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Ward, H. S.; Crawford, R.

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WIND-INDUCED VIBRATIONS AND BUILDING MODES

by

H. S. Ward and R. Crawford

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VIBRATIONS CAUSEES PAR LE VENT ET MODES DE CONSTRUCTION

SOMMAIRE

Les auteurs de cet article exposent succintement les méthodes utilisées pour déterminer les fréquences et les modes des vibrations causées par le vent sur des bâtiments ayant de nombreux étages. Une étude a été effectuée sur trois bâtiments ayant respectivement dix, trente-huit et quarante-sept étages. On a utilisé un simple modèle théorique pour calculer les fréquences de vibration de ces bâtiments; ce modèle était basé sur l'hypothèse qu'il n'y avait aucune rotation des joints dans les charpentes des bâtiments. En comparant les valeurs théoriques et les valeurs mesurées, on a constaté qu'elles étaient sensiblement les mêmes; toutefois, il semble que ces valeurs seraient en meilleur accord si les modèles théoriques représentant les deux bâtiments les plus élevés étaient munis de joints rotatifs. On a eu recours à une auto-corrélation et à une analyse spectrale d'énergie pour les données de vibration afin d'obtenir une évaluation des caractéristiques d'amortissement des bâtiments. Les valeurs obtenues constituaient 1 à 2 % de la quantité critique d'amortissement.

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WIND-INDUCED VIBRATIONS AND BUILDING MODES

By H. S. Ward and R. Crawford

ABSTRACT

This paper outlines the methods that have been used to determine the frequencies and modes of vibration of multistory buildings from their wind-induced vibrations. Three buildings of ten, thirty-eight and forty-seven stories were investigated. A simple theoretical model was used to calculate the frequencies of vibration of the buildings; the model was based on the assumption that there was no joint rotation in the building frames. A comparison of the theoretical and measured values of the frequencies showed that this simple model was a realistic representation of only the smaller building. It is concluded that a model that includes joint rotation would be more realistic for the taller buildings. Autocorrelation and power spectrum analysis of the vibration records were used to obtain an estimate of the damping characteristics of the buildings. The values obtained were 1 to 3 per cent of the critical amount of damping.

Introduction

It is only in the last few years, as structures have become taller and economy has become more important, that structural designers have been made to realize the importance of determining the dynamic characteristics of buildings. Although for many years buildings have been designed for wind and earthquake loads, the approach has been based on quasi-static concepts, despite the obvious fact that it is a dynamic problem. Before a dynamic approach can be developed very far there is a need for more information on the actual and predicted performances of buildings.

Since the 1930's the Coast and Geodetic Survey (1936) has been using wind-induced and machine excited vibrations to measure the periods of vibration of buildings, and similar information has been obtained in Japan (Takeuchi and Nakagawa, 1960). Most of the measurements to date have been obtained by using frequency controlled vibrators that exert an oscillating force of a few tons. With this method steady state vibrations at a number of frequencies are measured and it is then possible to determine the frequency response of the structure. The development of sophisticated vibration generators in California (Hudson, 1962) has meant that the measuring techniques need not be too elaborate, since the amplitudes of vibration are comparatively large.

The authors have used the method of wind-induced vibrations, and they have shown (Crawford and Ward, 1964) that with appropriate instrumentation and analysis techniques it is possible to determine the dynamic characteristics of buildings by this method. The object of this paper is to report some further results, obtained from observations of a ten-story, reinforced concrete building and two steel-framed buildings of 38 and 47 stories.

Because of the late development of interest in the dynamic properties of structures, there has been little opportunity to correlate theoretical calculations and measured values of these properties. Because these properties are often of the utmost importance in the earthquake and wind resistant design of structures, it is imperative that every opportunity should be taken to try to understand the dynamic behaviour of buildings. Housner and Brady (1963) have studied existing information, and have concluded that some of the existing empirical formulae used in building codes to determine the periods of vibration of buildings can be in error by as much as 100 per cent.

Another conclusion was that some modern steel frame buildings behave essentially as if they had rigid floor girders. Calculations have been made to test this assumption for the buildings described in this paper, and it is shown that fair agreement was obtained for the ten-story building, but that there was an overestimation of the fundamental frequency of the order of 50 per cent for the multi-story structures. Some possible causes for the discrepancy are discussed, and the measured values are compared with some of the derived formulae obtained by Housner and Brady.

There is perhaps less known about the damping properties of buildings than of any other dynamic characteristic. An attempt has been made to determine damping values from the records of the wind-induced vibrations of the building by two methods: power spectrum analysis and auto-correlation analysis. In one instance the results have been compared with those obtained by man-induced vibrations. The methods need to be applied to more buildings before general conclusions can be drawn, but the initial findings indicate damping values of the order of 1 to 3 per cent of critical.

Description of the Buildings

The three buildings investigated have been constructed within the last five years· One of the buildings, the Sir Alexander Campbell Building is the headquarters of the Canadian Post Office Department at Ottawa; the other two buildings, the Canadian Imperial Bank of Commerce Building and CIL House, are located on Dorchester Boulevard in Montreal.

The Post Office Building is 266 ft by 74 ft, eleven bays by three bays in plan dimension, and 147 ft 6 in. high. There are ten floors above ground, including a penthouse and one basement; and typical story height is 12 ft 1 in. The frame and floor slabs are reinforced concrete, and the external walls are non-load-bearing 4-in. or 8-in. brick walls.

The building rests on groups of piles 22 in. in diameter, each of 125 tons capacity, that pass through 20 ft of clay and 20 ft of gravel and sand to solid limestone rock. The columns are rectangular and oriented so that the stiffest axis is parallel to the long dimension of the building; the columns are approximately twice as stiff about this axis as they are about the one at right angles to it. The column stiffness is approximately constant from the foundation to the fifth floor, where the values are approximately halved and then remain constant to the roof.

The Canadian Imperial Bank of Commerce Building is 140 ft by 100 ft, seven bays by four bays in plan, and rises 603 ft above the street level. There are 44 stories above ground level with a typical story height of 12 ft 5 in., except for the first floor and the five mechanical floors around the fifteenth floor level and the top. A substructure three floors in depth covers the entire site of 245 ft by 185 ft.

The building is founded on bed rock 48 ft below street level, with footings designed for 25 tons/sq ft bearing capacity. It has a structural steel frame with high tensile bolted field connections. Lower column sections are composed of 320-lb core sections, with cover plates up to 7 in. thick and 28 in. wide. The summation of the column stiffnesses differ by about 20 per cent about the two principal axes, and the columns are arranged so that they provide the greatest resistance about the axis parallel to the long dimension. Sections of the columns are changed every three floors, and the second moment of area of the columns between the ground and the first floor is approximately 12 to 15 times that of the corresponding columns between the forty-third floor and the roof.

Typical floor construction consists of corrugated metal deck units supported by purlins and topped by reinforced concrete. The curtain wall is constructed of precast concrete faced with slate. Exterior columns and framing adjacent to internal shafts are fireproofed in concrete, and other internal framing has sprayed asbestos fireproofing.

