁾ derived as follows.

If we divide the freedom of the quantum field $\phi(p)$ into higher frequency modes with $|p| > \Lambda$ and lower frequency modes with $|p| < \Lambda$ by introducing the cutoff scale Λ in the Euclidean formalism, then the Wilsonian effective action at this scale, $S_{\text{eff}}[\phi; \Lambda]$, may be defined by integrating out the higher frequency modes in the partition function:

$$Z = \int \prod_{|p| < \Lambda_0} d\phi(p) e^{-S_0[\phi; \Lambda_0]} = \int \prod_{|p| < \Lambda} d\phi(p) e^{-S_{\text{eff}}[\phi; \Lambda]}. \quad (1)$$

Here S_0 denotes the bare action with the bare cutoff Λ_0 . This effective action contains the general operators invariant under the original symmetries in the bare action, for example the chiral symmetry of our present concern.

The Wegner-Houghton RGE determines the variation of the Wilsonian effective action under the infinitesimal change of the cutoff Λ . For example, the RGE for the

D -dimensional scalar field theory is found to be given exactly as

$$\begin{aligned} \frac{\partial S_{\text{eff}}}{\partial t} = & DS_{\text{eff}} + \int \frac{d^D p}{(2\pi)^D} \phi_p \left(\frac{2-D-\eta}{2} - p^\mu \frac{\partial'}{\partial p^\mu} \right) \frac{\delta S_{\text{eff}}}{\delta \phi_p} \\ & - \frac{1}{2} \int \frac{d^D p}{(2\pi)^D} \delta(|p|-1) \left\{ \frac{\delta S_{\text{eff}}}{\delta \phi_p} \left(\frac{\delta^2 S_{\text{eff}}}{\delta \phi_p \delta \phi_{-p}} \right)^{-1} \frac{\delta S_{\text{eff}}}{\delta \phi_{-p}} - \text{tr} \ln \left(\frac{\delta^2 S_{\text{eff}}}{\delta \phi_p \delta \phi_{-p}} \right) \right\}, \end{aligned} \quad (2)$$

where $t = \ln(\Lambda_0/\Lambda)$ is introduced as a scale parameter. The 1st line of the RGE represents the canonical scaling of the effective action, while the 2nd line comes from the radiative corrections, which correspond to the tree and the 1-loop Feynman diagrams including only the propagators with the momentum of the scale Λ .

In practical analyses it is necessary to simplify this RGE by some approximation. Here we examine the RGE in the so-called local potential approximation.^{11), 19)} In this approximation the radiative corrections to any operators containing derivatives are ignored in the RGE (2). Therefore, only the potential part of S_{eff} , V_{eff} , may be evolved with the shift of Λ . It should be noted that the wave function renormalization is ignored in this scheme. The RGE for the scalar theory is given explicitly by

$$\frac{\partial V_{\text{eff}}}{\partial t} = DV_{\text{eff}} - \frac{D-2}{2} \phi \frac{\partial V_{\text{eff}}}{\partial \phi} + \frac{A_D}{2} \ln \left(1 + \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2} \right), \quad (3)$$

where $A_D = 2/(4\pi)^{D/2} \Gamma(D/2)$ is the factor from the momentum integration. This equation offers us a set of the infinitely many beta functions for the general couplings appearing in V_{eff} . However, it should be noted here that each beta function may be evaluated through just one loop corrections with the general effective interactions. The non-perturbative nature of the RGE is assumed to be maintained by solving infinitely many coupled renormalization equations. Actually, it is known that this approximated RGE is quite effective in the case of scalar theories.^{14), 15)} The generalization of this RGE to include fermions has also been studied in relation with the triviality-stability bound for the Higgs boson mass in the Standard Model.²⁰⁾

