Winners and Losers in Housing Markets*

Nobuhiro Kiyotaki†
Princeton University, CEPR and NBER

Alexander Michaelides‡
LSE, CEPR and FMG

Kalin Nikolov§
Bank of England and LSE

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†Department of Economics, Princeton University, Princeton, NJ 08544. Email: kiyotaki@princeton.edu.

‡Department of Economics, London School of Economics, Houghton Street, London, WC2A 2AE, UK. Email: A.Michaelides@lse.ac.uk.

§Bank of England and Department of Economics, London School of Economics, Houghton Street, London, WC2A 2AE, UK. Email: Kalin.Nikolov@bankofengland.co.uk.

Abstract

This paper is a quantitatively-oriented theoretical study into the interaction between housing prices, aggregate production, and household behavior over a lifetime. We develop a life-cycle model of a production economy in which land and capital are used to build residential and commercial real estates. We find that, in an economy where the share of land in the value of real estates is large, housing prices react more to an exogenous change in expected productivity or the world interest rate, causing large redistribution effects between net buyers and net sellers of houses. Changing the financing constraint, however, has limited effects on housing prices.

JEL Classification: E20, R20, R30.

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1 Introduction

Over the last few decades, we observe considerable fluctuations in real estate values and aggregate economic activities in many economies. In Japan, both the real capital gains on real estate during the prosperous decade of the 1980s and the losses during the depressed decades of the 1990s and the early 2000s are in the order of multiple years worth of GDP. Recent fluctuations in housing prices in many countries raise concerns. To what extent are these housing price fluctuations consistent with fundamental conditions? How do the fluctuations affect the welfare of different groups of households? In this paper, we develop a life-cycle model to investigate how housing prices, aggregate production and the wealth distribution react to changes in technology and financial conditions. After checking whether the model is broadly consistent with the life-cycle of home ownership and consumption, we use the model to assess which groups of households gain and which groups lose from changes in fundamentals.

To develop a theoretical framework, we take into account the limitation on the supply of land and the limitation on the enforcement of contracts in real estate and credit markets. Land (or location) is an important input for supplying residential and commercial real estates. Because the supply of land is largely inelastic and because the real estate price includes the value of land, the real estate price is sensitive to a change in the expected productivity growth rate and the real interest rate in equilibrium. We also consider incomplete contract enforcement to be an essential feature of an economy with real estate. Often, because landlords are afraid that the tenant may modify the property against their interests (or disputes may arise over splitting the modification costs), landlords restrict tenants' discretion over the use and modification of the house, and tenants enjoy lower utility from renting the house compared to owning and controlling the same house. If there were no other frictions, then the household would buy the house straight away. The household, however, may face a financing constraint, because the creditor fears that the borrowing household may default. The creditor demands the borrower to put his house as collateral for a loan and asks him to provide a downpayment from his own net worth.

In this paper, we take the importance of land for production of tangible assets (including real estates), the loss of utility from rented housing and the tightness of collateral constraints as exogenous parameters, and examine how they affect household consumption and housing over the lifetime, housing prices and aggregate quantities.[1] For this purpose, we develop an overlapping generations model of a production economy in which land and capital are used to produce residential and commercial tangible assets. We are also interested in the way households cope with idiosyncratic and uninsurable shocks to their labor income, which helps to generate realistic housing choices and wealth distribution.

The interaction between the collateral constraint and the loss of utility from renting a house turns out to generate a typical pattern of consumption and housing over a life-cycle. When the household is born (or becomes independent) without any inheritance, it cannot afford a sufficiently high downpayment for buying a house; the household rents and consumes modestly to save for a downpayment. When the household accumulates some net worth, the household buys a house subject to the collateral constraint, which is smaller than a house that would be bought without the collateral constraint. As net worth further rises, the household upgrades along the housing ladder with the collateral constraint continuing to be binding. At some stage, the household finds it better to start repaying the debt rather than maximizing the size of the house. When the time comes for retirement, possibly with idiosyncratic risk attached, the household moves to a smaller house, anticipating a lower income in the future.

In equilibrium, due to the limitation of land supply, the supply of tangible assets tends to grow more slowly than final output causing an upward trend in the real rental price and the purchase price of the tangible asset. The more important is land for producing tangible assets compared to capital (other parameters being equal), the higher is the expected growth rate of the rental price, and therefore the higher is the housing price-rental ratio. This is true for a country like Japan or a metropolitan area. In such an economy, the household needs a larger downpayment relative to wage income in order to buy a house, and tends to buy a house later in life, resulting in a lower home-ownership rate.

Moreover, in an economy where land is more important for producing tangible assets

[1] Here, the importance of land for production of tangible asset is defined as the elasticity of tangible asset supply with respect to land for a fixed level of the other input. See equation (2) later.

compared to capital, we find the housing price to be more sensitive to exogenous changes in fundamentals such as the expected growth rate of labor productivity or the world interest rate, along the perfect foresight path from one steady state to another. Davis and Heathcote (2007) note that housing prices are more sensitive in large U.S. metropolitan areas where arguably a "land constraint" may be more binding. Del Negro and Otrok (2007) find empirical evidence that is consistent with this prediction: using a factor decomposition of recent house price changes in the U.S., they attribute a higher percentage change to local factors in states where the share of land in the real estate value is larger.[2]

In contrast to the change in productivity growth and the world interest rate, we find that financial innovation which permanently relaxes the collateral constraint has a surprisingly small effect on housing prices, despite increasing the home-ownership rate substantially both in the transition and in the steady state. In our economy, tenants or credit-constrained home owners are relatively poor and own a small share of aggregate wealth as a group. As a result, the effect of relaxing the collateral constraint on housing prices is largely absorbed by a modest conversion from rented to owned units.[3]

In addition to the effect on the housing price and aggregate output, the exogenous changes in the productivity growth rate and the interest rate affect the welfare of various households differently, causing winners and losers in housing markets. This distribution effect on wealth and welfare is substantial, since housing wealth forms the largest component of nonhuman wealth for most households. As a general rule of thumb, net house buyers (such as young