The CIL House is 168 ft by 112 ft, six bays by three bays in plan, and rises 430 ft above the street level. There are thirty-four floors above ground level, with a typical story height of 11 ft $8\frac{3}{4}$ in., except for the first floor and the mechanical floors at the tenth and thirty-second floor. A substructure 4 floors in depth makes the total height of the structure equal to 478 ft.

The building is founded on the same bed rock as the Canadian Imperial Bank of Commerce Building. It has a structural steel frame with welded connections, and welding fabrication was also used to manufacture the heavy columns; flange plates up to 28 in. by $7\frac{1}{4}$ in. and web plates up to 17 in. by 5 in. were used. The summation of the column stiffnesses differ by about 20 per cent about the two principal axes, and the columns are arranged so that they provide the greatest resistance about the axis parallel to the long dimension. Sections of the columns are changed every two floors, and the second moment of area of the columns between the ground floor and the first floor are approximately seven to eight times that of the corresponding columns between the 32nd floor and the roof.

In the basements, ground, mechanical floors and roof, the floor construction is concrete slabs formed in place; elsewhere the floors are constructed of 3-in. steel deck with a $2\frac{1}{2}$-in. concrete fill. The curtain wall is of light-weight aluminum construction, supported at every floor by steel outriggers. Beams and columns for areas with concrete slabs are fireproofed with concrete, and in all other areas the steel is fireproofed with sprayed asbestos.

All three structures are used as office buildings and are sub-divided by internal partitions. In the Canadian Imperial Bank of Commerce Building, the partitions were designed so that they were kept free of the frame through the use of expansion joints. The permanent partitions in the CIL House are of light-weight slag aggregate blocks. The partitions in the Post Office Building are of light-weight metal stud construction or 3-in. block; they are not intended to provide any lateral strength to the building. Rough calculations suggest that partitions may cover about 5 per cent of the floor area of each of the buildings.

The three buildings are in areas that have a history of earthquakes, and they are designed for earthquake loads. The details of the earthquake design are not known to the authors, but they would probably be based on the National Building Code of Canada (in this case the loads would be equivalent to the zone 3 loads of the 1961 Uniform Building Code). In the Canadian Imperial Bank of Commerce and CIL House the lateral strength has been provided by the columns and wind girders. Each of the buildings contains reinforced concrete cores used for elevator shafts.

Measuring Wind-Induced Vibrations

Six Willmore Mark II seismometers were used to record the wind-induced vibrations of the buildings. They are sensitive electromagnetic transducers with a fixed coil and a heavy magnet that acts as the moving mass. The seismometer can be set to measure in the horizontal or vertical direction; in the horizontal direction the natural period of the suspension can be adjusted in the range 0.6 to 5.0 sec.

The electrical output from the seismometers was fed through d-c amplifiers into a portable 7-channel FM tape recorder. The maximum voltage that could be recorded by the tape recorder without overloading was one volt rms. It was found that much of the signal emanated from the fundamental mode of vibration. In order that this component should not predominate at the expense of the higher modes,

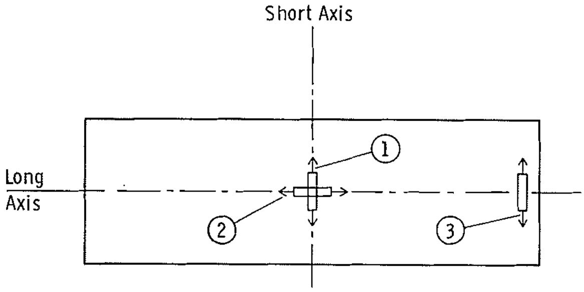

Fig. 1. Locations of Willmores at which measurements were taken in the Canadian Imperia Bank of Commerce and C.I.L. House.

the natural period of the transducer was chosen so that the frequency of the fundamental mode lay on the sloping section of the velocity response characteristic of the transducer.

With the appropriate natural period and electrical damping to 0.65 of critical, the velocity response of the transducer was flat over the frequency range of the higher modes of vibration. Gains of from 2 to 100 between the transducers and the tape recorder were often adequate to obtain a recorded signal of one volt rms for the wind-induced vibrations.

The program of measurements was arranged so that the torsional vibrations and the lateral vibrations about the two main axes of the buildings could be determined. In the Post Office Building the seismometers were set up in a central location on the 9th, 8th, 7th, 6th, 4th and 2nd floors. The transducers were oriented in the direction of each main axis in turn, and records of the vibrations were taken for approximately one hour about each of the two axes.

The seismometers were set up at three typical locations on each floor where measurements were to be taken; a diagrammatic sketch of these locations is shown in Figure 1. In Locations 1 and 2 the main signals are provided by the lateral modes of vibration, and in Location 3 a large signal is provided by torsional vibrations.

For identification of the torsional modes, seismometers were occasionally set up in Location 3 at each end of a floor and the difference of the signals was recorded, thus eliminating the lateral mode of vibration from the signal and doubling the value for the torsional mode. The usual procedure was to make simultaneous measurements at six levels, and then move down the building to five new levels, with one overlapping level for continuity between sets of measurements. During the period when measurements were taken the wind speed varied between 5 and 30 mph.

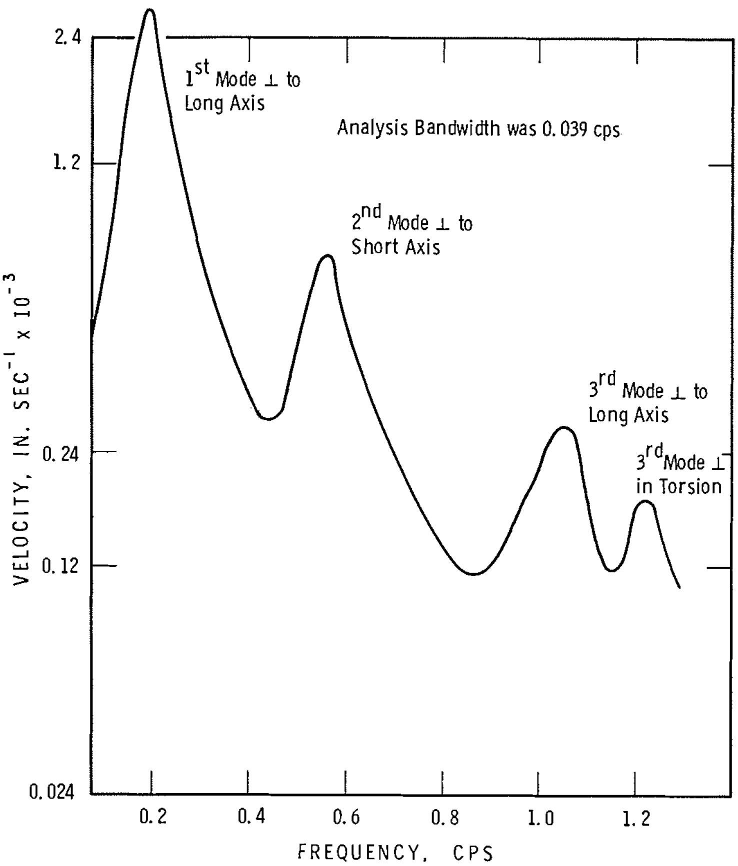

FIG. 2. Fourier analysis of vibrations measured on the 43rd floor of the Canadian Imperial Bank of Commerce.

