WINGED EDGE POLYHEDRON REPRESENTATION

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Prepared for:
Advanced Research Projects Agency
October 1972

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OCTOBER 1972


## DOCUMENT CONTROL DATA-R \& D

Securify classificarion of thle, body of abstract and indexinu annotafion nust be enfered when the uverall seport fs clastilled)

| ohiginating activitr (Corporate author) | 2e. REPORT SECURITY CLA3SIFICATION |
| :---: | :---: |
| Stanford University | Unclassified |
| Computer Science Depariment | 2b. Group |
| Stpuford, California 94305 |  |

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WINGED EDGE POLYHEDRON REPRESENTATION.

## Bruce G, Baumgapt

Abstract: A wiaged edge polyhedpon pepresentation ls stated and a set of opimitives that opeserve Euler's F-E+V $=2$ couation are explainea, pesent use of this reppesentation in aptificial Intelllgence fop computer graphlcs and wopld modeling is lliustrated and its intended futupe adolication to computep vision is discifibed.

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Thls peseapoh was supoopted in part by the adyaneed Reseepch ppojects Agency of the offle of the Socpetary of Defence under contract SD-183,

FIGURE 1.1 - Exanples of World I'odel Scenes.
\&


1. INTRODUCTION,

In opder to get a comouter to dal with the physlcal world it must have a data pepresentation on which computations involving space, tims, shape, size, and the apopapance of thlngs gan be done. It is my curpent prejudice that oolyhodra provide the opoder starting dolnt fop bullding such a ohysloal world representation. At Stanford Aptificial intellegence, Blnford and agin have started instead with spine-cposs section models as alternate aporoach to the same protems [refepenoe 1]. Other peseapchers with somewhat different goals, are attempting to build semantic, predioate calculus, problem
 paper Is about a body, face, edge, vortex polynedron model that is for modelling objects and scenes of oojects fop the sake of computer vision.

Although the data structure to be discussed is not language dedenaent, the terminlogy and examples wllifollow ALGOL and LISP. Also, the reader is assumed to have aome acquintance with the ldeas associated with the following technical tepmsi

A: block, node, itam, elament, atom.
B: llnk, Dolnter, addesse, refepence.
C: datum, content, value.
D: llst, ilng, stack, odi, tree.
E: dynamic pres storage 8 memopy allocation.
A ferough presentation of these terms and ldeas can be found in chapter two of volume one of Knuth's cookbook. 'The art of computer Programting [Reference 7J. The word "ring" used informally ln this padep will always mean a double polntep ring with a headi and as in LISP, words of memory hadpen to be able to hold two dolnteps.

FIGIRRE 1.2 - A Polyhedron Nodel of a Nechanical drm.


1

1. A. introduction to Wopld Modeling.
! Wlil introduce my requipements por a computer model of the ohysical wopld in terms of lis pola as memory, as a memory, a worle model has contents and an addressing meohanlsm. Tho kincis of date that! wish to holy in my world model are:

CONTENT REQUIREMENTS

> 1. Topological data.
> 2 . Goometplo data.
> 3 . photomotplo data,
> 4. Parts tpe data.

Topologicel data has to do with the notion of nelghbophood; a worla model has data on what is next to what. A face, edpei vertex model is essentially dedicated to surface todologys matters of volume topology are not storad but rather must be computed. Geometric deta has to do with notions sueh ss loous, longth, area and yolume. Photometple date includes the loous and nature of llant souposs as well as gata on how surfaces pefleot, absorb and scattop ilght, papts tees data has to do wlith the notlon that objeots are comoosed of parts, which $f$ construe as data on the structure of the phyoleal world pathep than as ourely artifact of having structipad wopld datal that is, I profor to have the specification of how an ontity is broken into darts be oxternal to my wopld model. The kinds of data not included ape somantlo deta fother than body names): ohysleal data such as mass, Inaptia tensors, electrioal propertins and so onl and cultupal date such as whether an object is a toy, tool, op wapon: with any artletle, pellolous or market value.

Next the kinds of addeasing mechanisms l wish to have, cop equivalently the inout-outout modes of the model) are:
accessing requirements

1. ADD日aiance given camerai return an image of what the wopld would look like from that camera.
2. Recognition - given an Image, retupn the objecte from the wopld modei that appear in thet lmage.
3. Camera Solution - giyon a pocegnized imade, find the loeation 8 oplentation of the camera.
4, porceotion - given Images, from solved.cameras, olace new bodies into the model for doitlons of tho lmages that have nue yet been rooognlzed.
E. Soatial Accessing a givan a locus and padius, retupn the dortions of objects in that sohepe.

Claply, these ape the high level acoessing pegulpaments whloh ere the posions for having a world model and the design goals for model bullding.

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FIGURE 1.3 - A Camera iodel.

FIGURE 1.4 - Logical and Plasical Paster Sizes.


1. B. Introduction to a Camera Model.

As the accessing pequipements imply, worid model pequices a spocial ontity callod a camera which is ueed to model image fopmation. Although the camera model is Imoortant here for a comolete speciflcation of the data, it may be sklpoed on a flpst peading. The Dartloular camera model ! have been using lately, ls oxpressed by elghteen peal numbers linvolving nine degrees of freodom. Flist there is the camepa lens centep loousi

EX, CY, CZ, In world coopdinates.
Aflaed to the lens center is the camera frame of reforence with unlt vectops $i, j$ and $k$, when the lmage fopmed by the camera is placed in corpespondence to a display screen, as lllustrated in figupe 1.3, the Unit veeiop 1 maos into the plantward dositive $x$ of the disolay screen, and the unit vector $J$ mads Inte the upward positive y of the alsplay sepeen, and the unlt vector $k$ comes out of the displey sereen to form aplght handed oooplinate system, Together the three unit vectops ef the oamera are the theoe by theee potation matrix:

> ix, iy, lz
> jX: jy, jZ In world coordinates.
> $K X, K Y, K Z$

Next, there ars three scales which fetermine the copsespondenoe betioen woild size and mage slze. Obsepve thet the wopld ls measured In physical units of leneth like meters op faet whlio computer lmages
 thus tine conversion scales must be in terms of logical lmage unlts wer physicei wopld units. in an actual telovision camera a minute Image (say 9 mm by 42 mm Is formed on vicilsen tube and that image has a Daptioulap number of pows arid oolumns. it is the llttie lmage on the vidicon that we pretend to madel by the six parametersi

$$
\begin{array}{ll}
\text { LOX, LOY, LDZ } \\
\text { POX, PDY, FOCAL } & \text { Logicel phayter elzo. } \\
\text { Phulcel raster sizo. }
\end{array}
$$

Where the numbor named FOCAL, is the rocal plans distanoe which in this monol (with diĝant objoots) oan safoly de caikjed with the long peval longth and can be given in mlllimeters foonvontiona! lens pun $12,5 \mathrm{~mm}$ to 75 mm fop $\mathrm{j}^{\prime \prime} \mathrm{TV}$. Fhe integtr LDZ is an aptliact so that the unlts oame out coprectiy in the $z$ dimonsion. Thus the seales factors are dsfined:

```
SCALEX ~ -FCCALmLOX/POX:
SCALEY * GOCAL LOY/PO'T
SCALEZ F FJCALMLOZ_
```

This simpla camera medel is lised te gompute vertex Image cocpolnates, a more daborase ohysleal camepa medel can be pound in Sobel [reference 9].

BAcL 7

FIGURE 1.5 - A Renaissance Camera Model.