[2] Davis and Palumbo (2008) find that the share of land in the value of houses has risen in U.S. metropolitan areas and they argue that this contributes to faster housing price appreciation and, possibly, larger swings in housing prices. Glaeser et. al. (2005) find that land use restrictions are needed to explain recent high housing prices in Manhattan. van Nieuwerburgh and Weill (2006) also argue that the increase in the dispersion of housing prices across regions can be quantitatively generated from an increase in the dispersion of earnings in the presence of planning restrictions. We ignore the restrictions on land use and planning, even though they further increase the natural limitation of land in supplying structures. Other factors that might be empirically relevant for house price determination (such as owner-occupied housing as a hedge against rent risk, the effects of inflation and money illusion) are not considered in our framework; see Sinai and Souleles (2005) and Brunnermeier and Julliard (2008).

[3] This is different from Ortalo-Magne and Rady (2006) who show that relaxing the collateral constraint increases housing prices substantially. We will later discuss further why our results differ from theirs.

worker-tenants) lose while net house sellers (such as retiree-home owners) gain from the house price hike.[4] The gap in welfare changes between winners and losers in the housing market is larger in an economy where land is more important for producing tangible assets compared to capital, since in such an economy housing prices react more in response to an identical shock. Because the welfare effect depends on the underlying shocks causing house price changes, we need a general equilibrium framework with heterogeneous agents to analyze the effect of the shock on the welfare of different households.

Our work broadly follows two strands of the literature. One is the literature on consumption and saving of a household facing idiosyncratic and uninsurable earnings shock and a borrowing constraint, which includes Bewley (1977, 1983), Deaton (1991), Carroll (1997), Attanasio et. al. (1999) and Gourinchas and Parker (2002). Huggett (1993), Aiyagari (1994), and Krusell and Smith (1998) have examined the general equilibrium implications of such models. The second strand is the literature on the investment behavior of firms under liquidity constraints. In particular, Kiyotaki and Moore (1997) is closely related since they study the dynamic interaction between asset prices, credit limits and aggregate economic activity for an economy with credit constrained entrepreneurs. When many households borrow substantially against their housing collateral and move up and down the housing ladder, these households are more like small entrepreneurs rather than simple consumers.

Our attention to housing collateral is in line with substantial micro evidence in the UK (Campbell and Cocco (2007)) and the US (Hurst and Stafford (2004)) which suggests that dwellings are an important source of collateral for households. Given the empirical findings that connect housing prices, home equity and aggregate consumption, there has been sub-

[4]The household is a net house buyer if the expected present value of housing services consumption over the lifetime exceeds the value of the house currently owned. The present living population as a whole is a net seller of the existing houses to the future population (that is not born yet). But, the value of this aggregate net selling position is quantitatively very small, because the discounted value of selling the existing houses to the next unborn population in 70 to 80 years from now is negligible. In comparison, the redistribution within the present population between young and old, or between tenants and home owners, is much larger. Thus, unlike some popular arguments, the wealth effect of housing prices on aggregate consumption is negligible (aside from the liquidity effect), because the positive wealth effect of the net house sellers is largely offset by the negative wealth effect of the net house buyers.

stantial research on building models that capture these relationships, either with a representative agent (Aoki et. al. (2004), Davis and Heathcote (2005), Lustig and van Nieuwerburgh (2005), Kahn (2007), Piazessi et. al. (2007)), or with heterogeneous agents (Ortalo-Magne and Rady (2006), Fernandez-Villaverde and Krueger (2007), Chambers, Garriga and Schlagenhauf (2004), Iacoviello (2005), Iacoviello and Neri (2007), Nakajima (2005), Rios-Rull and Sanchez (2005) and Silos (2007)). Distinguishing features of our analysis include an investigation of the interaction between household life-cycle choices and the aggregate economy, an explicit account of the role of land as a limiting factor in a production economy and evaluating welfare changes across heterogeneous households stemming from shocks to fundamentals.

Section 2 lays out the model and section 3 presents long-run observations relevant for housing markets. Section 4 investigates the individual and aggregate predictions of the model using calibration and Section 5 performs the welfare evaluations.

2 The Model

2.1 Framework

We consider an economy with homogeneous product, tangible assets, labor, reproducible capital stock, and non-reproducible land. There is a continuum of heterogeneous households of population size $\overline{N}_t$ in period t, a representative foreigner, and a representative firm.

The representative firm has a constant returns to scale technology to produce output (Y_t) from labor (N_t) and productive tangible assets (Z_{Yt}) as:

$$Y_t = F(A_t N_t, Z_{Yt}) = (A_t N_t)^{1-\eta} Z_{Yt}^{\eta}, \quad 0 < \eta < 1, \tag{1}$$

where A_t is aggregate labor productivity which grows at a constant rate, $A_{t+1}/A_t = G_A$. Tangible assets (Z_t) are produced according to a constant returns to scale production function using aggregate capital (K_t) and land (L):

$$Z_t = L^{1-\gamma} K_t^{\gamma}, \quad 0 < \gamma < 1. \tag{2}$$

The tangible assets are fully equipped or furnished, and can be used as productive tangible assets (such as offices and factories) or houses interchangeably:

$$Z_t = Z_{Yt} + \int_0^{\overline{N}_t} h_t(i)\, di, \tag{3}$$

where $h_t(i)$ is housing used by household i in period t. With this technological specification of tangible assets, the firm can continuously adjust the way in which the entire stock of land and capital are combined and can convert between productive tangible assets and housing without any friction.[5] The parameter $(1-\gamma)$ measures the importance of land for the production of tangible assets compared to capital, which would be equal to the share of land in property income if there were separate competitive rental markets for land and capital. Thus, we often call $(1-\gamma)$ as "the share of land in the production of tangible assets" hereafter. Typically, the share of land in the production of tangible assets is higher in urban than in rural areas, because land (or location) is more important for production with the agglomeration of economic activities.[6] We assume that the aggregate supply of land L is fixed. The capital stock depreciates at a constant rate $1-\lambda \in (0,1)$ every period, but can be accumulated through investment of goods (I_t) as:

$$K_t = \lambda K_{t-1} + I_t. \tag{4}$$

tangible assets built this period can be used immediately.