ANALYSIS OF THE RECORDS

Determination of Modes and Frequency of Vibration. The main object of the work was to determine the frequencies and modes of vibration of the building. This was achieved through the use of an Ampex 7 track loop-recorder, a Honeywell-Brown analyser and a few components of an analogue computer. The Honeywell-Brown analyser was used to perform Fourier analyses or power spectrum analyses of the signals played back from the loop-recorder, and the analogue computer was used to determine the phase relation between the vibrations of the different floors.

A typical result of a Fourier analysis of the 43rd floor vibrations of the Canadian

Imperial Bank of Commerce Building is shown in Figure 2. To bring the records to a more convenient frequency range for analysis the original record, taken with a tape speed of $1\frac{7}{8}$ in./sec, was played back at 60 in./sec, recorded on the loop-recorder at $7\frac{1}{2}$ in./sec and then played at 60 in./sec; thus a speed-up ratio of 256 was achieved. With this ratio the effective bandwidth of the analyser, relative to the original signal, could be as narrow as 0.0026 cps, though a width of 0.04 cps was generally used. A comparison of the amplitudes of the signal from the loop-recorder and those of the original record showed that they agreed within ± 3 per cent.

When the frequencies of the different modes had been determined by Fourier analysis, a phase-comparison circuit was set up on the analogue computer. The object of the circuit, shown in Figure 3, was to form the sum and the difference of two signals that represented the wind-induced vibrations of different floors. When switch 1 was down, the sum of the signals was formed; with it up, the difference of the signals was obtained.

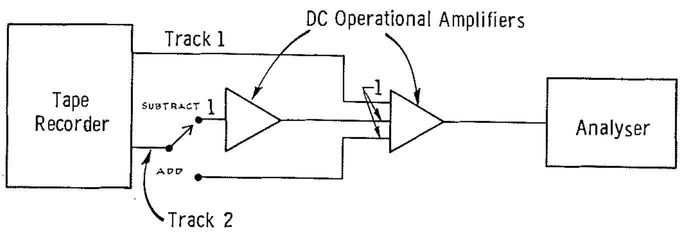

FIG. 3. Phase—detector circuit.

The output from this circuit was fed into the analyser and a frequency sweep carried out automatically so that the phase relations for all the modes at a particular floor could be obtained. If the output recorded on the chart of the analyser was greater for the summation of the two signals than for the difference, then the signals were in phase, and vice-versa. In this manner it was possible to investigate the phase relation for each mode as a function of building height. Displacements were calculated from the velocity data, on the assumption of sinusoidal vibration.

Determination of the Damping of Buildings. Very little is known about the damping properties of buildings. An attempt was consequently made to determine the damping of the three buildings by two separate techniques for analysing wind records. One of the methods, based on a narrowband frequency analysis of the wind-induced vibrations, assumed that the response of the buildings to the wind was a random vibration of a lightly damped system. In this instance the damping can be obtained from a power spectrum analysis of the signal, providing the wind excitation is random.

The method is based upon the determination of the half-power points of a resonance peak in the power spectrum. This situation is shown in Figure 4 where the half-power bandwidth is represented by the distance between A and B and f_n is the resonant frequency of one of the modes. The half-power bandwidth is equal to f_n/Q where Q is equal to the inverse value of $2 \times$ (fraction of critical damping.) In order to provide adequate resolution of the half-power bandwidth the power spectrum analyses were performed with a bandwidth equal to $f_n/4Q$. When the analysis bandwidth was varied in the range from $f_n/4Q$ to $f_n/8Q$ there was no variation in the half-power bandwidth and this supports the assumption that the vibrations are random. The power spectrum analysis method was used on the three buildings and on a 19-story building that had been investigated previously (Crawford and Ward, 1964).

The second technique, which looks very promising, was an autocorrelation analysis of the wind-induced vibrations. This was done for two of the buildings. The autocorrelation function was obtained by filtering out all the modes except the one under investigation. An analogue-digital converter was then used to provide a digital computer with 2000 samples of the vibration record. The autocorrelation function was computed for 200 time increments and a digital-analogue converter permitted the results to be plotted directly on graph paper. The autocorrelation functions look like exponentially-damped cosine curves, and the damping value was obtained by the ogarithmic decrement method.

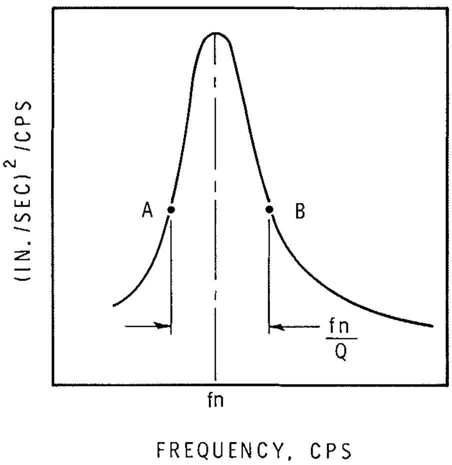

FIG. 4. Determination of damping from a power spectrum analysis.

A third method, used only at the Post Office Building, made use of a technique recently reported by Hudson *et al* (1964) in which the building is forced to vibrate in one of its natural modes by man-induced vibrations. This was achieved by providing the man with a visual display of the building vibration in order to enable him to synchronize his movements so that they could build up the level of vibration of the fundamental mode. The visual display was provided by feeding the signal from a seismometer into a portable recorder. When the vibration had been built up to a suitable level, the motion was allowed to decay and the damping value was again obtained by the logarithmic decrement method. The technique was successful when the man was positioned on the top floor on a quiet day when there was very little wind.

The final form of analysis used on the vibration records was obtained by the use of a Brüel & Kjaer, Type 160, probability density analyser, again using recording techniques to bring the signal above the 5 cps limit of the analyser. This method of analysis has shown that the lateral wind-induced vibrations have a gaussian ampli-

TABLE 1

MEASURED FREQUENCIES OF VIBRATION OF THE BUILDINGS

A—Canadian Imperial Bank of Commerce B—C.I.L. House
C—Post Office Building

BUILDING C

Mode of vibration	1st	2nd	3rd	4th	5th
Frequency of lateral vibrations perpendicular to long axis (cps)	1.44	4.14	6.41	9.06	11.29
Frequency of lateral vibrations perpendicular to short axis (cps)	1.69	4.69	7.03	9.27	12.34
Frequency of torsional vibrations about vertical axis (cps)	3.59	5.85	7.87	10.15	

BUILDING B

Mode of vibration	1st	2nd	3rd	4th	5th	6th	7th
Frequency of lateral vibrations perpendicular to long axis (cps)	0.224	0.683	1.25	1.85	2.50	3.02	3.75
Frequency of lateral vibrations perpendicular to short axis (cps)	0.254	0.722	1.28	1.87	2.50		
Frequency of torsional vibrations about vertical axis (cps)	0.293	0.830	1.22				

BUILDING A

Mode of vibration	1st	2nd	3rd	4th	5th	6th
Frequency of lateral vibrations perpendicular to long axis (cps)	0.215	0.625	1.09	1.66	2.07	2.56
Frequency of lateral vibrations perpendicular to short axis (cps)	0.215	0.586	1.03	1.52	1.92	2.38
Frequency of torsional vibrations about vertical axis (cps)	0.254	0.704	1.21	1.78	2.22	2.76

tude distribution, but in heavy winds the torsional vibrations tend to be sharply peaked.