4. $\quad$ I. C. Introduction to Body, Faop, Edge, Voptex (BFEV) Modeling,

This introduction to BFEV modeling will be Inlopmal and specifle to the winged edge model ppesented in part-ll of thls papep. Many of the besic numerioal pelatons which make BFEV models important are steted in ALGOL notation without proof.

The Veptex.
A vertex is an instance of polnt In Eucildaan theee space, The important thing about a vertex is its worlo locus iwfth Gomponent names XWC,YWC,ZWC standing for wopld-coordinates). Vertex locil are tha basic geometric data prom which lensth, aroa, volume, face vectors and image positions oen be computed, also a vertex may exist simultanoulsy in one or more image spaces. an image space (with oomponent names XPP,YPP,ZPP standing fop perspectiveoppojeoted) is always three dimensional and is determined with pespect to alven camera centered coordinate system frith component names XCC,YCC,ZCC standing for camera-coopdinates). The third lmage cumponent, ZPP, is taken Inversely Dpoportional to the distance of the vertex fiom the camepa Image diane, ZCC. Using the camera of the pievious section. The transformation of a vertex wople locis to a camera centered locus is:

```
X - XHC - CX:
Y + YWC - CYB
Z - ZHC - CZ;
XCC + IY*X + IY*Y + IZ*Z{
YCC - JX*X + JY*Y + JZ*Z;
ZCC * KX*X + KY&Y + KZ&Z;
```

The flest three assignment statoments are the tpansiation to the camera frame's origin, the second three assignments are the potation to the eamera frame's orientation. Next the persoective ppojection transformation is computede

```
XPP - SCALEX*XCC/ZCC;
YPF - SCALEYOYCC/ZCCS
ZPP . SCALEZ fZCC;
```

The XPP and YPP assignments are depived by means nf simliap tplangles, es is belng done by the man in figuse 1,51 the ZDD essignment is for preserving the deoth information and the collnearlty of the world in tice perspectiva projected lmage space, A3 given, the pp frame is pight handed and vertlees in front of the camera's image olane will have negative ZDDi Zod values near -FOCAL ape elose to the camera and values adopoaching zero are fap away.

A ilnal matter with pespect to vertices ls thelp valence. The valence of vertex is the number of edges that meet t the vortex. A vertex valence of three, for examole, Indicates a erthedral copner.

1. C. Intpoduction to BFEV Modeling, (continued).

The Edge.
For a start, the structure of an edge need be thought of as little mope than two vertices; the topological subtlety of edges will be explained later. However, two vertices do define the lmaoptant georetric ecge data called the 20 llne cooffloients. Named $A A_{\text {, }} B B$, CC: these coeffleients are computed from the perspective locus of the edge's endpolnts as follows:

```
AA - Y1 - Y2;
8B - X2 - X1:
CC * X1*Y2 - X2*Y1;
```

These coefficients appear in the 20 equation of the line that contairis the edge:
$\theta=A A X+B B Y+C C ;$
Wher the edge coefficients are normalized:

the line equation gives the distance, of a doint $X, Y$ from the line:

$$
0 \cdot A A * X+B B * Y+C C ;
$$

The oistance is actually ASS(Q), slnce 0 is negative on ons side side of the line; also if one were standing on the olane at ooint $X_{1}, Y_{1}$ facing xa,y2 the $Q$ dosltive half-Dlane would be on your left and the o riegative half plane woula be on your right.

An important oderation on two edges is to detect whether or not they intersect; this can be decided by checking first whether the endcoints of one edge are in the opoosite half planes of the other edge, arid second whether the endoolnts of tr., latter edge are in the opdosite half planes of the first. When bo conditions obtain, then the intersection point can be found:

$$
\begin{aligned}
& T+(A 1 * B 2-A 2 * B 1) ; \\
& X+(B 1 * C 2-B 2 * C 1) / T ; \\
& Y-(A 2 * C 1-A 1 * C 2) / T ;
\end{aligned}
$$

An actual compare for Intersoction should initially checre por the identity case, and for edges wlth veptex in common.
I. C. Introduction to BFEV Medeling, (continued).

The face.
A face is a inite peolon of plane onclosed by stealght lines, A safe formal pace structure could be bullt by deflining a tplangle as theoe non-oolinear vertlces and then insi=ting that all faces be triangle inteplops. Unhadolly, BFEV faces are usually padpesented as list of vertioes and edges (op by something neaply - quivalents for the sake of saving memory space. such 'list' faces ape not monolithic but tend to suffor soeclal cases and oathologies such as:

```
Colncident op erossing odges,
    Holes and Disjolntness,
    Concavity (s Convexity).
    Non-coolanarlity.
```

Like edges, faces have characterlstic ooefflelents, face coofficlents satisfy the equation of a plane in whion the face ls emboded:

$$
A A * X+B B \# Y+C C * Z Z=K K \text {. }
$$

The equation could be divided by $K K$, but that is undesipable because the AA, BB, CC are mope useful as unit ropmal vector, in which case KK is the distance of the oflgin fipom the plane, Given the looll of these non-colineap yeptices, the coefficients of a plane can be corputed by Kramer's rule as follows:
$K K \quad+\quad X_{1} *(Z 2 * Y 3-Y 2 * Z 3)$
$+Y 1 *\left(X 2 * z 3-Z 2 * x^{3}\right)$
$+Z_{1 *}^{*}(Y 2 * \times 3-X 2 * Y 3):$
$A A$. (Z1*(Y2-Y3) + Z2*(Y3-Y1) + Z3*(Y1-Y2)) $:$
$B B$ * $(X 1-(z 2-z 3)+X 2 *(z\}-z 1)+X 3 *(Z 1-Z 2))$ )
$C C \quad * \quad X 1 *(Y 3-Y 2)+X 2 *(Y 1-Y 3)+X 3 *(Y 2-Y 1))\}$
and normalized:
$A B C+S Q R T(A A+2+8 B+2+C C+2) 1$
$A A * A A / A B C ;$
$B B+B B / A B C ;$
$C C=C C / A B C ;$
$K K+K K / A B C ;$

If the given vertices V1, VZ. V3 had benn taken golng counfer clockwlse about the face as viewed from the exterior of the solfd, ther the following polations obtalns

$$
\begin{aligned}
& A A * X+B B y Y+C C * Z<K K I m p l \mid e s X, Y, Z \text { above the olane. } \\
& A A+X+B B=Y+C C=Z=K K \text { implios } X, Y, Z \text { in the olane. }
\end{aligned}
$$

Face coefficlants prove usoful in both world and lmage coopdinates.

Counter to the usual usuage, l defino the word "body" to mean an entity more soecific than a oolyhedron; the ldea being that a dolynecion ls peopesented oy the whole structure of bodes, faces, edges and vertices. Bodies may have location, opientation and volume in soace, Bodies may be conected to faces, edges and vertices, which ray or may not form a complete polyhedron. it is tyoical to have only one body to a dolyhedron when repesesting a pigid object like a slegge hammer and several bodes to a dolyhedron when redesenting a flexible object like a man. Furthermore, the body concedt is used to hanole the notion of parts and abstract pegional objects such as a oarking lot, Fop oxample, the Stanford Al Parking Lot is refresented by a body that has thpee papts: the Neur, mid and far Lots. The Near Lot then has aisles and lanes and lamo Islands; a lamp island has a curb and two lampss lamp has a base, stem and top. inis parts structure is carried in body nodes, Finally, the word "object" will be used to refer to ohysical objects such as a redwoodutree, building. or roadway,

## Preceding page blank

Figure 1.6
FACE PERIMETER - a face is surfounded oy odges and vertices.