The representative firm owns and controls land and capital from last period and issues equity to finance investment. As the firm increases the size of tangible assets with capital

[5] Davis and Heathcote (2005) use a production function in which only a fixed flow of new vacant land can be used for building new houses. Perhaps, in reality, the allocation of land and capital is not as flexible as in our model but not as inflexible as in Davis and Heathcote (2005). We also assume there is no productivity growth in the production of tangible assets, because Davis and Heathcote (2005) calculate the growth rate of productivity in the US construction sector to be close to zero (-0.27 percent per annum). We ignore labor used in this sector for simplicity.

[6] We will not attempt to explain why agglomeration arises. We should not confuse the share of land $(1-\gamma)$ with the scarcity of land (or marginal product of land), because scarcity not only depends upon the share of land, but also on labor productivity, the capital-land ratio and the capital-labor ratio. We will later discuss how the share of land in the production of structures is related to the share of land in the value of tangible assets in Section 3.4.

accumulation, it will be convenient in subsequent analysis to assume that the firm maintains the number of shares to be equal to the stock of tangible assets.[7] Let q_t be the price of equity before investment takes place and let p_t be the price of equity after investment takes place in this period. Let w_t be the real wage rate, and r_t be the rental price of tangible assets. The firm then faces the following flow-of-funds constraint:

$$Y_t - w_t N_t - r_t Z_{Yt} - I_t + p_t Z_t = q_t Z_{t-1} \tag{5}$$

The left hand side (LHS) is the sum of the net cash flow from output production, minus investment costs and the value of equities after investment. The right hand side (RHS) equals the value of equity at the beginning of the period (before investment has taken place).

The owners of equity pay p_t to acquire one unit and immediately receive r_t as a rental payment (including imputed rents). Next period, the owner earns q_{t+1} before investment takes place. Therefore, the rate of return equals

$$R_t = \frac{q_{t+1}}{p_t - r_t}. \tag{6}$$

There are no aggregate shocks in this economy except for unanticipated, initial shocks. As a result, we assume that agents have perfect foresight for all aggregate variables, including the rate of return.

From (5) and (6) under perfect foresight, the value of the firm (V_t^F) to the equity holders from the previous period is equal to the present value of the net cash flow from production and the rental income of tangible assets produced:

$$\begin{aligned} V_t^F &\equiv q_t Z_{t-1} = Y_t - w_t N_t - r_t Z_{Yt} - I_t + r_t Z_t + (p_t - r_t) Z_t \\ &= Y_t - w_t N_t - r_t Z_{Yt} - I_t + r_t Z_t + \frac{1}{R_t} V_{t+1}^F \end{aligned} \tag{7}$$

The firm takes $\{w_t, r_t, R_t\}$ as given and chooses a production plan $\{N_t, Z_{Yt}, Y_t, I_t, K_t\}$ to maximize the value of the firm, subject to the constraints of technology $(1), (2), (3)$ and (4).

[7] This means the firm follows a particular policy of equity issue and dividend payouts. However, alternative policies do not change allocations because the Modigliani-Miller Theorem holds in our economy under perfect foresight and would only complicate subsequent expressions.

Since the production function of output is constant returns to scale, there is no profit from output production. Therefore, the value of the firm equals the value of the tangible asset stock. Given that the number of equities are maintained to equal the stock of tangible assets (by assumption), the price of equities equals the price of tangible assets. Hereafter, we refer to the shares of the firm as the shares of tangible assets.

Households are heterogeneous in labor productivity, and can have either low, medium, or high productivity, or be retired. Every period, there is a flow of new households born with low productivity without any inheritance of the asset. Each low productivity household may switch to medium productivity in the next period with a constant probability δ^l. Each medium productivity household has a constant probability δ^m to become a high productivity one in the next period. Once a household has switched to high productivity it remains at this high productivity until retirement. All the households with low, medium and high productivity are called *workers*, and all the workers have a constant probability $1-\omega \in (0,1)$ of retiring next period. Once retired, each household has a constant probability $1-\sigma \in (0,1)$ of dying before the next period. (In other words, a worker continues to work with probability ω, and a retiree survives with probability σ in the next period). The flow of new born workers is $G_N - \omega$ fraction of the workforce in the previous period, where $G_N > \omega > \delta^i$ for $i = l, m$. The productivity level of the individual household is private information. All the transitions are i.i.d. across a continuum of households and over time, and thus there is no aggregate uncertainty on the distribution of individual labor productivity. Let N_t^l, N_t^m and N_t^h be populations of low, medium and high productivity workers, respectively, and let N_t^r be the population size of retired households in period t. Then, we have:

$$
\begin{aligned}
N_t^l &= (G_N - \omega)\left(N_{t-1}^l + N_{t-1}^m + N_{t-1}^h\right) + (\omega - \delta^l)N_{t-1}^l, \\
N_t^m &= \delta^l N_{t-1}^l + (\omega - \delta^m)N_{t-1}^m, \\
N_t^h &= \delta^m N_{t-1}^m + \omega N_{t-1}^h, \\
N_t^r &= (1-\omega)(N_{t-1}^l + N_{t-1}^m + N_{t-1}^h) + \sigma N_{t-1}^r.
\end{aligned}
$$

We choose to formulate the household's life-cycle in this stylized way, following Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992) and Gertler (1999), because we are mainly interested in the interaction between the life-cycles of households and the aggregate economy. The three levels of labor productivity give us enough flexibility to mimic a typical life-cycle of wage income for our aggregate analysis.