CALCULATED FREQUENCIES OF LATERAL VIBRATION

In order to calculate the frequencies and modes of vibration of a system it is necessary to know the distribution of stiffness and mass and the amount and type of damping; although if damping is small, as it is in this case, it can be neglected. The distributions of mass and stiffness in the buildings have been evaluated on the basis of the assumptions described in the following paragraphs.

In any building, the greatest percentage of the weight of the structure is concentrated at each floor level. A fair representation of the distribution of the mass in a building is obtained, therefore, if it is assumed that it is a lumped-mass system with

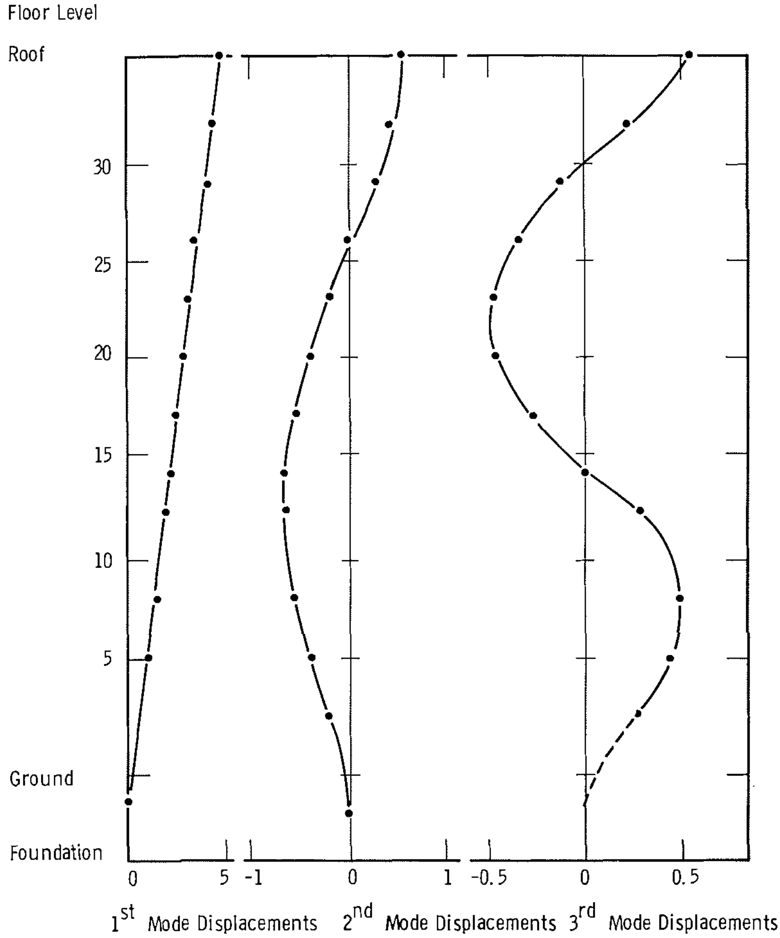

FIG. 5. Mode shapes for lateral vibrations perpendicular to the short axis, C.I.L. Building.

the masses concentrated at each floor level. This assumption was made, together with the further assumption that all the masses of each floor for a given building were equal. This latter condition is probably true for all floors except perhaps the first floor and the mechanical floors, where the mass may be as much as twice the typical floor mass.

Housner and Brady (1963) found that for space-frame buildings up to 26 stories the best agreement between measured frequencies and theoretical values was obtained when it was assumed that the horizontal members were rigid compared to the columns. They surmised, however, that for buildings with forty or more stories the effect of the flexibility of the girders would become more significant. The theoretical values for the frequencies obtained in this paper were based on the assumption of rigid girders to see whether the behaviour noted by Housner and Brady did extend to taller buildings. In this instance it is possible to set up a tri-diagonal stiffness matrix following the procedure described by Norris *et al* (1959). The coefficients in

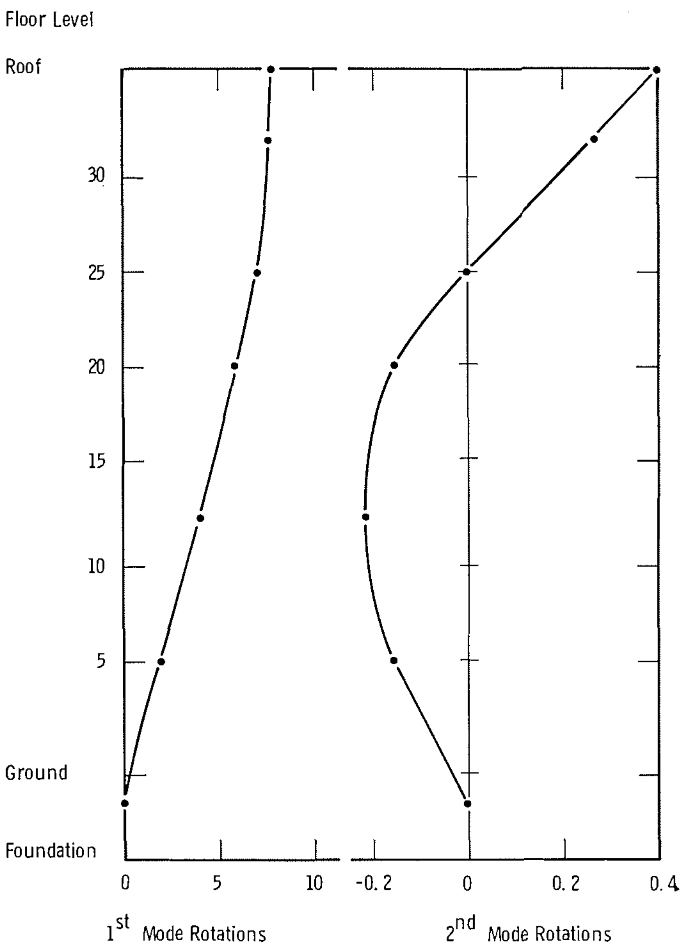

FIG. 6. Mode shapes for torsional vibrations about the vertical axis, CIL. Building.

the matrix are functions of the second-moment of area of the columns, the distance between floors, and Young's modulus of the material making up the columns. No allowance was made in the stiffness matrix for the stiffness of the elevator cores.