Now let us consider application of this formulation to the massless fermions coupled by the gauge interaction. For example we may take the action of the massless QED as the bare action

$$S_0 = \int d^4 x \left\{ \bar{\psi} \not{\partial} \psi + e \bar{\psi} \not{A} \psi + \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2\alpha} (\partial_\mu A_\mu)^2 \right\}, \quad (4)$$

where α is the gauge parameter. As is well known, the gauge invariance is no longer maintained, once the momentum cutoff is made. Therefore the generic gauge non-invariant operators are generated in the Wilsonian effective action. Then we must encounter the rather complicated problem of picking up the special RG flows corresponding to the gauge invariant theories in the infinite-dimensional coupling space. Recently it has been discussed how to deal with the gauge theories in the framework of the Wilson RG by using the modified Slavnov-Taylor identities.²¹⁾

Here, however, we simply ignore the corrections to the operators including the gauge fields, as well as imposing the local potential approximation, as the first step towards the analysis of the chiral symmetry breaking phenomena. Then we may avoid the intriguing problem of the gauge invariance, since no gauge non-invariant operator appears in the effective action. This approximation is indeed so rough as to make the beta function of the gauge coupling vanish identically. Therefore it cannot be supposed to give any picture of the real dynamics of the gauge theories. However, on the other hand, the so-called ladder approximation used in the SD approach also totally ignores the vertex corrections as well as the corrections to the gauge kinetic functions. Therefore it would be meaningful to examine the RGE in this scheme in comparison with the SDEs in the ladder approximation. The effect of the running gauge coupling will be discussed in §4. Although this approximation scheme is rather crude, it is seen that the chiral critical behavior may be described well. Actually, it is found that this approximation is even better than the ladder approximation applied to the SD equations.

The Wilsonian effective action to be solved with the RGE in this scheme is now reduced to the form

$$S_{\text{eff}}[\psi, \bar{\psi}; \Lambda] = \int d^4x \left\{ \bar{\psi} \not{\partial} \psi + V_{\text{eff}}(\psi, \bar{\psi}; \Lambda) + e \bar{\psi} \not{A} \psi + \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2\alpha} (\partial_\mu A_\mu)^2 \right\}, \quad (5)$$

where $V_{\text{eff}}(\psi, \bar{\psi})$ denotes the general potential composed of the chiral symmetric multi-fermion operators. These multi-fermion operators are induced by exchange of the “photon” with higher momentum. The so-called gauged NJL model is often examined in the SD approach, and the phase diagram in the two-parameter space of the gauge coupling and the four-fermi coupling has been examined.³⁾ However, from the RG point of view, this coupling space should be regarded as a subspace of the infinite-dimensional coupling space of the Wilsonian effective action. It should be noted also that these multi-fermi operators are irrelevant or non-renormalizable, and therefore, are not considered in the perturbative QED. However, they cannot be simply discarded in the strong coupling region. It will be seen in the next section that the four-fermi coupling turns out to be relevant near the criticality and plays a crucial role in the critical dynamics of the chiral symmetry breaking.

§3. Critical dynamics of the chiral symmetry

In this section we examine explicitly the Wegner-Houghton RGE in the approximation discussed in the previous section. The form of the effective potential V_{eff} written in terms of the fermions is found to be restricted to a polynomial composed of the following parity and chiral invariant operators, which are mutually independent:

$$\begin{aligned} \mathcal{O}_1 &= (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 = -\frac{1}{2} \left\{ (\bar{\psi}\gamma_\mu\psi)^2 - (\bar{\psi}\gamma_5\gamma_\mu\psi)^2 \right\}, \\ \mathcal{O}_2 &= (\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_5\gamma_\mu\psi)^2, \\ \mathcal{O}_3 &= \left\{ (\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_5\gamma_\mu\psi) \right\}^2. \end{aligned} \quad (6)$$

Therefore the 4-fermi part of the effective potential may be written as

$$V_{\text{eff}}(\psi, \bar{\psi}; t) = -\frac{G_S(t)}{2\Lambda^2} \{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\} + \frac{G_V(t)}{2\Lambda^2} \{(\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_5\gamma_\mu\psi)^2\}. \tag{7}$$