Figure 1.7
VERTEX PERIMETER - a vertex is surpounded by edzes and faces.


Figuro 1.8
EDGE PERIMETER - an boge is surfounded by 2 faces s veptlces. VERTEX


VERTEX

1. C. lntroduction to BFEV Modeling, (oontinued).

FOUF KINDS OF GFEV ACCESSI'VG.

```
1. Accessing by name and serlal number.
2. Parts-Tree Accessing.
3. FEV Sequential Accessing.
4. FEV Feplmeter Accessing.
```

A BFEV mocel has four kinds of accessing, ihe most conventional BFEV access is potrlioval oy symbolic name which pequipes a symol tabie. Next, botween bodies there is parts-Tpee accessing. At the top of the Parts-Tree is a spectal body named the worla to whleh all the other sodies are ettached; thus the worle body serves as an OBLIST node, Given a particular body, a list of its sub-papts can de petrieved as well as its suera-part or "supart", a suoart is the whole entity to which a part belongs, the world being its own surart.
within eaci booy thepe ls face, edge and vertex soquential accessing. जlven a oody, all its faces, or edges, or vertices need to be readily availad!e since oerspective projection loods thru all the vertices, and the process of disolay clipoing loops thre all the ejges, and the act of checking for body intersection looss theu all the taces. In LIS ${ }^{\circ}$, ene might provide feV sequential accessing by placing a list of faces, a list of odges and a list of vertices on the property list of eacn body, so that a cube mignt be reoresented as:
(CEFPROP CUEE (F1 F2 F3 F4 F5 F6) FACES)
(DEFPROP CUQE (E1 E2 E3 E4 ES EG ET E E9 E1A E11 : 12)EDGES)
(DEFPROP $\operatorname{ZUEE~(V1~V2~V3~V4~V5~V6~V7~V8)~VERTICES)~}$
Finally, among the faces, edges and vertices of a body there is corireter accessing. Faces have a depimeter of edges and vertices [flgupe i.6]i less commonly used, vertices nave a perimetar of edaes and iaces [figure 1.7]; and of particular note, edges have a derifeter always formed oy two facej and two vertices, [flgure 1.8]. Perifeter accessing peaulres that given a face, edge or vertex, that the peririeter of that entity be readlly accessible. Since the surface of a dolyhedron is orientable, that ls has a well defined inside and outside, (klein bottles with their crosscaps will not ba modeled). suct perlmeter lists can be ordered (say clockwise) with pespect to the extepiop of the dolyhedron. Pepineter accessing is mentioned in Guzran [peference 6] and Falk [peference 4] and is the underlylng basis of part-ll of this paoep which presents a polyhedron model bulit for accessing and alsering face, edge and vertex perimeters.

FIgure 2.1 - BASIC NODE STRUCTURE.

| BODY-BLOCK | FACE-BLOCK | IEDGE-BLOCK | JVERTEX-BLOCK |
| :---: | :---: | :---: | :---: |
| -3. derticodart | -3. | -3, | -3. XHC |
| -2, | -2. | -2, | -2. YHC |
| -1, | -1. | -1. | -1. ZWC |
| D. type | D. type | 0. tyoe | 0. type |
| +1, nfaceppiaco | +1. niacoidiace | +1. niacoidiace | +1. |
| +2, ned, ped | +2. Dod | +2, ned, Ded | +2. pod |
| +3: nut,put | $+3$. | +3, nut,ovt | +3. nut,dovt |
| +4, | +a. | +4. ncw, dow | +4. |
| +5. | +5. | +5. ncewodecw | +5. |
| +6. | +6. | +6. | +6. |

Figure 2.2 - THE WINGED EDGE.
(as viewed from the exterlop of a solld).


PAKT-A., THE WINGEE EJUE JATA ST:JNTJRE,
11. A. Winged Egge Jata Structure.

Boales, Faces, Eages and Veptices ape peoresentec oy blocrs of contiguously aodressed words. A single block size of ton aerds is adeguate. A single word, like a LISP node, can nold tivo adaresses or a floating polnt number. The BFEV blocks are dointec :t cy the adaress of their word numbered zepo which contains coatroi dits Incicating whether the block ls a body, face, edge or vertex. figura 2.1 illustrates the block format that is being presented as an exanole of a winged edge data structure; a minimal numper of words for each block is indicated.

The basic geometric datum is the vertex locas, which is stored in three words of each vertex block at positions -3, -?, -1; these dositions are named XWC, YiC, ZWC respectively; the letters "wC" s'candina for "world coordinates".

The basic topological data are the three rings of the body; (a ring of faces, a ring of odges, and a ring of vertices) and the wingededge polnters (eight such pointers in each edge block, and one suct delinter in each face and vertex olock). The face, edge and vertex ping dolnters are stored at positions $+1_{0}+\frac{\bar{c}}{},+3$; eacn dosition has two names: NFACE, :NED, NVT for the left dolnters respectively; and PFACE, PEC, fit for the pight. A face, edga or vertex can only belong to one cody and so there is only one bosy node in a glvan face, edje or vertex ring: and that body node serves as the heas of the ring. The reason for double pointer pings is for the sake of padid celetion; other rinor advantages would not justify the use of doudie plings.

The eight wivged pointers of an odge block lnclude: two Dointers to the faces of that edge, two dolnteps to the vertices of that edoe, and four dointers to the next adges ciockwisa and counter clockwise in each of that edige's two faces; these last four pointers are called the wings of that edge. As figure 2.2 suggests, four of these eight polnters are stored In the same dositions and referred to by the sary names as the face and vertex ring pointers; namely the NFACE, PFACE, NVT and PVT oointers. Thepe are four ways in whicn a Dair of faces ans a eair of vertices can be placed lnto tme tio face Dositions and two vertex dositions of an edge; by constraining ehese choices two blts are impllcity encoded, ora bit is called the easge Darity, and the other is called the supface zapity; these viss are exolained later. Finally, the single winged edge pointor found in faces and vertices is kedt in the position named PED and it ooints to one of the edges belonging to that face or vertex.

Athougn the vertices in figure 2.2 are shown with three edges, vertices may have any number of edges; those other zorential edges would not ne aifectly connected to $E$ and so were not snown.
a summary of winged edge operations.

11. b, The Winged Egge Ooerations.

Uyraric Stopage Allocation,
At ine very botton, of what is becoming a pathep deez mast of Driritlves within primitives, are the two dynanic storage allacation furctions GETBLK and RELRLK. GETELK allocates fron i to kK words of merory scace in a contiguous block and peturns the machine audess ef the $+i r s t$ word of tinat block. RELBLK peleases the indicater bleck to the availaole free memory space. (lt is sad that the machines of our day do rot come witn dynajlc fpee storagel. A good reference for implemerting such dynamie storage, mentioned earliep, is knuth [reference 7]. Altnough a fixed block size of ten or fewer words car. be rade to hanole the 3 friventities, grandiose and fickle reseapch. apolications (as Nell as memory use optimization) oeman the flexibility of a vapiable nlock size.