A further factor that affects the stiffness of a structure is the degree of fixity at the foundation. In this work it was assumed that the buildings were on a rigid base; this assumption is probably valid for the two taller buildings because they are founded on solid rock, but it may not be valid for the smallest building. Another effect that was investigated was the amount of rotation at the base of each column.

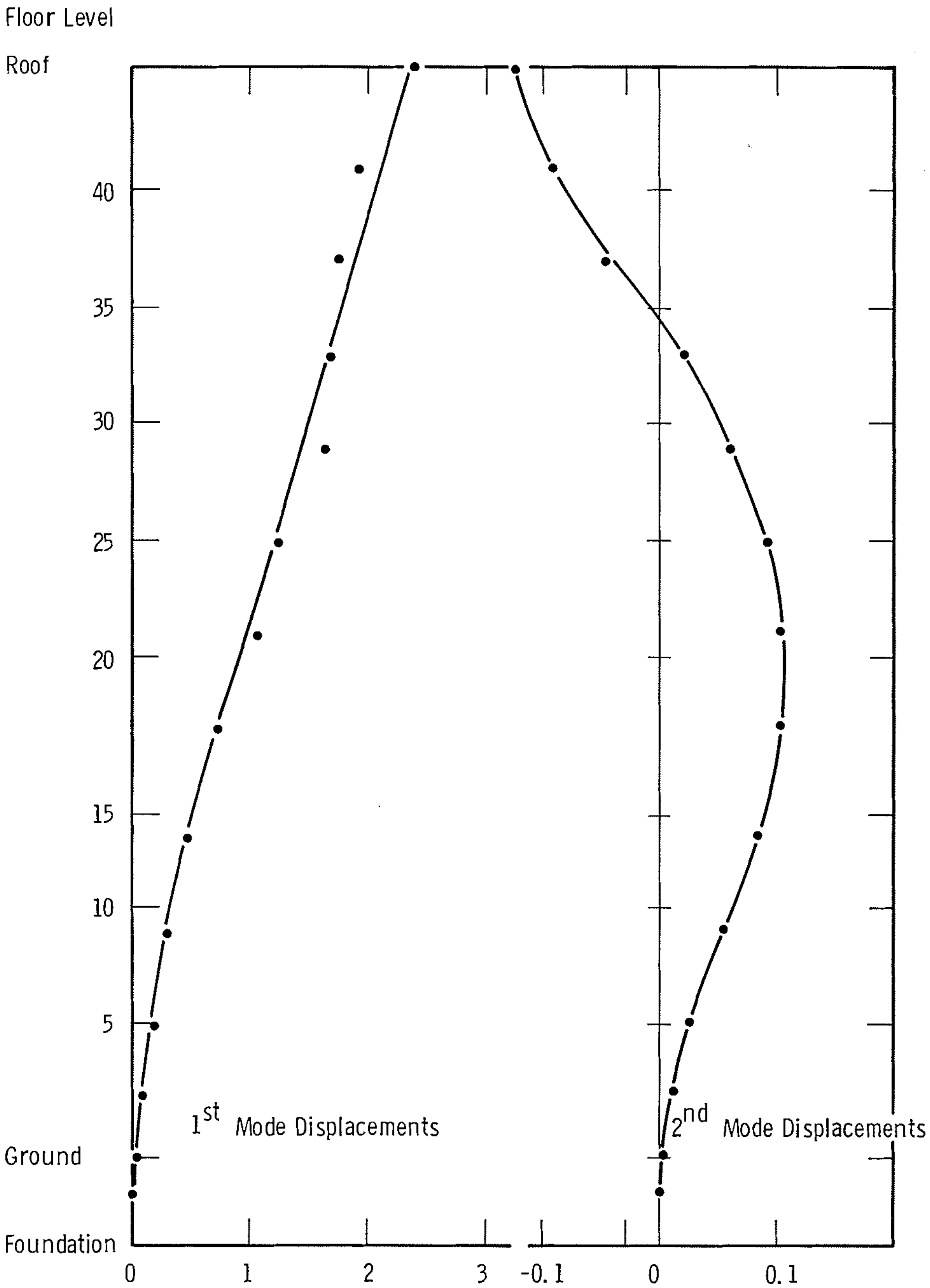

FIG. 7. Mode shapes for lateral vibrations perpendicular to the long axis, Canadian Imperial Bank of Commerce.

It was found that there was little difference between the periods obtained from assuming fixed or pinned conditions.

The time required to evaluate the eigenvalues using an IBM 1620 computer was proportional to the cube of the number of stories; the times taken ranged from 5 min for the Post Office Building to 3 hr 40 min for the Canadian Imperial Bank of Commerce Building. Calculations of the mode shapes would have taken nearly twice as long and were not undertaken.

Results for the Modes and Frequencies of Vibration

The frequencies of the different modes observed for the three buildings are shown in Table 1; some of the mode shapes are plotted in Figures 5 to 8. In each of these the abscissa co-ordinates are relative. Thus for a given figure they indicate the relative magnitude of the displacements in the different modes of vibration. The abso

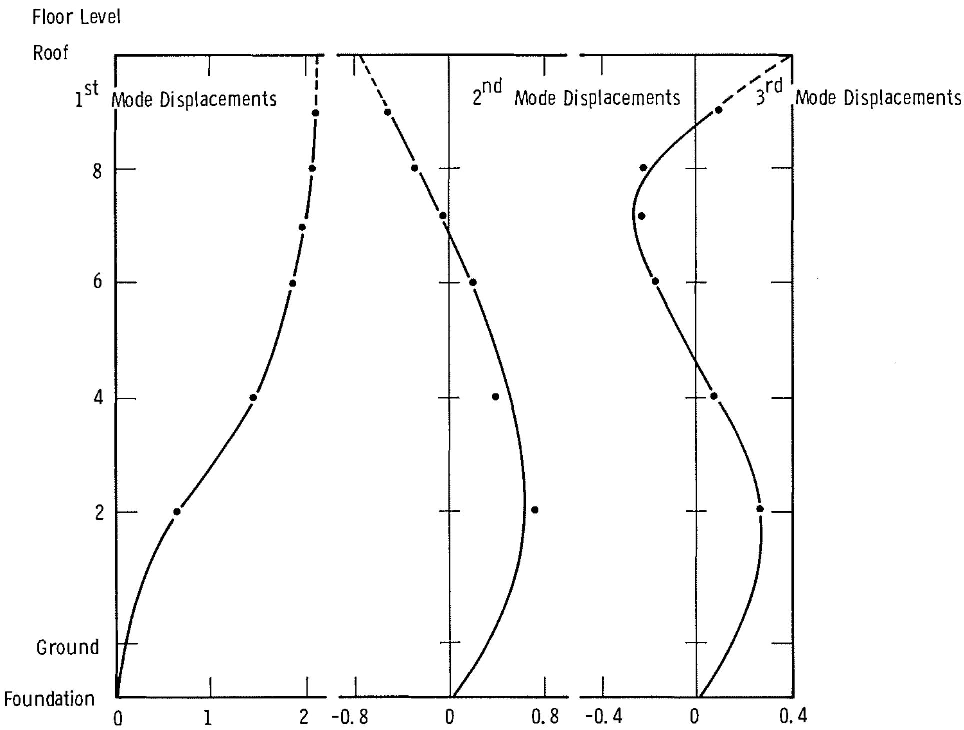

Fig. 8. Mode shapes for lateral vibrations perpendicular to the short axis, Post Office Building.

lute values of the displacements depend upon the direction and frequency content of the wind, but some typical values for the displacement and rotation in the fundamental mode, of the top floors of the three buildings, are shown in Table 2. During most of the work the wind speeds were based upon local weather office information, but recently it has been possible to obtain more precise information on the wind conditions in Montreal by means of an anemometer installed at the top of a 235-ft radio antenna on the roof of the Canadian Imperial Bank of Commerce Building.