Hereafter let us call G_S the “scalar four-fermi coupling” and G_V the “vector four-fermi coupling”.*)

Now let us evaluate the radiative corrections to these four-fermi operators, since it will be sufficient to examine only these couplings for the purpose of understanding the critical dynamics. By using the propagator of the gauge field in the Landau gauge ($\alpha = 0$), the RG equations for the four-fermi couplings are found to be

$$\begin{aligned} \frac{d}{dt}g_S &= -2g_S + \frac{3}{2}g_S^2 + 4g_Sg_V + g_S\lambda + \frac{1}{6}\lambda^2, \\ \frac{d}{dt}g_V &= -2g_V + \frac{1}{4}g_S^2 - g_V\lambda - \frac{1}{12}\lambda^2, \end{aligned} \tag{8}$$

where we have introduced the rescaled couplings, $g_S = G_S/(4\pi^2)$, $g_V = G_V/(4\pi^2)$, $\lambda = 3e^2/(4\pi^2)$. Here we should note that no multi-fermi couplings more than four-fermi take part in the radiative corrections for the four-fermi couplings, owing to the 1-loop nature of the RGE. Therefore we may obtain the RG flows within the 2-dimensional coupling space (or 3-dimensional, if the gauge coupling is also taken into account) irrespective of other couplings.

In Fig. 1 the Feynman diagrams representing the one-loop corrections to the four-fermi couplings are shown. Let us call the corrections given by the diagrams in the dashed box in Fig. 1 the “ladder” type, and the others the “non-ladder” type hereafter. If we approximate the RGE by restricting to the “ladder type” correction, then the beta function for the scalar four-fermi coupling is found to be given by

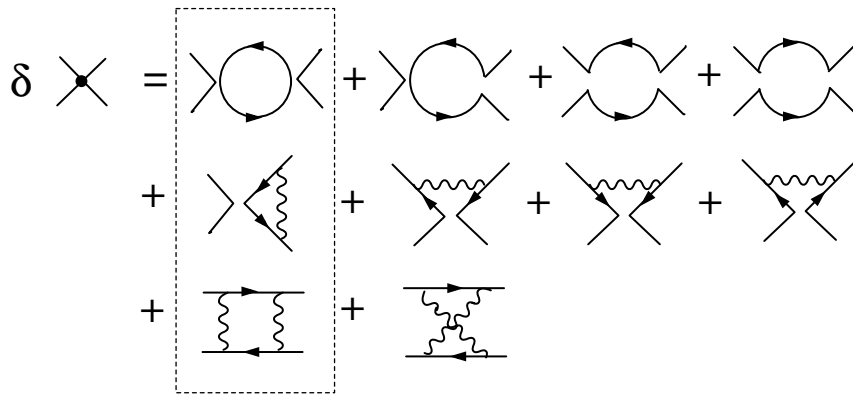


Fig. 1. Feynman diagrams of the radiative corrections to the four-fermi couplings g_S and g_V considered in the RGE (8). The diagrams surrounded by the dashed box show the “ladder” type corrections.

*) The sign of the scalar four-fermi coupling introduced here is the conventional one in the literature.

$$\frac{d}{dt}g_S = -2g_S + 2(g_S + \lambda/4)^2, \tag{9}$$

where it is noted that the RGE for the scalar four-fermi decouples from that for the vector four-fermi coupling.

Before examining the full RGEs (8), let us consider the RG flows in the subspace of $\lambda = 0$. The beta functions give the “fixed points” at $(g_S^*, g_V^*) = (0, 0), (1, 1/8), (-4, 2)$. The point $(0, 0)$ is the IR trivial fixed point, and $(1, 1/8)$ is the UV fixed point on the critical surface.*) The RG flows are found as is shown in Fig. 2. It is seen that there are two phases divided by a critical surface. The chiral symmetry is supposed to be spontaneously broken in the upper region in Fig. 2. Then we may realize that the chiral symmetry breaking is caused essentially by the strong scalar four-fermi interaction, not by the vector four-fermi interaction.