BFEV Make $夭$ Kill Operations.
Just above the frea storage poutiries are the four paips of rake ano kill opepations. The MKB operation cpeates bodj block and attaches it as a sub-part of the given body. The worlo fosy always exists so that MKB(NORLD) wlll make a oody attached to the world. in this oaper, the terms 'attach' and 'detach' pefer to opepations on the earts-tpee linkases. The FEV make operations: mKF, rike, mkv create the copresponding feV entlites and olace tnem ir tneir respective FEV pings of tha given booy. In the cuprent implementation, the FEV makers set the type bits of the gatity, end
 FEV counter in the given body's node, (the font. Ecnt, vert node positions ape shown in figure 2,3). The kill operations: KLS: KLF, KLE, aro KLV; celete the entity from its pirg cor removo it from the Darts-tree), pelease its space by ca!ling RELBLK, and then decrement the appropilate counters. The body of the entity is needed oy the kill primitives and can be provide dipectly as an argument op if rissing, will oe found in the data by the drimitive itself.

Fetch Link and Store Link Operations.
Each of the fetch link and stope link oparations namad in the surnary is a single machine Instruction that arcesses the coprespondins link position in a node. Once EFEV nodes exist, wlth their rings and partsatree already in olace; the fetch and stcre link. odepations are useo to construct or modify a polyhedron'z zurfzee. At thls lowest level, constructing a polyhedron regulires onpee steos: first the two vertex and two face dointers are rlaced inte each edye in counter clockwise order as they apdear when that edge is viewea fror the exterior of the solldi socond an edge pointer is uleced in eacr. face and vertex, sc that one can later get from a given face or vertex to one of its edges; and tilird the edge wings aro linkea so thai all the orderes oepimeter accessing operations describeo below will wepk. Wing linking is facilitated by the wlivg oderaticn.

| FIGURE 2.4-MIDPOINT EXAMPLE (sen toxt on Dage 20). |
| :---: |
| INTEGER PROCEDURE MIDPOINT (INTEGER E): BEGIN "MIDPOINT" <br> INTEGER B,ENEW,VNEW,V1,V2; |
| ```a Create a new edge and vertex: B - BODY(E); VNEW + MKV(B): ENEW * MKE(B);``` |
| ```a get vertices and faces connected fo edges: PVT,(PVT(E),ENEW); PVT,(VNEW,E): NVT, (VNEW,ENEWI; PFACE,(PFACE(E), ENEW); NFACE,(NFACE(E),ENEW):``` |
| ```a get edges conNECTED TO VERTICES; IF PED(V)=E THEN PED.(ENEW,V): PED,(ENEW,VNEW)S``` |
| ```* LINK THE WINGS TOGETHER; WING(NCCW(E),ENEH)/WING(PCW(E),ENEW): NCW,:E,ENEW);PCCW.(E,ENEW): PCW,(ENEW,E)INCCW,(ENEW,E);``` |
| - Place vnew at midpcint position; <br> V1 - PVT(ENEW): V2 - NVT(E): <br> XWC(VNEW) - (XWC(V1)+XWC(V2)) 0.5; <br> YWC (VNEW) - (YNC(V1) +YWC(V2)) - 0.5; <br> ZWC(VNEW) - (ZWC(V1) + ZWC(V2)) • ©.5; <br> RETURN(VNEW): <br> END "MIDPOINT": |

The wing Link Operatien.
The kJNG ooeration stores edge pointers into edges so that the face perimeters and vertex perineters are made; anc so that surface paplty is oreserved. Given two edges whlch have a vertex and a face in common, the wiNG operation olaces the first edse in the profer pelationshlo (PCW, NCCW, NCW, or PCCW) with pesject to the second, and the second in the proper relationsh!o with resoect to tho first. The INVERT operation sinas the vertex, face, clockwise wing, and counter clockwise wing pointers of an edge, INVERT opeserves surface parlty, but flips edge parlty.

The Midooint Examole.
In figupe 2.4 an example of how the oderations given so far coula be used to code a midpoint pplmitive is shown. fhe examole miapoint primitive takes an edge argument and splits it in two by making a new edge and a new vertex and by olacing them into the oolyhearon with the topology shown in the diagram. Then the midoolnt locus is computed and the new veptex is return. The ALGOL notation Usea is SAlL, which allows defining the character "al as a COMMENT delimitar and allows defining XWC as a real number from the spectal erray named MEMORY. The MEMORY array In SAlL is the job's actual machine memory space and gives the user the freedom of accessing any kore in his coro image.

Tha Partg-Tree Operations.
As shown in figure 2.1, each body node has two oartsetpee links named PART and COPART. The PART link is the head of a list. of sub-Darts of the body. When a body has no sub-darts the part link is the negative of that body's colnter: that ls the body goints at Itseif, when a body has parts, the fipst cart is pointed at by PART and rine second is pointed at by the copart link of the first and so on unt!i a negative puintep is retrieved which indicates the end of the parts llst, ine negative oolntep at the end of a sarts list points back to tre orginal body, which is the supra-part or "supart" of all those bodies in that list.

The parts may bo accessed by its IInk names PART and COPART. A|sc the SUPART of a hody peturns the (dositive) dointer to the supart of a body. The gofy operation returns the body to which a fase edge op vertex belongs: this might be found by Coring a FEV ring until a bocy node is reached, but for the sake of speed each edge (as shown in flgupe 2.3) has a pBocy I Ink which points back to the body te which the edge belongs, and since each face and vertex doints at an edge; the budy of an FEV entity can be petrieved by fetcring only one of two links.

Part Tree Operations (continued).
The parts-tpee is altered by the DEY(B) odaraticn wich peroves a body $\}$ fom its supart and leaves it hansing free; and the ATT, o1, BC) operation which places a free body R1 irto the aarts list of a body BZ, Since bodies are made attached to the world ocdy and generaliy kept attachec to something, two furthor oartsetree operations are grovided, compounding the first two in the recessary rarner, The DETACH(?) odepation DET's 3 from lts current owner and ATT's it to the worldi and the ATTAGH(B1,R2) operation aill EET ji fror its supart and attach it to a new supart. In nermal fone world) circuinstances one only neads to use ATTACH to tuila thines.

Perireter Fetch and Store Operations.
Thepe apa seven perimeter fetch prifitives, whict arien छ̈iven an eage and one of its links will fetch another link ir acertain fashion, Using the winged edge data structupe these primitivas are easily implemented in a fow machine instructions which iest the tyoe bits ano tyoically do one or two compares, clockwise ard aounter clockwlse are alwass determined from the outside of a polynedron looking jown on a particular faca, edge or veptex, 1 aroloziza for the hich redundancy on the next dage, tut felt that it was neaessary to rake the explanazions indedendent for reference,



FIGunt 2.6 - Vertex Ferimeter accessins with resoect tu ocie e.


The Perimeter Fetch Operations.
$E$. ECW(E,F): Get Edge Clockinse from
E E ECW(E,F); Get Edge Counter Clockwise from E aoout F's oepimeter.
. Given an edge and a face belonging to that edze, the ECW
fetch primitive returns the noxt edele clockilsa belonging to the given face's derimeter and the ECCW fetch primitive returns tha next edge counter clockwise belonging to the given face's perimster,

E * とCW(E,V): Get Edga Clockwise from E about V's perimeter.
E - ECCh(E,V); Got Edge Counter Clockwise from E about V's derineter.
Given an edge and a vertex belonging to that edge, tha $\overrightarrow{E C W}$ fetch pimitive retupns the noxt odge clockwlse belonging to the given veptex's derimeter and the ECCW fetch orimitive returns the next edge counter clockwise belonging to tine given vertex's perimeter.
$F$ * FCW(E,V): Get the face clockwise from $E$ about V.
$F$ - $F C C W(E, V)$; Get the face counter clockwise from $E$ about $V$.
Given an edge and a vertex belonging to that edge, the FCW fotch ppinltiva peturns the face clockwise from the given edje about the glven veptex and the FCCW fetch primitive peturns tine face counter clockwise from the given edge about the glven veptex.