The calculated results for the natural frequencies of lateral vibrations of the buildings are shown in Table 3. Most of these results refer to the case where the columns are assumed fixed at the foundations; the calculations showed that there was very little change in the values of the frequencies when pinned conditions were assumed.

The stiffnesses of the columns in the steel buildings were obtained by calculating the second moments of areas of their sections; the concrete section was used in the calculations of the reinforced concrete building. The weights of the floors used in the calculations were obtained from the structural design loads.

TABLE 2

DISPLACEMENTS OF THE TOP FLOORS OF THE BUILDINGS

A—Canadian Imperial Bank of Commerce B—C.I.L. House
C—Post Office Building

BUILDING A

Wind speed (mph)	15-25 *	25-40*
Deflection perpendicular to long axis (ins.)	1.8×10^{-3}	5.2×10^{-3}
Deflection perpendicular to short axis (ins.)	1.66×10^{-3}	1.4×10^{-2}

* Wind Speed Recorded by Anemometer on Radio Antenna

BUILDING B

Wind speed (mph)	5-10	20-30
Deflection perpendicular to long axis (ins.)	7.0×10^{-4}	
Deflection perpendicular to short axis (ins.)	6.0×10^{-4}	8.5×10^{-3}
Angular rotation about vertical axis (degrees)		2.27×10^{-4}

BUILDING C

Wind speed (mph)	5-10
Deflection perpendicular to long axis (ins.)	0.3×10^{-5}
Deflection perpendicular to short axis (ins.)	0.49×10^{-5}

RESULTS FOR THE DAMPING CHARACTERISTICS OF THE BUILDINGS

A power spectrum analysis of the output of the anemometer (wind speed) on top of the radio antenna, with the wind blowing at a steady 15 mph, is shown in Figure 9. The wind records have been steady on each occasion when readings have been taken from the anemometer, a fact that could be related to its isolated position. It is possible than an anemometer placed at roof level might have greater power than is shown in Figure 9 for the frequency range from 0.1 to 0.3 cps. A typical probability density analysis of both wind speed and wind-induced vibrations, shown in Figure 10, indicates that the wind-induced vibrations are random.

The results obtained by the power spectrum analysis method for the damping values of some of the modes of vibration of four buildings are shown in Table 4. The results for a fourth building (Crawford and Ward, 1964), are also included.

TABLE 3

CALCULATED FREQUENCIES OF VIBRATION OF THE BUILDINGS

A—Canadian Imperial Bank of Commerce B—C.I.L. House C—Post Office Bldg.

Building	Motion Perpendicular to	Boundary Condition at Support	1st Mode cps	2nd Mode cps	3rd Mode cps	4th Mode cps	5th Mode cps	6th Mode cps	7th Mode cps	Highest Mode cps
A	Long Axis	Fixed at Foundation Level	0.306	0.787	1.24	1.75	2.38	2.95	3.37	35.3
A	Short Axis	Fixed at Foundation Level	0.259	0.616	0.989	1.36	1.80	2.32	2.71	32.9
B	Long Axis	Fixed at Foundation Level	0.387	0.949	1.50	1.98	2.57	3.23	3.87	36.0
B	Short Axis	Fixed at Foundation Level	0.332	0.812	1.31	1.75	2.29	2.89	3.46	37.9
B	Long Axis	Pinned at Foundation Level	0.384	0.933	1.48	1.96	2.53	3.16	3.84	35.3
C	Long Axis	Fixed at Foundation Level	1.33	3.76	6.63	9.25	11.5	13.1	14.2	19.5
C	Short Axis	Fixed at Foundation Level	1.11	3.02	5.29	7.49	9.16	10.9	11.7	16.4

Figure 11 shows the results of one of the trials performed at the Post Office Building when man-induced vibrations were used to excite the top floor of the building. Figure 12 shows the normalized results for the autocorrelation function of the velocity of vibration of the fundamental lateral mode of vibration perpendicular to the long axis for the 41st floor of the Canadian Imperial Bank of Commerce Building. The damping values obtained by these methods are shown in Table 4.

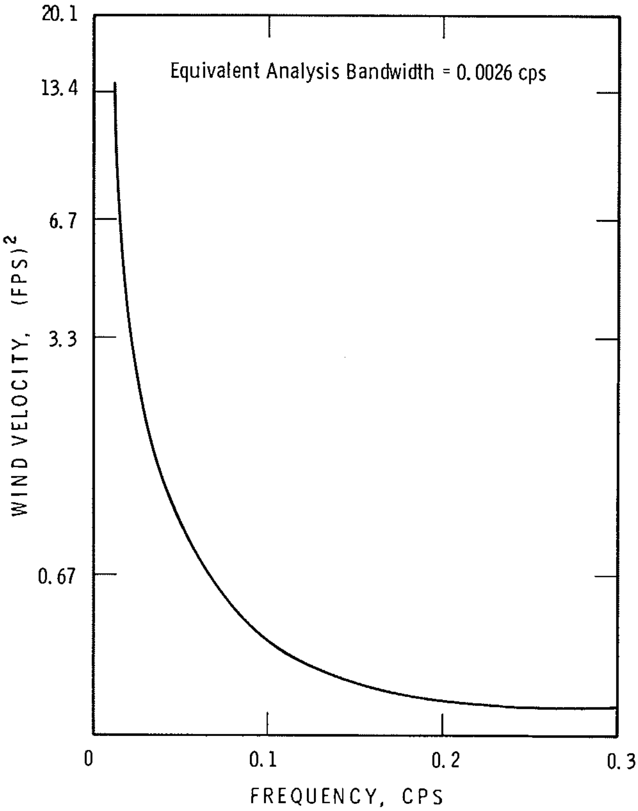

FIG. 9. Power spectrum analysis of a steady 15 mph wind.