It is also easy to evaluate the exponents which are important physical quantities in the critical dynamics. By linearizing the RG equations around the UV “fixed point”, the dimensions of the relevant four-fermi coupling and the irrelevant four-fermi coupling are found to be 2 and $-5/2$, respectively. The relevant four-fermi operator is given by the combination of $\mathcal{O}_{rel} = \mathcal{O}_1 - (1/8)\mathcal{O}_2$. The renormalized trajectory in Fig. 2 is given by the straight line passing through the non-trivial fixed point. Indeed we may deduce from the RGEs (8)

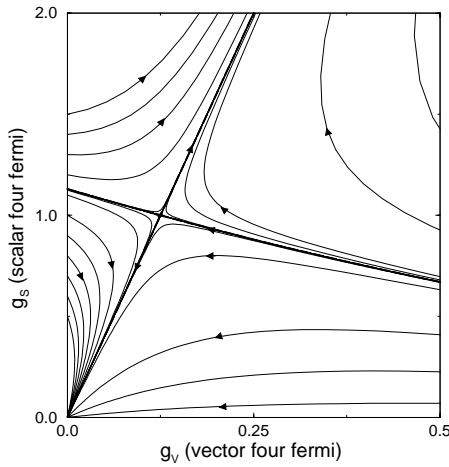


Fig. 2. RG flow diagram of the four-fermi couplings (g_V, g_S) in the $e = 0$ plane. The critical surface and the renormalized trajectory are indicated by the dashed lines.

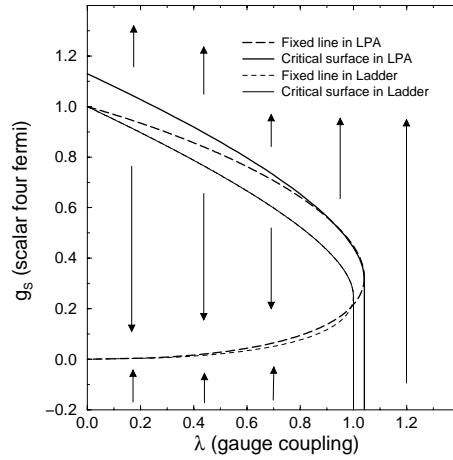


Fig. 3. The fixed lines projected on the (λ, g_S) plane and the critical couplings of the scalar four-fermi obtained from the beta functions given in Eq. (8) and in Eq. (9).

) Strictly speaking, $(g_S^, g_V^*) = (1, 1/8)$ is not a fixed point, since the beta function for the eight-fermi coupling does not vanish there. However, it turns out to be a non-trivial fixed point for space-time dimensions satisfying $2 < d < 4$. We do not consider the point $(-4, 2)$, since it seems to be created by the approximation.

$$\frac{d}{dt}(g_S - 8g_V) = -\left(2 + \frac{1}{2}g_S\right)(g_S - 8g_V), \quad (10)$$

which implies that once the combination of $(g_S - 8g_V)$ is vanishing at a point, then it remains null along the renormalization flow. Therefore the renormalized trajectory is precisely given by the line $g_S = 8g_V$. Namely, the effective four-fermi operator at low energy is just \mathcal{O}_{rel} irrespective to the phases.