V . VCW(E,F); Get the vertex clockwise from E about F. $V$ - VCCW(E,F); Ggt the vertex counter clockwise from E about F.

Given an edge and a face belonging to that edge, the VCM fetch primitive peturns the vertex clockwise from the aiven edse about the given face apd the VCCW fetch primitive petupns the vertex counter clockwise from the given edge about the given face.

F © UTHER(E,F): Get the other face of an esge. $V$-UTKER(E,V): Get the other vertex of an edge.

Given an edge and one face of that edge the ciHER fetch Drimitive peturns the other face belanging to that edge. biven an edge and one vertex of that edge the oTw[R fetch primitive petupns the other vertex belonging to that odge.
II. C. Elaborations on Winced Ede Structure.

Ir this section. some variations on the basic winged edge structure ara given. These variations arise as adeotatisne for ry aoolication, any as unimplemented ideas for inorcvenonts. The adaptations, shown in figure 2.3. include adding serial numbers and ALl links to all the facts, edges and vertices. the serial miners crcuije another way of addressing and are especially useful curlnat inflate arc output. The ALT link is used for jointing to adilitional fut terfcrary data; the most elaborate alt data has to do with eases during a hladen line elimination. Sacrificing memory space for sued and flexibility, the face and edge coefficients are stares in each no ce, and the inge coordinate (XDD,YDD,ZiD) and display coordinates
 coordinates mosel a carrara and the display soorainjtes refer to location on a jisilay console. Having two tiers cf image coordinates allows strolling about the modeled image without crarijlr.j tref caldera for heaven fopbidter, having to redo a hater line elimination;, The remaining so far unmentioned names include: th. Tjoirt link in vertices sick is for shadow end hider er lino operations, the the ? word in faces which contains photometric data, writhe LOCUR and F:GME links of a boon node which eire io a locatior-orientaticr matrix and an ASCII dint nate respecialely.

Sacrificing speed for the sake of memory, Ene effect of having rest of the extra data mentioned above car be atilaved bu recomputing it rather than fetching it. Furthermore, the ifinga cate stricture can be made slightly smaller by eliminating tee fare and vertex pings. Faze and vertex sequential accessing carisitll ae join by ravirc two marking sits in each face and vertex, and by tire :o in o thai the ede rind looking at the two faces and two vertices af each EOE E for ones that are not freshly harked. It would be nice ; such ecorcmizirs could de cone below the level of the operations.

Hesices optimizations, the next improvement ideal facula like to atterrot would be to split the notion of a body inti the two notions of a "nard" and a "cell". Parts would rave the darts tree ans nares that bodies now rave, whereas a cell would have lo lure and face structure. !n this hypothetical Cell, Face, Edge, Ueptex ( $\quad$ (iv) riocel, each face could point tc a cell on either side of it, tin cell with the lower serial number (or something) being con tithed as exterior. Coll nimger zero would de the infinite void of tree e so ace in nhict. everstaire is embedded. The trouble vicing CFE: is that the impertart matter af a dolytiecron surface has to be salvage; it ran not de atanoonec, cecause models without good surface perresfrtations can not presici dofaarance, which is one of my reajiremem.s.


```
SUMMARY OF POLYHEDRON DRIMITIVES.
a. euler primitives.
    1. BNEN M MRFV; make a cody, face s vertex,
    2. KLEFEV(G);
    3. VNEW - MKEV(F,V);
    4. ENEW - MKFE(V1,F,V2);
    5. VNEW - ESFLIT(E);
    6. F - KLFE(EvEN);
    7.E - KLEV(VVEW);
    8. V - KLVE(EVEW):
    9. B - GLUE(F1,F2);
10. BNEW + UNSLUE(E);
kill a body f all its pieces.
    make edge & vertex.
split an edye.
kil| face 8 edge lgaving a face.
kill edge & vertax laving an edge.
kill vertex s edge leaving a vartex.
glue two faces together.
unglue along a seam containligg E.
B. SULID PRIMITIVES.
    1. VPEAK - PYRAMID(F);
    2. F + PRISM(F):i
    forma pyramid on a face.
    form a rectangular prism.
    3. F . EWPRISMOID(F)
    4. F . CCWPR!SYOID(F);
    5. ROTCOM(F);
    form a clockwlse prismois.
complete a solld of rotatisn.
    6. FVDUAL(B); form face vertex dual of a bojy.
    7. BNEN - MKCOPY(B);
    8. EVERT(B);
    8. EVERT(B);
    10. 31 - BIN(91, B?):
    make a cooy of a body.
    turn a body surface Insite out,
    form unlon of body intepiops.
form intersection of body intepiors.
C. GEOMETRIC PRIMITIVES.
    1. translate(0.r);
    2. ROTATE(Q,q);
    3. gILATE(G,२);
    4. REFLECT(O,R);
```

D. IMAGE PRIMITIVES.
1. FRUJECTOR(CAMEKA, NORLD):
2. ELIST-CLIPER(NINOJW,NORLO);
3. UCCULT(WORLJ):
4. SHADOW(SUN. $\operatorname{CoRLD}$ );
5. TV - MKVID(HINDOW, HORLD):
6. 220 - MKR2U(WINDON, WCRLD):
7. P2O - CAREYE(TV);
Urider construction, Oct 1972.

IIl. Hhimltives on polyhejra,
In this section a number of orimitives for cioing things to Dolynedra are explained, Although these primitives are curpently implemented using the winged edge data structure, they do not pequire a particular polynedron representation. Indeed, many of these prifitives ware originally Imolementad in a leap dolyhadron peppesentation very similar to that of falk, feldman and paul [reference 5]. Thus, the primitives of thls section ape on a level logically independent from the operations of the previous section.

Ansthep aspect of these primitives is that they san re used as the basis of a "yraphics language" or nore accurately as a jackaye of subroutines for geometric modelling, In this vein, the orinitives are curpently collected as a dackage called GEOMES for ceomatric Modeling Embedded in SAIL; and as GEOMEL, Geometrlc Modelirg Embedded in LISP. A thipd language, called GEOMED, arises out of the sommand language of a geometric model editor based on the drimitives.

The primitives are shown in four groups in the sumpary. The first group, the Euler primitives, were Inspired by coxeter's groof of Euler's formula, section 10.3 of ireference 2j. Althougin ine proof only pequired three primitives, additional ones of the sare ilk were develoded for conveniance. The second groud is composej of some dolyhedron ppimitives that were coded using the Ejler drimitivas. The thipa group is for crimitives that move bodes, faces, edjes anc vertices; or comnute geomatrlc values such as length and volume, Tris group ls underdeveloped for two peasons: one, because ! have done these computarlons ad hoc to date; and two, bacause they lmoly the subject of animation which ls large and difficult and not af central imoortance to vision. Witn the exception of the camera, ty worlds are nearly (but not absclutely) static. A less impoverisned jeometric. groud will be presented in the future. The final groud, has three well developed drimitives for making 20 images; anj several Drimitivas that when finished wlll pealize part of the vision syster that 1 am trying to bulld.
lll. A, Eulop Ppimitives.
As mention above, the Euler pplmitives are based on the Eulep Equation $F-E+V=2 * B-2 * H$; where $F, E_{1} V, B$ and $H$ stand for the number of faces, edjes, vertices, bodes and handies that exist. the term "handie" comes from topology, and is the number of well formed holes In supfeces a sphere has no handles, a torus has one handie, and an IBM flowcharting template has 26 handies. The Euler equation restricts the dossible todologies of FEV graphs that can be Dolyhedra; although such Eulerian polyhedra do not recessaplly corpespond to what we nopmally all a solid classical solyhedron. Strict adherence to constructing a dolyhedion that satisfles Eulep equation $F-E+V=2 * 3$ - $2 * H$ would require only four primitives:

1. Make Body, face and Voptox
है Make Edge and Vertex.
2. Make face and Edge.
3. Glue two faces of one body.

Howevar, the four corresponding destructive orimitivas are also Dossible and desirable:

1. Kill Body, Face and Veptex
2. Kill Edge and Vertex.
3. Kill Face and Edge.
4. Unglue along a seam.
4.' Unglue along a sam.
```
+F -E +V = 2*B - 2*H
```

+F -E +V = 2*B - 2*H
-1....-1....-1......
-1....-1....-1......
...+1 -1............
...+1 -1............
-1 +1................
-1 +1................
+2 -N +N..........-1
+2 -N +N..........-1
-2 +N -N....+1......

```
-2 +N -N....+1......
```

And ifally the operation of splltting an edge at a midpoint into two edges oecame so important in fopming p-jolnts during hifden line elimination that the ESPLIT primitive was introduced in olaca of the equivalent KLFE, MKEV, MKFE sequence. are tolerated using the Euler primitives, some non-classical oolyhedra trarsitional states are called:

Somlnal Polytiodion.
Wire Polyhedron.
Lamina Polyhedron.
Shell Polyhedron.
Face with wire Spurs on its perimeter.
A serinal polyhedron is like a doints a wire polyhedren is lineap with two ends like a single diece of wire; lamina and shell dolyhedra are surfaces, and the picturesque phrase about spurs is a restriction on how faces are dissected into more faces: These terms will be explainec in mope detall when they are needed.

$$
\begin{aligned}
& +F-E+V=2 * B-2 * H \\
& +1 . . . .+1, . . .+1, . . . . \\
& \text {...-1 +1,........... } \\
& \text { +1 -1................... } \\
& -2+N-N . . . . . . . . . .+1 \\
& -2+N-N, . . .-1, . . . .
\end{aligned}
$$

IIl. A. Euler Primitives.

1. EVEW MKBFV: Make Seminal Eody.

The MKBFV opimitive pgtupns a body with one face and one Vertex and no edses, othep bodies ape formed by adoiying orimitives to the seminal MKEFV hody. The seminal body is initially attached as a part of the world.
2. KLBFEV(BNEW): Kili Body and all its pleces.

The KLBFEV drimitive will detach and delete fom momory the body glven as an argument as well as all its faces, edoes, vartices anc suo-parts.

## 3. VNEW - MKEV(F,V); Make an edge and a vertex.

The MKEV, primitive takes a face, f, and a veptex, $y$, of $F^{\prime}$ s Derifeter and it creates a new edge, ENEN, and a new vertex, VNEW. ENEh and VNEW Epe called a "wire spur" at $V$ on f. IIKEV peturns the newly rade vertex, VNEN: ENEW can be peached since PEO(V'EW) is always ENEW, Only one wire spur is allowed at $V$ on $F$ at a time.

When apolied to the face of a seminal body, MKEy fopms the speelal dolyhedpon called a "wire" and returns the new veptex as the "negative" end of the wire. A wirc dolyhedron is illustrated in figupe 3.1. When apolled to the negatlve end of a wire, MKEV extends the wife: however if apolied to any other vertax of the wire, MKEV refuses to change anything and merely petupns its vertex apgument.


```
FIGURE 3.4 - TNO EXA:MPLES USING EULER PRIMITIVES. (see page 30).
a make a cuge;
INTEGEK PROCEDURE MKCJRE;
BEGIN "MKCUBE"
    INTEGER G,F,E,V1,V2,V3,V4;
a CHEATE SEMINAL DOLYHEDRON;
    G - MKPFV; F - PFACE(B); VI + PVT(B);
    XWC(V1)w+1; YNC(V1)w+1; ZWC(V1;*-1;
a MAKE SEMINGL POLYHEDRON INTO A LAMINA POL.YHEORON;
    V2 - MKEV(F,V1); XWC(V2)--1;
    V3 - MKEV(F,V2); YWC(V3)-a-1;
    V4 * MKEV(F,V3); XWC(V4)a+1;
    F + NiKFE(V1,F,V4);
a MakE FOUR SPURS ON THE LAMINA;
    V1 . MKEV(F,V1); ZWC(Vi)
    V2 * MREV(F,V2):
    VZ + MKEV(F,VZ):
    V4 * MKEV(F,V4);
a JOIN SFUHS TO FCRM FINAL FACES;
    E. - MKFL(V1,r,V乍;
    E - MKFE(V2,F,V3);
    E - MKFE(V3,F,V4):
    E - MKFE(V4,F,V1);
    HET!JRN(H);
END "mkcube";
~ FCKM A PYRAMID OV A FACE;
INTEGEK PROCFDUHE PYFAMID (IITEGER F);
BEGIN "PYRAMIL"
    inTEGER v,VO,E,Ef,pEak,EX;
    #EAL X,`.B; INTEGER I;
    \lambda+Y+Z+1+2;
- get a vertex of the face and make a spur to a pe.ak;
    E&ET&FEC(F);
    VE - VCN(EO, 5;;
    PEAK - MKEV(F,V者);
a CONNLET THE OTHER vEPTIEES gF the face to thf. peak;
    NHILE. TRUE こ0
    !5心l:
        v - ハCCW(E,F);
                X-X+XWC(V); Y-Y+YWC(V); F
                !'\CPEM(1);
                I= v=Vz THEV OOME;
                E ECCW(E,F);
                Ex - UKFE(PEAK,F,V);
    E..);
o rosilinN the pfok verpex at the center of the face;
    XNC(PEAK)-X/!; YA((FEAK)+Y/l; ZNC(PEAK)+Z/!;
    =ETHAN(F:AK);
A:= "PYM&かiJ";
```

4. ENEW $-\operatorname{MKFE}\left(V_{1}, F, V_{2}\right)$;

The NKFE primitive can be thought of as a face spi it. iven a facs and two of its vertices. MKFE forms a now face on the cinckwlse side of the line $\sqrt{ }$ th to $V 2$ leaving the old face on the counter clockwise side, V1 becomes the PVT of Eided, V2 secomes the NVT of ENEW, F becones the PFACE of ENEW and FNEW oecomes the iNFACE of ENEW; also ENEW becomes the PED of $F$ and FNEW.

Figure 3.3 - MKFE and KLFE.

AFTER FaKLFE(ENEL):

AFTER EVEWんMKFESV1,F, 保;


MKFE is also used to join the two ends of a wipe oolynedpon to form a "lamina"; or the two ends of wire spurs to split a face; or an end of a wire soup and a regular perimeter vertex to solit a face. A "lamina dolyhedpon" has only two faces and thus no volume.

## EULEK EXAMPLES,

The use of the drimitives discussed so far is illustrated by the example subpoutines in flgupa 3.4 on daje 29. the mare cube sutpoutine stapts by placin's a seninal vertex at (1, 1,1); tnen a dife of thpee edges is made using the MKEV primitive. As the coje inolies, HKEV olaces its new vertex at the locus of the old one. The ends of the wipe ape joined with a MKFE to fopm a lamina polyhedpon, then a spur is placed on each of the vertices of the lamina, and finaliy the spups ape Joined.