Each household derives utility from the consumption of output (c_t) and housing services (h_t) of rented or owned housing, and suffers disutility from supplying labor (n_t). (We suppress the index of household i when we describe a typical household). We assume that, when the household rents a house rather than owning (as an owner-occupier) and controlling the same house, she enjoys smaller utility by a factor $\psi \in (0,1)$. This disadvantage of rented housing reflects the tenant's limited discretion over the way the house is used and modified according to her tastes. The preference of the household is given by the expected discounted utility as:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t, \left[1 - \psi I(rent_t)\right] h_t\right) - v(n_t, \varepsilon_t) \right] \right), \ 0 < \beta < 1, \tag{8}$$

where $I(rent_t)$ is an indicator function which takes the value of unity when the household rents the house in period t and zero when she owns it.[8] Disutility of labor $v(n_t, \varepsilon_t)$ is subject to idiosyncratic shocks to its labor productivity ε_t. The value of ε_t is either high (ε^h), medium (ε^m), low (ε^l), or 0, depending on whether the household has high, medium or low productivity, or is retired, and follows the stationary Markov process described above. $E_0(X_t)$ is the expected value of X_t conditional on survival at date t and conditional on information at date 0. For most of our computation, we choose a particular utility function with inelastic labor supply as:

$$u\left(c_t, h_t\right) = \frac{\left(\left(\frac{c_t}{\alpha}\right)^{\alpha} \left(\frac{[1-\psi I(rent_t)]h_t}{1-\alpha}\right)^{1-\alpha} \right)^{1-\rho}}{1-\rho}, \tag{9}$$

and $v_t = 0$ if $n_t \leq \varepsilon_t$, and v_t becomes arbitrarily large if $n_t > \varepsilon_t$. The parameter $\rho > 0$ is the coefficient of relative risk aversion (as well as the inverse of the elasticity of intertemporal substitution) and $\alpha \in (0,1)$ reflects the share of consumption of goods (rather than housing services) in total expenditure. We normalize the labor productivity of the average worker to

[8] We assume that, in order to enjoy full utility of the house, the household must own and control the entire house used. If the household rents a fraction of the house used, then she will not enjoy full utility even for the fraction of the house owned.

unity as:

$$N_t^l \varepsilon^l + N_t^m \varepsilon^m + N_t^h \varepsilon^h = N_t^l + N_t^m + N_t^h. \tag{10}$$

We focus on the environment in which there are problems in enforcing contracts and there are constraints on trades in markets. There is no insurance market against the idiosyncratic shock to labor productivity of each household. The only asset that households hold and trade is the equity of tangible assets (and the annuity contract upon this equity). An owner-occupier can issue equity on its own house to raise funds from the other agents. But the other agents only buy equity up to a fraction $1 - \theta \in [0, 1)$ of the house. Thus, to control the house and enjoy full utility of a house of size h_t, the owner-occupier must hold sufficient equity s_t to satisfy:

$$s_t \geqslant \theta h_t. \tag{11}$$

We can think of this constraint as a collateral constraint for a residential mortgage — even though in our economy the mortgage is financed by equity rather than debt — and we take θ as an exogenous parameter of the collateral constraint. Because the tenant household does not have a collateral asset, we assume the tenant cannot borrow (or issue equities):

$$s_t \geq 0. \tag{12}$$

We restrict tradeable assets to be the homogeneous equity of tangible assets in order to abstract from the portfolio choice of heterogeneous households facing collateral constraints and uninsurable labor income risk. Because we analyze the economy under the assumption of perfect foresight about the aggregate states, this restriction on tradeable assets is not substantive (because all the tradeable assets would earn the same rate of return), except for the case of an unanticipated aggregate shock. Although we do not attempt to derive these restrictions on market transactions explicitly as the outcome of an optimal contract, the restrictions are broadly consistent with our environment in which agents can default on contracts, misrepresent their labor productivity, and can trade assets anonymously (if they wish).[9]

[9] The outside equity holders (creditors) ask the home owners to maintain some fraction of the housing equity to prevent default. There is no separate market for equities on land and capital upon it, because

The flow-of-funds constraint of the worker is given by:

$$c_t + r_t h_t + p_t s_t = (1-\tau) w_t \varepsilon_t + r_t s_t + q_t s_{t-1}, \tag{13}$$

where τ is a constant tax rate on wage income. The LHS is consumption, the rental cost of housing (or opportunity cost of using a house rather than renting it out), and purchases of equities. The RHS is gross receipts, which is the sum of after tax wage income, the rental income from equities purchased this period, and the pre-investment value of equity held from the previous period.[10]

For the retiree who only survives until the next period with probability σ, there is a competitive annuity market in which the owner of a unit annuity will receive the gross returns q_{t+1}/σ if and only if the owner survives, and receive nothing if dead.[11] The retiree also receives the benefit b_t per person from the government, which is financed by the tax revenue on wage income of the workers as

$$b_t N_t^r = \tau w_t (N_t^l + N_t^m + N_t^h). \tag{14}$$

Because the productivity of each household is private information and a low productivity worker can pretend to be retired, the viable retirement benefit does not exceed after-tax

people prefer to control land and capital together in order to avoid the complications. Cole and Kocherlakota (2001) show that, if agents can misrepresent their idiosyncratic income and can save privately, the optimal contract is a simple debt contract with a credit limit. See Lustig (2004) and Lustig and van Nieuwerburgh (2005b) for analysis of optimal contracts with tangible assets as collateral.

[10] When the worker is an owner-occupier of a house of size h_t and issues equity to the outside equity holders (creditors) by outstanding size of $(h_t - s_t)$ in period t, she faces the flow-of-funds constraint:

$$c_t + [p_t h_t - q_t h_{t-1}] + r_t (h_t - s_t) = (1-\tau) w_t \varepsilon_t + [p_t (h_t - s_t) - q_t (h_{t-1} - s_{t-1})].$$

The LHS is an outflow of funds: consumption, purchases of the owned house over the resale value of the house held from last period, and rental income paid to the outside equity holders of this period. The RHS is an inflow: after-tax wage income, and the value of new issues of outside equity above the value of outside equity from the previous period. By rearranging this, we find that both the home-owner and tenant face the same flow-of-funds constraint (13), in which only the net position of equity matters.