Next let us examine the RG equations (9) given by the “ladder type” corrections. For each gauge coupling λ there are the UV fixed point and the IR fixed point, which are found to be

$$g_S^*(\lambda) = \left(1 \pm \sqrt{1 - \lambda}\right)^2 / 4, \quad (11)$$

where $+$ is for the UV fixed point and $-$ is for the IR fixed point. They form a line of fixed points in the (λ, g_S) space, as shown in Fig. 3. The phase boundary is also shown in Fig. 3. The upper region is considered as the chiral symmetry broken phase. The critical gauge coupling is given by

$$\lambda_{\text{cr}} = 1. \quad (12)$$

Indeed this critical surface just coincides with that obtained by solving the SDE in the ladder approximation.^{2),3)}

The anomalous dimension of the four-fermi coupling g_S , $\gamma_G = 2 + \text{dim}[g_S]$ near criticality is immediately deduced from the RGE (9) as

$$\gamma_G = 4g_S^*(\lambda) + \lambda = 2\left(1 + \sqrt{1 - \lambda}\right), \quad (13)$$

which is also found to coincide with the result by the ladder SDE.³⁾ Therefore it is seen that our approximation used to derive the RGEs (8) is certainly better than the ladder approximation. Moreover it is easy to take all the corrections shown by Fig. 1 including the “ladder type” ones in our framework. It should be noted that the sum of corrections of the “ladder” diagram and the “crossed ladder” given in the last line of Fig. 1 is found to be free from gauge parameter dependence. Thus this extension of the approximation beyond the “ladder” is significant to obtain gauge independent results.¹⁸⁾ We would like to stress here that the exact RG equations allow us to examine the critical dynamics with remarkably simple calculation, which is in clear contrast with the situation in the SD approach.

Now we are in a position to go beyond the “ladder” by examining the full RG equations (8). The fixed line $(g_S^*(\lambda), g_V^*(\lambda))$, which is given by the solution of the 3rd order equation in turn, is shown in Fig. 3 and also in Fig. 4 by projection into the (λ, g_S) -plane and (g_V, g_S) -plane, respectively. It is seen that the critical gauge coupling is now found to be slightly larger than the value in the “ladder” approximation, ($\lambda_{\text{cr}} = 1.0409$). In Fig. 4 the critical surface separating the two phases is also indicated by the cross sections at various gauge couplings up to the critical one. The critical surface given in the case of the “ladder” type should be compared with the cross section between the critical surface and the $g_V = 0$ plane, which is found as shown in Fig. 3. It is seen that the phase boundary obtained with

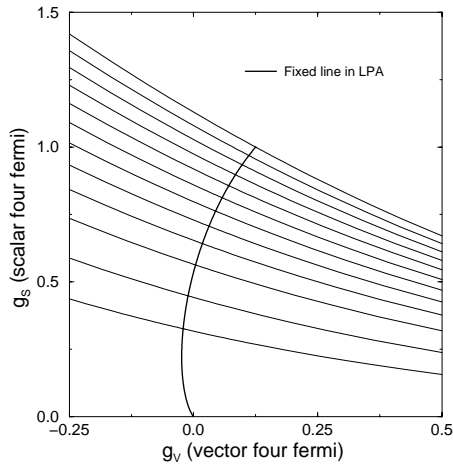


Fig. 4. Cross sections of the critical surface at $\lambda = 0, 0.1, 0.2, \dots, 1.0, \lambda_{cr}$ and the fixed line projected on the plane.

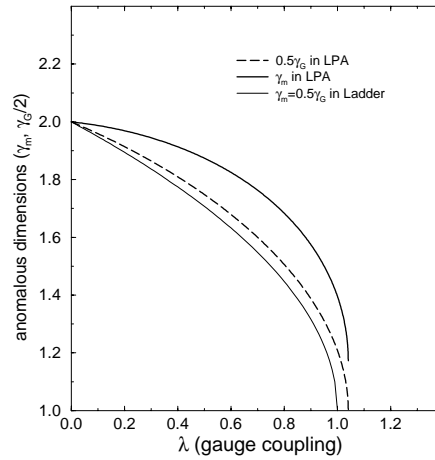


Fig. 5. Anomalous dimensions γ_m and γ_G obtained in our LPA scheme and in the ladder approximation.

our scheme is shifted towards the outside compared with the phase boundary, which has been previously seen in the ladder SD approach.