The pyramid exatole ls more palistice since polynedra are not generatedex nihil, but rather apise out of the vision poutines and the geometric editor. PYRA:HID takes a face as an argument iwhich is assured to have no spurs) and runs a spur from one veptax to the ricale of the faces, then all the pamaining vertices of tae face are Joined to that snur to form a nypamid.

Ill, A. Euler Primitives. (Continued).
5. VAEW - ESHLIT(E); Edge Split.

This primitive splits an edge by making a new vertex and a new edge, its implementation is very similar to the miodeint examrie on cage 19. ESPLIT is heavily used in the hidden line eliminator.

```
6. F * KLFE(ENEN');
Kill face fog ge.
```

This prlinitive kills a face and an edge leaving cue face. sire this primitive is irtenced to be an inverse of MEre, the Naacp of EAEM is killed. forever the NFALE and PFACE of an edge rave be sheared by using the INVFRT(E) primitive. See figure 3,3 fer K! fE.
7. E - KLEV(VNEN);

Kill Edge vertex.
This primitive kills an edje and a vertex leavine one ease. This primitive will eliminate spurs made with MKEV ard niocoints made with ésfl.! T; in a ape form lt would have to leave vertices with a valence greater than two antowched, however it in fact "umaypamios" then with a series of KLFE's and then kills the remaining sour.

ع. V - KLVL(EVEW);
kill vertex Ease.
This primitive kills a vertex and an edge leaving ane vertex. This primitive is tap face-vertex dual of Klfe, namely instead of killing NFACE of E and fixing uv PFACE's perimeter, KLEV kills the NVT of $E$ and fixes we $\operatorname{DyT}$ of E's Derimater.
9. b-GLUE(F1,F2); Glue two faces.

This primitive glues two faces together forming one ned jody out of trio ola ones (ste body of fa survives) of forming a handle on the given jody, the number of oates in the two faces must he the same ane their orientation should be opposite exterior te exterior!. *IC. BNEN - UNGLIE(E); Unglue along seam. *not implemented.

This primitive unglues alone the seam containing E. The Uhulut primitive requires that a loon of egest se napkedas a "e The alcrig which unglue will form two opposite faces. tine rijras are made in the temporary aye sit In $t^{\text {he }}$ edge nodes of the given beds. If the cut forms two disjoint bodies then a new dozy is -ave on the NFACE side of the original E argument.


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11!. B. SOLID PRIMITIVES.

1. VPEAK - PYRAMID(F);
2. F - PRISM(F);
3. F - CWPRISMIOD(F):
4. F - CCWPRISMIOD(F);

These four primitives ape called the "sweed primitives", because they fopm a simple polyhedpoit from a face in a fashion that apdears like sweeding the face along. The sweep primitives fwith the excedtion of PYRAMIO do not change the location of the aiven face but merely copy its peplmetep, fopming new faces and edges between the old perlmeter and the now perimeter, The pyramid dpimitive has already apdeared as an example on page 29.

Starting with a nine sided face lamina, the rocket in pigure 3.6 was fopmed from the bottom by sweeding two ppism stages, then two counter clockwise prlsmoid stages, and then two clockwise prismoid stages, and finally one pyramid to fopm the nose cone. ine fins were made by prism sweeding everythlfd face of the first stage.


FIGL'RE: 3.6 - Rockets made with sweep primitives.


FIGURE 3.7 - Solid formed by rotation.
$\{$



Euclid's construction of a dodecahedron from a cube.



FIGLRF. 3.8 - Dual of a Dodecahedron.
111. b. Solld Ppimitives. (continued).
6. FVDUAL (B):
7. ENEWaMKCOPY(B:;

These two primitives illustrate the extremes from a class of tiscellaneouz drimitives, FVDUAL is a worthless curosity and MKCOPY is quite useful but uninteresting. FVDUAL(B) of a body changes all the faces of a body into vertices and all the vertices into faces, in the winged edge data stpucture this merely requires compjating a locus for each face (its center), re-"tyolng" faces and vertiees, and then swapoing the face and vertex link positions in each face, edse and vertex of the body.
figure 3.8 illustrates Euclid's construcijen of a dodecanedion from a cube. The unlt cube is fopmed, then ailits ejges are midoointed and translated 6.2 units into the throe daips of Darallel faces; then the mldoolnts are lifted ol 3 units off tha olane of each face of the cube; then MKFE is apolied six times to sollt the eisht sifed faces into five slded faces; giving a dodscahedron (neaply regular). ADolying the fVDUAL to the dodecahearon yields the icosanearon.

Ill. B. Solld Primitives. (continjed).
5. EVEfT(s);
9. 31 -GUN(31, B2);
19. B1-EIN(E1,E2);

These thpee orinitivas perform the iogolaar operations on Dolyhedpon intepior volumes. EjEfT(B) turns a body inside out, thus charging a cube into a room, as a solid into a bubole, cejects with infinite "interiors" are dermissitie; such polyhedra ape impossitele in rany classical develooments of solid geometry witich make the interior of a dolynetron to be the region of space with finite velure, by definition. the body union is PUN, which allows El to survive if the interiors of the todies are not disjoint. a body with two disjoint polyradrons is shunned. The booy intersection is siv. which allows 81 to survive if the interiors of the bodies are not aisjoint.


TWO BODIES


BODY UNION

FIGURE 3.9

C. GEOMETRIC PRIMITIVES.

```
1. TRANSLATE(Q,Q); Q argument is a body, face, edge op veptex.
2. RUTATE(Q,R);
3. DILATE(Q,R);
4. REFLECT(O,R);
```

The four Euclidean transformations are translation, rotation, reflection and dilation; and as first mentioned in Klain's Erlangen program, 1872, these four primitives form a groud. the primitives may de appliad to bodies, faces, edges or vertices in opder to chanse vertex mopld locii. Thus a body is the set of vertices in its vertex ping, a face ls the set of vertices on its perimeter, an adge is the two vertices which are its ends, and a single vertex is itself; but there are special cases having to do with faces. IIn GEOMED a special counter, nesative fent, is maintained in wipe sweer faces in order to make solids of potaticns. The second argument $R$ is a dolnter to a transfopmation array in world coordinates of four pows and three columins:

| $X W C$, | $Y W_{i} C$, | $Z W C$ |
| :--- | :--- | :--- |
| $I X$, | $I Y_{1}$ | $I Z$ |
| $J X_{1}$ | $J Y_{1}$ | $J Z$ |
| $K X_{1}$ | $K Y_{0}$ | $K Z$ |

For trarslation, only the XNC, YiNC and $Z W C$ are involved and all the vertices ara translated in the obvious faghion:

$$
X-\ddot{x}+X N C ; \quad Y-Y+Y W C ; \quad Z-Z+Z W C ;
$$

Whereas for rotation (dilation and reflection) the innermost corputation apolied to eacn vertex is:

```
X - X + XNC: Y Y Y Y YWC; Z Z F Z + ZWC;
XX * IX#X + IY*Y + IZ*Z;
YY - JX#X + JY*Y + JZ#Z;
ZZ + KX*X + Kyav + kZ#Z;
\lambda - XX - XWC; Y M YY - YNC; Z Z ZZZ - Z;NCi
```