Discussion of Results

The measured and theoretical values of the periods of the fundamental modes of lateral vibration of the buildings are shown in the first two lines of Table 5. The greatest discrepancy between the two sets of results is of the order of 73 per cent, and the closest agreement occurs for the Post Office Building for motion perpendicular to the long axis when the error is of the order of 9 per cent. The theoretical values of the periods are greater than the measured values for the Post Office Building (this is most likely explained by the effect of the elevator cores); the opposite is true for the other two buildings.

The close agreement between one set of the calculated and measured results for the Post Office Building indicates that the simple theoretical model considered here

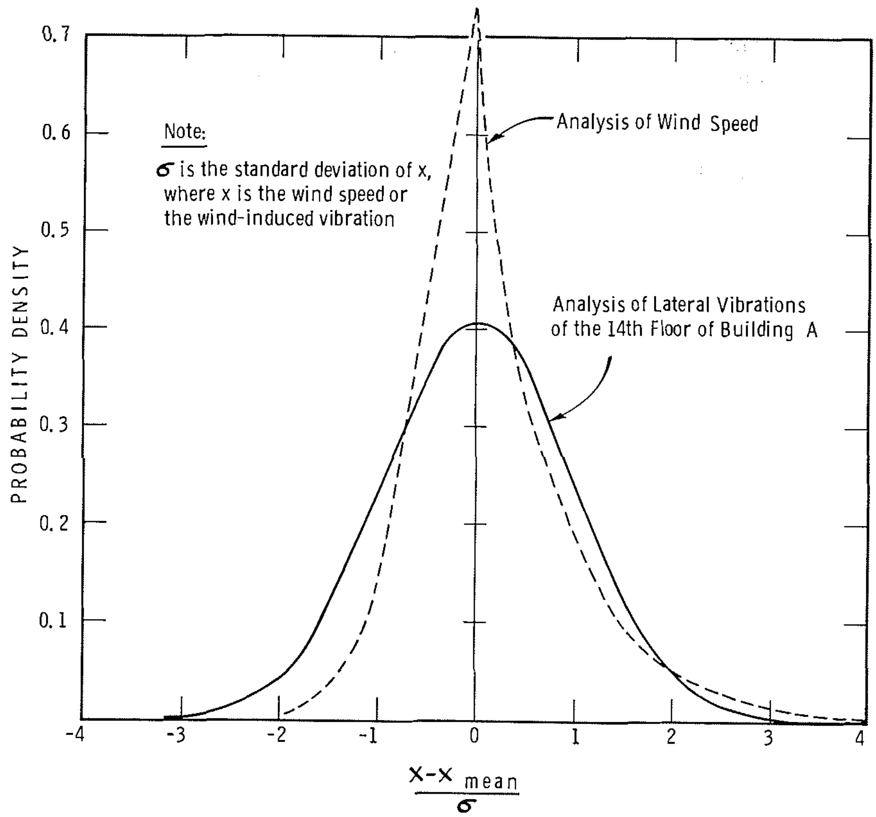

FIG. 10. Typical probability density analysis of wind speed and wind-induced vibrations.

TABLE 4

DAMPING VALUES OF THE LATERAL MODES OF VIBRATIONS FOR MOTION PERPENDICULAR TO THE LONG AXIS

A—Canadian Imperial Bank of Commerce B—C.I.L. House C—Post Office Bldg.
D—National Health and Welfare Bldg.

Building	Mode	Damping Value from Power Spectrum Analysis % Critical	Damping Value from Man-induced Vibrations % Critical	Damping Value from Autocorrelation Analysis % Critical
A	Fundamental	1.8		1.6
A	Second	3.2		
A	Third	2.3		
B	Fundamental	1.2		1.8
C	Fundamental	1.0	1.1	
D	Fundamental	1.3		

is probably a satisfactory approximation of the actual behaviour of the structure. In the case of the two taller buildings, however, the discrepancy between the theoretical and measured values of the frequencies shows that the assumption of rigid girders is not realistic. Because the theoretical frequencies based on this assumption are too

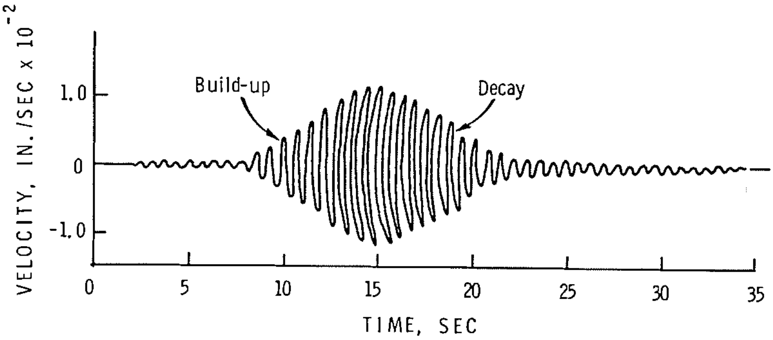

FIG. 11. Man-induced vibrations of the top floor of the Post Office Building.