[11] When the retiree who owned the house dies, then the house is sold and the proceeds are split among the surviving annuity holders.

wage income of the low productivity worker[12], or:

$$b_t/w_t = \tau \frac{G_N - \sigma}{1-\omega} \leq (1-\tau)\varepsilon^l.$$

The flow-of-funds constraint for the retiree is

$$c_t + r_t h_t + p_t s_t = b_t + r_t s_t + \frac{q_t}{\sigma} s_{t-1}. \tag{15}$$

Each household takes the equity from the previous period (s_{t-1}) and the joint process of prices, and idiosyncratic labor productivity shocks $\{w_t, r_t, p_t, q_t, \varepsilon_t\}$ as given, and chooses the plan of consumption of goods and housing, and the equity holding $\{c_t, h_t, s_t\}$ to maximize the expected discounted utility subject to the constraints of flow-of-funds and collateral.

The representative foreigner makes purchases of goods C_t^* and equities of tangible assets S_t^* in the home country (both C_t^* and S_t^* can be negative), subject to the international flow-of-funds constraint against home agents as:

$$C_t^* + p_t S_t^* = r_t S_t^* + q_t S_{t-1}^*. \tag{16}$$

The LHS is gross expenditure of foreigners on home goods and equities, and the RHS is the gross receipts of foreigners. Instead of explicitly deriving the foreigners' behavior, we posit the reduced form demand function for home equities of the representative foreigner as an increasing function of the gap between the rate of return on home equities and the rate of return on the foreign asset, R_t^*, as:

$$S_t^* = S^*(R_t,\ R_t^*) = \overline{S}^* + \xi(R_t - R_t^*),$$

where $\xi > 0$ is the sensitivity of demand with respect to the gap in the rates of return, and $\overline{S}^*$ is the parameter which summarizes the other determinants of their demand. One special case is a small open economy in which $\xi \to \infty$, and another special case is a closed economy in which $\overline{S}^* = \xi = 0$.

[12] The firm observes each worker's labor contribution to its production, but it does not observe whether the worker works elsewhere as well. The firm pays uniform payroll taxes before paying wages to the workers. Although the government does not observe the productivity of each household, it observes whether the household works or not, at least with some probability by random monitoring. We assume that the penalty of getting caught for cheating is sufficiently high, so that no worker receives the benefit while working.

Given the above choices of households, the representative firm and the foreigner, the competitive equilibrium of our economy is characterized by the prices $\{w_t, r_t, p_t\}$ which clear the markets for labor, output, equity and the use of tangible assets as:

$$N_t = \int_0^{\bar{N}_t} n_t(i)\, di = \varepsilon^l N_t^l + \varepsilon^m N_t^m + \varepsilon^h N_t^h = N_t^l + N_t^m + N_t^h, \tag{17}$$

$$Y_t = \int_0^{\bar{N}_t} c_t(i)\, di + I_t + C_t^*, \tag{18}$$

$$Z_t = \int_0^{\bar{N}_t} s_t(i)\, di + S_t^*. \tag{19}$$

and (3)[13]. Because of Walras' Law, only three out of four market clearing conditions are independent.

2.2 Behavior of Representative Firm

The first order conditions for the value maximization of the representative firm are:

$$w_t = (1-\eta) Y_t / N_t, \tag{20}$$

$$r_t = \eta Y_t / Z_{Yt} = \eta \left(\frac{M_t}{f_t Z_t} \right)^{1-\eta}, \text{ where } M_t \equiv A_t N_t \text{ and } f_t \equiv Z_{Yt}/Z_t, \tag{21}$$

$$1 - \frac{\lambda}{R_t} = r_t \gamma \left(\frac{L}{K_t} \right)^{1-\gamma} = \gamma \eta L^{(1-\gamma)\eta} \left(\frac{M_t}{f_t} \right)^{1-\eta} K_t^{\gamma\eta - 1}. \tag{22}$$

The first two equations are the familiar equality of price and marginal products of factors of production. The value of M_t is the labor in efficiency unit, and f_t is a fraction of tangible assets used for production. The last equation says that the opportunity cost of holding capital for one period – the cost of capital – should be equal to the marginal value product of capital. Thus we have

$$K_t = \left[\frac{\gamma\eta}{1 - \frac{\lambda}{R_t}} L^{(1-\gamma)\eta} \left(\frac{M_t}{f_t} \right)^{1-\eta} \right]^{1/(1-\gamma\eta)}, \tag{23}$$

[13] The name of individual household i is such that a fraction of new-born households named after the names of the deceased households and the remaining fraction of newborns are given new names for $i \in \left(\overline{N}_{t-1}, \overline{N}_t\right]$. In this way, the name of households are always distributed uniformly in $\left[0, \overline{N}_t\right]$ at date t.

$$Y_t = f_t \left[\left(\frac{\gamma\eta}{1 - \frac{\lambda}{R_t}} \right)^{\gamma\eta} L^{(1-\gamma)\eta} \left(\frac{M_t}{f_t} \right)^{1-\eta} \right]^{1/(1-\gamma\eta)} . \tag{24}$$

Because the production function of output is constant returns to scale, there is no profit associated with regular production. The resulting value of the firm is:

$$\begin{aligned} V_t^F &= r_t Z_t - (K_t - \lambda K_{t-1}) + \frac{1}{R_t} \left[r_{t+1} Z_{t+1} - (K_{t+1} - \lambda K_t) \right] + ... \\ &= \lambda K_{t-1} + \eta(1-\gamma) \left(\frac{Y_t}{f_t} + \frac{1}{R_t} \frac{Y_{t+1}}{f_{t+1}} + \frac{1}{R_t R_{t+1}} \frac{Y_{t+2}}{f_{t+2}} + ... \right). \end{aligned} \tag{25}$$

The first term of the RHS is the capital stock inherited from the previous period, and the second term is the value of land, which is proportional to the present value of the return to land which comes from output and housing service production. Thus, the equity holders as a whole receive returns from capital and land through their holdings of equities of the entire tangible asset.