At this roint,l shcuid now dresent a few general prinltives for setting up such transformation aprays, but l don't have them yat. The eroblem involves selecting frames of peferences, strength of trarsformation, axes of transformations, origins of frames and medes suct as aosoluta, relative or intarpolated. at oresent in my apolications these matters are hanaled ad hoc ithe most jenepal solution belng the zotoel and EUCLIC subpoutinas of GECMED). The heart of deriving a transformation array is to get a frame of reference fef anc an amount of potation DEL and to comnute the matrix oroclet:

```
R - (transpose(REF)cposs(DEL cross REF)):
```

For dilation (larger or smaller) cross DEL with a non-urity diagonal Taipix; for peflections flid the row signs on desired axes.
j. image frimitives.

```
    1. FRGJECTULG(CANEPA,GOKLD);
    2. ELIST-CLIPER(WINDU'*,WORLO);
    3. CCCULT(WJマLう);
    4. SHACON(SUN,NORLJ):
    5. IV - MKVIJ(NINDON,NORLO):
    6. :S20 - MKB20(NINDOW,NERLD);
    7. EzUS - CAREYE(TV);
```

* Urder construction, Oet 1972.
fROJECTUR compates the derspective projected locus of all tne vertices in a given world from a given samera. CLIPFR coroutes the Dortions of 30 lines that are vislble within a given disolay windew. OCCLL c cmpares all the atoes, faces and vertices in a aiven world; usirg their current projected coopdinates; faces, adges and vaptices that are not visiole from the lmplied camera's viewooint are marked as hidder: faces, odees and vertlces that apy visiole are mapked as visiole; and faces, edges and vertlces that were initially sartially visiole are broken up into visible and hiden dortions. The new faces, ecges and vertices introduced by zCCUL tre meriese so that they can be pemovad.

The following four primitives are stil: seing daveloded. Shacul will literally build a world with shadoins in it; shadow calls OCCLLT twice, once for the SUN and once for the camera, Tnere is no concedtual difficulty in doling many point sources, but : shall get one source working at a time. The MKVID prinitive gerepates TV intenslty rasters from she world model after OCCULT or SHADON has been adolied. The wrges primitive generates a 20 data structure of reglons and eages (which is almost a cody of the 30 structure trat has been presented, cut with soecial attention paid to p-jointsi; this 820 data siriscture is an image model. Finally, the CAREYE reifitive converts iV Intensity rasters into b2J image structure. A detailed alscription of these image primitives can not se jiven at this tife COCT 1972), because 1 haven't finished making then.
IV. ArHLICATIUNS.

The single adolicaticn around whicn the teometric moteling of tris paper ls beiry ouilt is for a computer television vision (TVV $\mathrm{i}_{\mathrm{i}}$ ) system for looking at real world scenes. l believe that a computer rust have a means of pefresenting what it is intended to see anc furtner that the iepresentation must have (in princinles an inverse relatior to a television image. my first premise is rarely questioped. the second premise is frequentity questioned. One alternative dosition is that so called "features" can oe extracted fron ar imaje and then used ty a heuristic droblom solvep to find an association oetween the percelved featuras ano previcus general knonleage; it is then stated that there is no need to go from the general knowledee or even from the so called linage "feazures" dack dowr to a television image, oven just in principle. i wish to state the odposite, there is a neen to go from the general rearesentation to a television itage $\quad$ n orser to devalod computer visicn vitnout $n_{\text {aving to solve ser pal other protlars of Artificial intelligance. }}^{\text {a }}$ AcDllations of goometric looeling other than talevirion vision mignt Include: architestural dradins, conouter animation, anc moceling for laser, radar, and sonar image systems.

6
?
IV. A. Nodeling: GEgrED - a drawing program.

GEUMEJ, jegmetric Model Editor, is.for making and editing solynoapa. The cominand language of GEOMED provides the Eulep orimitives in the context of a push down stack (the PADPDL) of oocies, faces, edges and vertices. The maln difference between an interactive program and a programming language being that the former carpiss along a working context so that most arguments any data do not nave to $=0$ explicitly namen.

| $V$ | - | make seminal vertex body |
| :--- | :--- | :--- |
| $E$ | - | make edge and vertex. |
| $J$ | - | nake edge and face. |
| $G$ | - | alue tio faces. |

In addition to the stack. GEOMED provides frames of peference for the Euclidean transformations; there is a worldframe, body frames, camera frames, relative frame and face frames. Also the strength of a Eucildean transformation can be halved op double, set dipectiy op entered numepically in several kinds of infis. and finally the transformation can be done once or pepeatedily by keying choros of Stanford's extra shift keys mamed "control" and "meta" wlth a ; : ( ) or character. These characters are not mnemonlcs but were chosen because of thier positions on the keyboard.


Finally, GEOME provices access to all the solid orimitives anc hioden line elimination, along wlth commands for the stack, incut, output, display, and switch toggling. The commands are detailec in the ooerating note, SAlLON-68, along with a list of GEONES and GEOMEL subroutines. Two examoles should suffice to illustrate how concise and illegible GEOMED command strings are:

```
1. V:NE;E(E:J+/*S-*
2. V:OE#E#E#E#E#E*E#J+:
(\:PS)SJSIS/SISISJSIGq
forms a torus.
```

Thus a colyhedron can be peoresented in a few characters iwhich must be corrifiled) one might nose that such a "representation by generation" coula provide a llak between semantic and geometpic rodels. The hard dipection is to get from a polyhedron model to the corrana string.
IV. b, Graphics: oCCULT - a hidden line ellminator.
eccult is a hidden Iine ellminator; it is neither a Watkins nop a warnock algopithm but is pathep a throw-back to the naivo idea of comparing gach edge with all the othep edges and having ways to dampon the potentially large number of comparlsons that misht ocoup.

There ape three kinds of dampening in oCCULT. The first fused in other hideen eliminatopsi is to get pld of the faces that have thelp tacks to the camepa and to consider for comearision only the edges with one dotentielly visible face. These edges are called "folds". The second kind of damoening, is to hide evepything connected to the hidden portion of an edge when a fold ciossing is discoveped, this is made possible by the winged edge primitives whleh allow dolyhedpon supfaces to be oasily traversed tooologleally and by the Euler primitives whioh allows the edges to be aulokly bpoken Into visiole and hidden poptions without losing their todology. the thipo kind of dampenlng involves having a pester of edge buckets to localiza the comparlsons.

The reason for dalag hidden line olimination in thls iashion 18 to get the topology of the image peglons and odges in a modeled scene Including the shadows. OCCULT Was used to make some of the flgures that appered eapliep in this papeps for examole the arm model In flgure 1,2 , (which pequired twelve seconds of popeig oompute timel. A paper on oCCULT should be avallable before the end of the year. 1972.

IV, C, Visloni CAREYE - video region-edge finder.
CAREYE, Cart Eye, is ine oldest, most pewritteng yet lesst finlshed papt of the apolication. At present lits best tplek is to take a relevision lmage and oonvert lt into video intensity contoup Ilnes similap so those discuised by Krakaup and Hopn pof M, I.T.J. From Vic, Video intensity Contoupsi the image goes thiough two Drocesses: flist, the oamera loous-oplentation fop the lmage ls solved by finding feature polnts in the image that coopesoond with known land mapk dolnt in the worldi and second, after the camera is solvedithe locus of peplously unknown pegions of the imege muet be added to the world modeli the third dimension of such unknown peolons being assumed to be very large, untli ovidence is found in suceeoding Images that make the pegion "ood out" of the background. These two orocesses are called Camera bocus Solving and Body locus Solving; CAMLS and BODLS: and are tho missing IInks In making dolyhedion models merely by looking at objeots and scenes of objects.

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