The exponents at the fixed line are similarly obtained. The exponent or the dimension of the relevant operator, which was found to be 2 in the previous analysis for $\lambda = 0$, decreases as the gauge coupling becomes larger. Then it eventually vanishes at $\lambda = \lambda_{cr}$, which is also seen directly from Eq. (8). In Fig. 5 the anomalous dimension of the relevant four-fermi coupling γ_G is presented in comparison with the “ladder” value given by Eq. (13).

Before ending this section let us mention the anomalous dimension of the fermion mass operator, which we denote γ_m . In order to evaluate it we may incorporate the mass term in the effective action. Then the beta function for the mass m may be derived by one-loop diagrams. It is found to be

$$\frac{d}{dt}m = m + \frac{2m}{1 + m^2}(g_S + \lambda/4). \tag{14}$$

Here it should be noted that the contribution from the “non-ladder”-type corrections vanishes. Therefore the vector four-fermi coupling does not appear in this beta function. The anomalous dimension on the fixed line is simply given by

$$\gamma_m = 2g_S^*(\lambda) + \frac{\lambda}{2}, \tag{15}$$

which is shown in Fig. 5 as well. In the “ladder” case it is seen that the anomalous dimensions satisfy the relation, $\gamma_m = \gamma_G/2$, which has been found also in the analysis of the ladder SDE.³⁾ In our analysis, however, γ_m turns out to be significantly larger than $\gamma_G/2$ and also than γ_m obtained in the ladder SD approach.

§4. RG flow with running gauge coupling

To this point the effect of the renormalization of the gauge coupling has been completely ignored. For this reason the obtained phase diagrams are not in agreement with realistic ones for gauge theories. In the SD approach, the so-called improved ladder approximation,⁶⁾ in which the gauge coupling is simply replaced by the running coupling subject to the perturbative RGE apart from the SD framework, has been often used. However, this prescription cannot be regarded as a systematic improvement of the approximation. On the other hand, in the Wilson RG framework it is possible to include the correction to the gauge coupling naturally by improvement of the previous approximation. This is a clear contrast with the improved ladder approximation.

If we attempt to treat the non-perturbative dynamics with strong gauge interactions faithfully in the Wilson RG framework, we will encounter the difficult problems such as extraction of the gauge invariant theories, development of simple approximation scheme, incorporation of the topological excitations and so on. However, as a primitive approximation, we may evaluate the Wilson beta function of the gauge coupling with the 1st order correction, namely the perturbative one. Then it is enough to solve the RG equations given by (8) in turn coupled with the perturbative RG equation for the gauge coupling λ . In Fig. 6 the RG flows for QED obtained in this manner are shown in the (λ, g_S) -plane. The critical surface separating the spontaneously broken and the unbroken phases is maintained, while the non-trivial fixed points disappear. Note that the point $(\lambda, g_S^*, g_V^*) = (0, 1, 1/8)$ is not a fixed point in 4 dimensions.

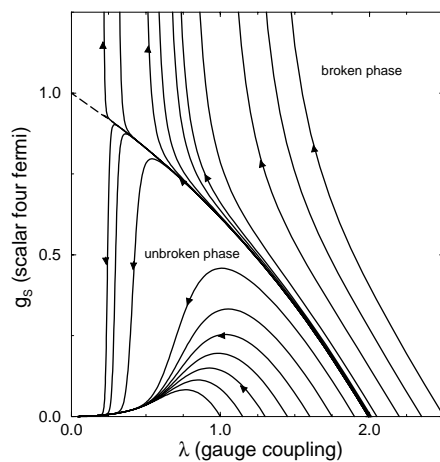


Fig. 6. RG flow diagram for QED. There appears the two phase structure. The upper (lower) region is supposed to be (un)broken phase.