2.3 Household Behavior

The household chooses one among three modes of housing - becoming a tenant, a credit constrained owner-occupier, and an unconstrained owner-occupier. The flow-of-funds constraint of the worker and retiree can be rewritten as

$$\begin{aligned} c_t + r_t h_t + (p_t - r_t) s_t &= (1-\tau) w_t \varepsilon_t + q_t s_{t-1} \equiv x_t, \\ c_t + r_t h_t + (p_t - r_t) s_t &= b_t + [q_t / \sigma] \, s_{t-1} \equiv x_t, \end{aligned}$$

where x_t is the liquid wealth of the household. Liquid wealth is the wealth of the household, excluding illiquid human capital (the expected discounted value of future wages and pension income). We call liquid wealth "net worth" hereafter.

2.3.1 The tenant

The tenant chooses consumption of goods and housing services to maximize the utility, which leads to:

$$\frac{c_t}{r_t h_t} = \frac{\alpha}{1-\alpha}.$$

Using the flow-of-funds constraint we can express housing and consumption as functions of current expenditure:

$$c_t = \alpha[x_t - (p_t - r_t)s_t],$$

and

$$h_t = \frac{(1-\alpha)\,[x_t - (p_t - r_t)s_t]}{r_t}.$$

Substituting these into the utility function we get the following indirect utility function:

$$u^T\,(s_t, x_t; r_t, p_t) = \frac{1}{1-\rho}\left[\frac{x_t - (p_t - r_t)s_t}{[r_t/(1-\psi)]^{1-\alpha}}\right]^{1-\rho}.$$

Due to the lower utility from living in a rented house, the tenant effectively faces a higher rental price than the owner-occupier for the same utility, i.e., $[r_t/(1-\psi)]$ rather than r_t.

2.3.2 The constrained owner-occupier

The constrained owner-occupier faces a binding collateral constraint as:

$$s_t = \theta h_t.$$

Thus he consumes $h_t = s_t/\theta$ amount of housing services, and spends the remaining on goods as:

$$c_t = x_t - \left(p_t - r_t + \frac{r_t}{\theta}\right) s_t.$$

The indirect period utility of the constrained home owner is now:

$$u^C\,(s_t, x_t; r_t, p_t) = \frac{1}{1-\rho}\left\{\left[\frac{x_t - \left(p_t - r_t + \frac{r_t}{\theta}\right) s_t}{\alpha}\right]^{\alpha}\left[\frac{s_t/\theta}{1-\alpha}\right]^{1-\alpha}\right\}^{1-\rho}.$$

2.3.3 The unconstrained owner-occupier

The collateral constraint is not binding for the unconstrained owner-occupier. Her intra-temporal choice is identical to the tenant's but she does not suffer from the limited discretion associated with renting a house.

$$u^U\,(s_t, x_t; r_t, p_t) = \frac{1}{1-\rho}\left[\frac{x_t - (p_t - r_t)s_t}{{r_t}^{1-\alpha}}\right]^{1-\rho}$$

2.3.4 Value functions

Let $\overline{A}_t$ be the vector of variables and a function that characterizes the aggregate state of the economy at the beginning of period t :

$$\overline{A}_t = (A_t,\ N_t^l,\ N_t^m, N_t^h,\ N_t^r,\ K_{t-1},\ S_{t-1}^*,\ \Phi_t(\varepsilon_t(i), s_{t-1}(i)))',$$

where $\Phi_t(\varepsilon_t(i), s_{t-1}(i))$ is the date t joint distribution function of present productivity and equity holdings from the previous period across households. Each household has perfect foresight about the future evolution of this aggregate state, even if each faces idiosyncratic risks on her labor productivity. The prices (w_t, r_t, p_t, q_t) would be a function of this aggregate state in equilibrium. We can express the value functions of the retiree, high, medium and the low productivity worker by $V^r(x_t, \overline{A}_t)$, $V^h(x_t, \overline{A}_t)$, $V^m(x_t, \overline{A}_t)$, and $V^l(x_t, \overline{A}_t)$ as functions of the individual net worth and the aggregate state.

The retiree chooses the mode of housing and an annuity contract on equities, s_t, subject to the flow-of-funds constraint. Then, the retiree's value function satisfies the Bellman equation:

$$V^r(x_t, \overline{A}_t) = \underset{j=T,C,U}{Max} \left(\max_{s_t} \left\{ u^j\left(s_t, x_t; r_t, p_t\right) + \beta\sigma V^r\left(b_{t+1} + \left[q_{t+1}/\sigma\right] s_t, \overline{A}_{t+1}\right) \right\} \right),$$

where $u^j\left(s_t, x_t; r_t, p_t\right)$ is the indirect utility function of present consumption and housing services when the mode of housing is tenant $(j = T)$, constrained owner-occupier $(j = C)$, or unconstrained owner-occupier $(j = U)$.

The worker chooses the mode of housing and saving in equities. The value function of a high-productivity worker satisfies the Bellman equation:

$$V^h(x_t, \overline{A}_t) = \underset{j=T,C,U}{Max} \left(\max_{s_t} \left\{ \begin{array}{c} u^j\left(s_t, x_t; r_t, p_t\right) + \beta[\omega V^h((1-\tau)\varepsilon^h w_{t+1} + q_{t+1}s_t, \overline{A}_{t+1}) \\ +(1-\omega)V^r(b_{t+1} + q_{t+1}s_t, \overline{A}_{t+1})] \end{array} \right\} \right).$$

The high productivity worker may retire with probability $1-\omega$ next period, and continues to work with probability ω.