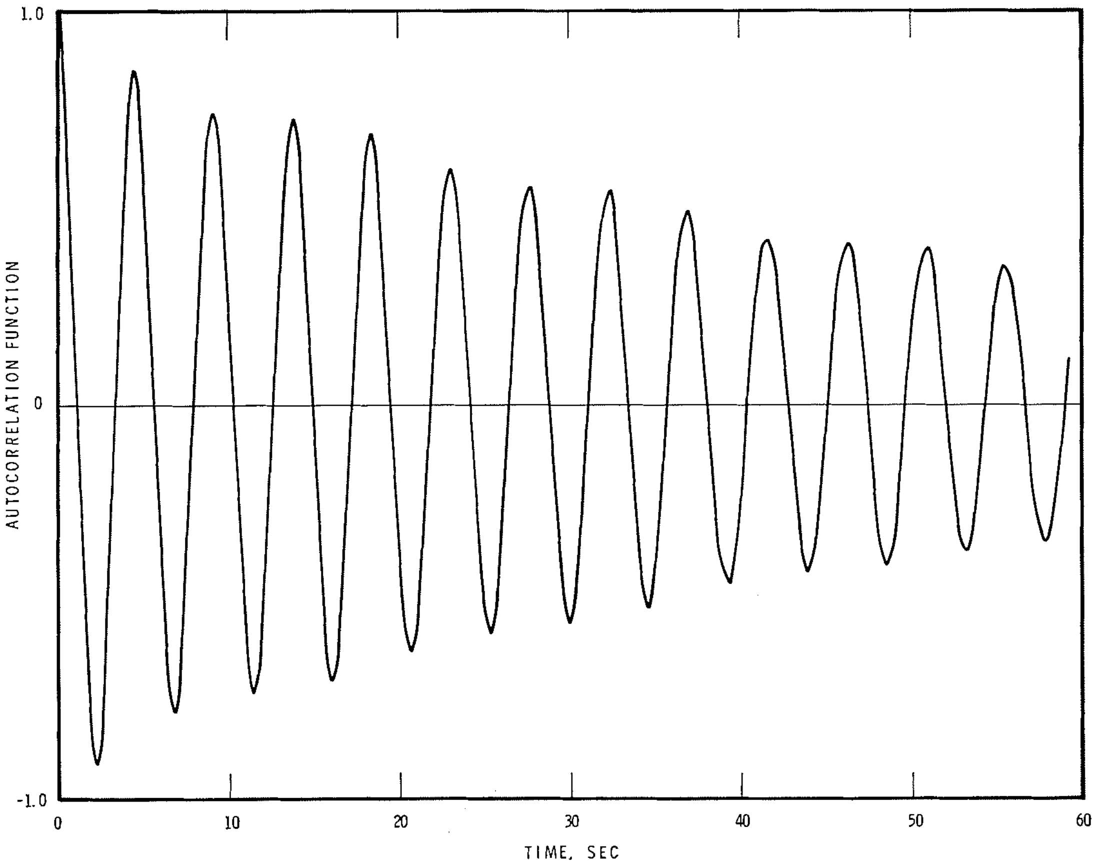

FIG. 12. Autocorrelation analysis of the fundamental mode of lateral vibrations for the 41st floor of the Canadian Imperial Bank of Commerce (motion perpendicular to the long axis).

high, it is obvious that a model which included joint rotation would provide better agreement with the measured values.

It is of interest to compare the measured values of frequency with those obtained from some empirical and theoretical formulae. The third and fourth lines of Table

TABLE 5

COMPARISON OF DIFFERENT CALCULATED VALUES OF THE FUNDAMENTAL PERIODS OF VIBRATION WITH THE MEASURED VALUES

A—Canadian Imperial Bank of Commerce B—C.I.L. House C—Post Office Building

Building	A		B		C	
Fundamental Period (sec)	Perpendicular to Long Axis	Perpendicular to Short Axis	Perpendicular to Long Axis	Perpendicular to Short Axis	Perpendicular to Long Axis	Perpendicular to Short Axis
Measured	4.65	4.65	4.46	3.93	0.69	0.59
Theoretical Value for Shear-type Structure with Fixed Columns	3.27	3.86	2.58	3.01	0.75	0.90
U.B.C. Formula $T = \frac{0.05\ H}{\sqrt{D}}$	3.25	2.76	2.25	1.84	0.84	0.46
U.B.C. Formula $T = 0.1\ N$	4.40	4.40	3.40	3.40	0.90	0.90
$T = 2\pi\sqrt{\frac{m}{K_N}}\ (0.63\sqrt{N})$					0.80	1.05
$T = 2\pi\sqrt{\frac{m}{K_N}}\ (0.8\sqrt{N})$	1.02	1.38	2.10	2.40		

5 contain the values of periods calculated from formulae in the Uniform Building Code (1961).

In these formulae

H is the height of the building, in feet,
D is the dimension of the building, in feet, in the direction of the motion, and
N is the number of stories in the building.

It can be seen that the formula $T = 0.1N$ gives a good estimate of the fundamental period for the two tall buildings, but not for the Post Office Building.

Housner and Brady (1963) developed formulae for the periods of space-frame buildings, and two of these are shown in lines 5 and 6 of Table 5. In these equations, m is the mass per unit area of the floors and K_N is the stiffness of the columns per unit area of the top floor. The formula in line 5 refers to the case where the column stiffnesses are constant through the height of the structure, and in line 6 a linear increase in stiffness is assumed when going from top to bottom of the structure. It may be seen that calculations for the three buildings, using the appropriate formulae, do not give good agreement with measured values.

With tall buildings it is quite probable that the higher modes of vibration will be important in the earthquake design of such structures, because their frequencies will be closer than the fundamental frequency to the peak frequencies found in strong-motion earthquakes (1 to 10 cps). The measured results in Table 1 show that the ratios of the natural frequencies of vibration to the fundamental frequency are of the order 1, 3, 5, 7, etc.

Mode shapes of buildings are also important in earthquake engineering design because they determine the manner in which earthquake loads are distributed through the height of the structure. The fundamental mode shapes of the Canadian Imperial Bank of Commerce Building and CIL. House are observed to be approximately linear functions of height. This result is not unexpected, considering that the stiffness varies considerably with height. In contrast, the Post Office Building, which has less variation in stiffness, shows more nearly the mode shape characteristic of a shear type building (Figure 8).

Torsional loads have been blamed for many failures of buildings during earthquakes, and most building codes require engineers to design buildings for such loads. Despite this, the methods of determining the loads appear to be "rule of thumb", since very little information is available on the actual torsional characteristics of buildings. Actual measurements of torsion could be used to alleviate this situation, however, because comparatively large amplitudes of torsional vibration have been recorded during the work.

The damping values presented in this paper should be regarded as tentative until more detailed knowledge is obtained of the spectrum of the wind loads acting on buildings. If this spectrum is assumed to be flat over the range of building vibrations, however, the methods described provide the damping characteristics of buildings.

Finally, it is suggested that a further aspect of the work needs to be investigated in the future: the effect of the amplitudes of vibration on the dynamic characteristics of buildings. The amplitudes that have been measured so far are quite small. It is conceivable that for amplitudes of a higher order of magnitude these characteristics might change.

Conclusions

It has been possible to determine the first few modes and frequencies of lateral and torsional vibration of multistoried structures from wind-induced vibrations. This information is directly useful in earthquake engineering design, but the primary objective is to provide guidance in the development of theoretical models suitable for computation.

In many of the attempts made to compare the assumed theoretical behaviour with the actual performance of multistoried structures, it has been found that there is an unsatisfactory margin between the two. Previous workers (Housner and Brady, 1963) have noted that for space-frame buildings up to 26 stories, the best agreement for the dynamic characteristics of the buildings was obtained by assuming the girders were rigid. If the buildings described in this paper are typical, however, it appears that for buildings between 40 to 50 stories it is necessary to include the effect of the flexibility of the girders to obtain good agreement. This anomalous behaviour of structures shows that there is still a great deal to be learned about their stiffness characteristics. One of the best ways of achieving this knowledge is to continue the comparison of the predictions of theory with the measured dynamic characteristics of buildings.

Damping values of the buildings obtained from analysis of the wind-induced vibrations indicate values from 1 to 3 per cent of critical damping. It is thought that further work needs to be done in determining more precisely the spectrum of the wind loading and the effect, if any, of the amplitudes of vibration on the dynamic characteristics of buildings.

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Building	Owner	Architect	Contractor
Canadian Imperial Bank of Commerce	Dorchester Commerce Realty Ltd.	Peter Dickinson (Arch.) Clifford & Lawrie (Consulting Arch.) Ross, Fish, Duschenes and Barrett (Supervising Arch.)	Perini
CIL	Dorchester University Holdings	Greenspoon, Freedlander & Dunn (Arch.) Skidmore, Owings & Merrill (Consulting Arch.)	Anglin-Norcross Quebec Ltd.
Post Office Building	Department of Public Works	Shore & Moffat	Geo. A. Crain & Sons Ltd.

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Building Physics Section
Division of Building Research
National Research Council
Ottawa 2, Canada

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