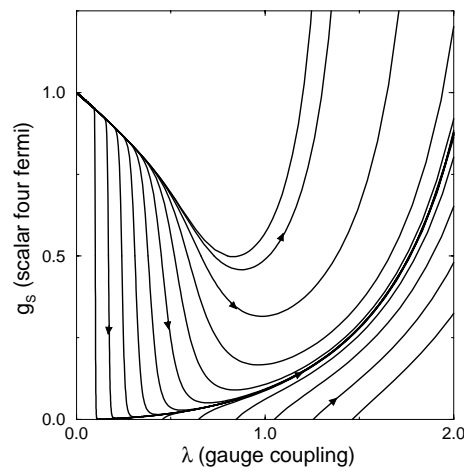


Fig. 7. RG flow diagram for the QCD like gauge theory. There appears no phase boundary.

The RG flows for the QCD-like asymptotically free gauge theory is shown in Fig. 7. These results should be compared with those obtained by solving the SD equations.²²⁾ It is seen that the phase structure is completely swept off. The entire region is considered to be in the broken phase of the chiral symmetry, since the 4-fermi coupling keeps growing in the infrared direction. The effective theories represented by the attracting line coming out from the trivial fixed point correspond to the continuum limit of QCD. Namely, this line gives the so-called renormalized trajectory of QCD. However, other flows of the gauged NJL models, especially starting at the critical point of the NJL model in the UV limit, do not converge on the renormalized trajectory. Therefore it may be supposed that these flows exhibit other renormalized trajectories. If this is the case, the gauged NJL model offers non-perturbatively renormalizable theories different from QCD. Indeed the existence of a non-trivial continuum limit other than QCD, or renormalizability of some kinds of the gauged NJL models, has been previously claimed.^{22),23)} In our framework of the non-perturbative RG, the renormalizability of the gauged NJL model may be shown by examining whether these flows are really the renormalized trajectories or not. Such studies will be reported separately.²⁴⁾

§5. Discussion

In this paper we examined the chiral critical behavior of the gauge theories in the Wilson RG framework. We considered the evolution of the effective potential composed of the chirally invariant multi-fermi operators by the exact RG equation with sharp cutoff in the local potential approximation. The RG flow of the four-fermi couplings was found to determine the phase structure. It is straightforward and remarkably easy to find the critical surface and the anomalous dimensions of the composite operators of fermions in this framework. Moreover, the critical dynamics obtained by solving the ladder SD equations are exactly reproduced by restricting the radiative corrections taken in the beta functions to the “ladder” type. While our RG equations also contain the “non-ladder” type corrections, which are necessary to obtain gauge independent physical results,¹⁸⁾ we have considered only the case of the Landau gauge in this paper.

However, we cannot assert with this analysis that the chiral symmetry is indeed spontaneously broken in the region considered to be the broken phase. In order to clarify this we need to evaluate the order parameters such as the dynamical mass of the fermion, the condensation of the composite operator of fermion, and so on. Evaluation of these order parameters is especially important in QCD, because they are the physical quantities that show the dynamical chiral symmetry breaking. In the Wilson RG picture, however, the spontaneous generation of the fermion mass itself seems to be even non-trivial, since the Wilsonian effective action remains chiral symmetric in evolution and, therefore, there is no room for the mass term to show up. These issues as well as the method to evaluate the order parameters in the Wilson RG framework will be also studied in a separate publication.²⁵⁾

In our analyses the RG equation for the gauge coupling was approximated by the perturbative one. Of course the fully non-perturbative treatment for the RGE is

required to see the dynamics of a strong gauge interaction in the infrared region. It is still an open question whether the Wilson RG approach gives a useful framework in this non-perturbative region. However, as shown in this paper, non-perturbative RG seems to have a good chance to seek for the dynamical chiral symmetry breaking phenomena in gauge theories further by going beyond the level examined so far in the SD approach.

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