The value function of a medium productivity worker satisfies:

$$V^m(x_t, \overline{A}_t) =$$
$$\underset{j=T,C,U}{Max} \left(\max_{s_t} \left\{ \begin{array}{c} u^j\left(s_t, x_t; r_t, p_t\right) + \beta[(\omega-\delta^m)V^m((1-\tau)\varepsilon^m w_{t+1} + q_{t+1}s_t, \overline{A}_{t+1}) \\ +\delta^m V^h((1-\tau)\varepsilon^h w_{t+1} + q_{t+1}s_t, \overline{A}_{t+1}) + (1-\omega)V^r(b_{t+1} + q_{t+1}s_t, \overline{A}_{t+1})] \end{array} \right\} \right).$$

Next period, the medium productivity worker switches to high productivity with probability δ^m, retires with probability $1-\omega$, and remains with medium productivity with probability $\omega-\delta^m$. The value function of a low productivity worker is similar to the value function of a medium productivity worker, except for m being replaced by l and h being replaced by m.

Growth in the economy with land presents a unique problem for the solution of the individual agent problem because wages grow at different rates from the rental price and the equity price even in the steady state. This means that we need to transform the non-stationary per capita variables in the model into stationary per capita units. In Appendix B, we describe how to convert the value functions of the household into a stationary representation.

2.4 Steady State Growth

Before calibrating, it is useful to examine the steady state growth properties of our economy. Let $G_X = X_{t+1}/X_t$ be the steady state growth factor of variable X_t. In the following we simply call the growth factor as the "growth rate". In steady state, the growth rate of aggregate output variables should be equal:

$$\frac{Y_{t+1}}{Y_t} = \frac{I_{t+1}}{I_t} = \frac{K_{t+1}}{K_t} = G_Y.$$

The growth rate of tangible assets need not be equal the growth rate of output, but it should be equal to the growth rate of productive tangible assets:

$$\frac{Z_{t+1}}{Z_t} = \frac{Z_{Yt+1}}{Z_{Yt}} = G_Z.$$

Then, from the production functions, these growth rates depend upon the growth rates of aggregate labor productivity and population as $G_Y = (G_A G_N)^{1-\eta}\, G_Z^{\eta}$, and $G_Z = G_Y^{\gamma}$. Thus

$$G_Y = (G_A G_N)^{(1-\eta)/(1-\gamma\eta)}, \tag{26}$$

$$G_Z = (G_A G_N)^{\gamma(1-\eta)/(1-\gamma\eta)}.$$

Because the supply of land is fixed, to the extent that land is an important input for producing tangible assets, the growth rates of output and tangible assets are both smaller than the growth rate of labor in efficiency units. Moreover, because tangible assets are more directly affected by the limitation of land than output, the growth rate of tangible assets is lower than the growth rate of output, when labor in efficiency units is growing.

In the steady state of the competitive economy, the growth rate of the real rental price and the purchase price of tangible assets is equal to the ratio of the growth rate of output and the growth rate of tangible assets:

$$G_r \equiv \frac{r_{t+1}}{r_t} = \frac{p_{t+1}}{p_t} = \frac{G_Y}{G_Z} = G_Y^{1-\gamma}. \tag{27}$$

To the extent that land is important for the production of tangible assets ($\gamma < 1$), the rate of increase of the rental price and the purchase price of tangible assets is an increasing function of the growth rate of workers in efficiency units in steady state. The wage rate grows in the steady state with the same rate as the per capita output as

$$G_w = \frac{G_Y}{G_N} = \left[G_A^{1-\eta} G_N^{-\eta(1-\gamma)}\right]^{1/(1-\gamma\eta)}. \tag{28}$$

Because the per capita supply of land decreases with population growth, the growth rate of the wage rate is a decreasing function of the population growth rate.

3 Observations and Model Implications

3.1 Empirical Observations

3.1.1 Types of Tangible Assets

Here, we gather some observations, which give us some guidance for our calibrations. Our model has clear implications about the amount of tangible assets and its split between a productive and a residential component. We use the U.S. flow of fund accounts (see appendix A) to compute the average quarterly tangible assets of the non-farm private sector to GDP (this includes the value of land) and this equals 3.3 for the 1952-2005 period, and is fairly stable. The fraction of productive tangible assets to total tangible assets (Z_{Yt}/Z_t) turns out to be around 0.41 (but this masks a downward trend from around 0.39 in 1991 to around 0.31 in 2005). The value of the total housing stock to GDP has an average value of around 1.94 but again this masks a marked increase from around 2.2 in 1991 to 2.6 in 2005.

3.1.2 Evolution of U.S. home-ownership rates and housing prices

There exists considerable variation in home ownership rates across countries and over time. Focussing on the recent U.S. experience, Figure 1 plots the home ownership rates (fraction of households that own and control houses as owner-occupiers) across different age groups from 1991 to 2007. The figure shows a general upward trend that starts after 1995 and basically reflects the choices of younger cohorts (see Chambers, Garriga and Schlagenhauf (forthcoming) for further discussion). Variations over time across different cohorts may reflect differences in financing constraints, and utility losses from renting, factors that we analyze in the theoretical model. At the same time as homeownership goes up, real house prices also increase by a substantial amount. Figure 2 plots the real (deflated by the urban CPI) house price both for the value-weighted Case-Shiller index and for the equally weighted OFHEO index (for purchase-only transactions). These are two first-order events and the model we develop will have clear implications for both.

3.2 Calibration

We consider one period of our model to be one year and think of the baseline economy as the United States.

3.2.1 Labor Income Process

Our welfare analysis will critically hinge on correctly capturing the skewed wealth distribution in the data. To deal with this problem we follow Castaneda, Diaz-Gimenez and Rios-Rull (2003) and construct a simplified version of their labor income process to capture the substantial earnings inequality in U.S. data, with the aim of generating endogenously a wealth distribution close to its empirical counterpart. We pick the probabilities of switching earnings states (δ^l, δ^m) and the individual labor income productivity levels ($\varepsilon^l, \varepsilon^m, \varepsilon^h$) to match six moments. The first moment is a hump-shape in labor income; we set the ratio of mean income of 41-60 year old to the mean income of 21-40 year old to be 1.3, based on PSID evidence. The other five moments are the five quintiles of the earnings distribution. All six moments are taken from Castaneda et. al. (p.839 and table 7, p. 845) but we have independently confirmed that even though these moments change in subsequent waves of the SCF (1995, 1998, 2001 and 2004), these changes are very small. Given that we normalize the average productivity to one, this means we have 4 parameters to match 6 moments. This results in setting $\{\delta^l = 0.0338, \delta^m = 0.0247\}$, while the ratio of the middle to low productivity is 4.51 and the ratio of high to low productivity is 15.75.

The probability of continuing to work (ω) is set so that the expected duration of working life is 45.5 years, while the probability of the retiree to survive (σ) implies an expected retirement duration of 18.2 years. The replacement ratio (b) is chosen so that the replacement rate for the workers with low or medium productivity is 40%, consistent with the data from the PSID (very high earnings workers similar to our ε^h types will be top-coded in the PSID). We set the growth rate of labor productivity (G_A) to two percent, and the population growth rate (G_N) to one percent.

3.2.2 Other parameters

Using the Cooley and Prescott (1995) methodology of aligning the data to their theoretical counterparts, Appendix C outlines how we calculate the share of productive tangible assets in the production of non-housing final output (η) from the NIPA data for the period 1952:Q1 to 2005:Q4. This share equals 0.258 which is a bit lower than the one used in other studies (between 0.3 and 0.4), because we treat the production of housing services separately (and this is a capital intensive sector).

A key parameter in our model is the share of land in the production of tangible assets $(1-\gamma)$. Thinking of the U.S. economy as our baseline, we set $\gamma = 0.9$ since Haughwout and Inman (2001) calculate the share of land in property income between 1987 and 2005 to be about 10.9%, while Davis and Heathcote (2005) also use $\gamma = 0.9$. Davis and Heathcote (2007) note that the share of land in residential housing values has risen recently in the U.S., and it is close to 50% in major metropolitan areas like Boston and San Francisco. We will run some experiments for the U.K., a country where we think land restrictions are more important than in the U.S.. Absent a model with regional variation in γ (an interesting topic for further research), we will use a lower γ to match aggregate features in the U.K.

with the aim of better understanding the influence of the share of land on the allocations in the steady state as well as in the transition.

The depreciation rate of the capital stock $(1-\lambda)$ is set at 10 percent per annum and the coefficient of relative risk aversion at 2. For the baseline, we consider a closed economy so that both $\overline{S}^*$ and ξ are set to be zero. Recent papers have calibrated α (the share of non-durables in total expenditure) at around 0.8 (Diaz and Luengo-Prado (2007) use 0.83 and Li and Yao (2007) use 0.8 based on the average share of housing expenditure found in the 2001 Consumer Expenditure Survey). We use a slightly lower number (0.76) since we think of housing as inclusive of other durables, while Morris and Ortalo-Magne (2008) provide evidence supporting this choice.

The fraction of a house that needs a downpayment (θ) is set at 20%, consistent with the evidence in Chambers et. al. (forthcoming) who estimate this to be 21% for first-time buyers in the early 1990s. We perform extensive comparative statics relative to this parameter since one of our goals is to better understand the role of collateral constraints on home-ownership rates, house prices and allocations.

3.2.3 Model Targets

We choose the discount factor (β) to generate a reasonable tangible assets to output ratio (3.3), and the fraction of utility loss from renting a house (ψ) to generate the number of renters observed in the data (36% in 1992). This yields $\beta = 0.9469$ and $\psi = 0.0608$ for the baseline economy.

3.3 General Features of Household Behavior

The household chooses present consumption, saving, and mode of housing, taking into account its net worth and its expectations of future income. $Figure\ 3A$ illustrates the consumption of goods, housing services and the mode of housing of the worker with low productivity as a function of net worth. In order to explore the stable relationship between the household choice and the state variable, we detrend all variables using their own theoretical trend as in *Appendix B*. When the worker does not have much net worth, $x < x_{1l}$, he does not have enough to pay for a downpayment of even a tiny house. He chooses to rent a modest house and consume a modest amount. In $Figure\ 3B$, the locus $s' = s(s, q, yl)$ shows the equity-holding at the end of the present period as a function of the equity-holding at the end of the last period for the low productivity worker. Everyone enters the labor market with low productivity and no inheritance ($s_0 = 0$). Because $s' = s(s, q, yl)$ locus lies below 45-dgree line for small enough s, as long as the worker continues to be with low productivity, he does not save, hoping to become more productive in the future. He continues to live in a rented house.[14]

$Figure\ 4A$ shows the choice of a worker in the medium productivity state. When she does not have much net worth to pay for a downpayment to buy a house, $x < x_{1m}$, she chooses to rent a place, a similar behavior with the low productivity worker. The main difference is that the medium productivity worker saves vigorously to accumulate the downpayment to buy a house in the future. In $Figure\ 4B$, the $s' = s(s, q, ym)$ locus (the transition of equity-holdings of the medium productive worker from this to the next period) lies above the 45-degree line for $s < sm^*$, so that the equity holding at the end of this period is larger than the last period. When the medium productivity worker accumulates modest net worth, $x \in [x_{1m}, x_{2m}]$ in $Figure\ 4A$, she buys her own house subject to the binding collateral constraint. Here, the size of an owned house is a sharply increasing function of net worth, because the worker maximizes the size of the house subject to the downpayment constraint.[15] When the medium productivity worker has substantial net worth $x > x_{2m}$,

[14] No saving by a low productivity worker is not always true. If the income gap between low productivity and higher productivity workers is small, the transition probability from less to more productive states is small, or the pension is very limited, then the low productivity worker saves to buy a house or for retirement.

[15] The size of the house at net worth $x = x_{1m}$ is smaller than the house rented at net worth slightly below x_{1m}, because she can only afford to pay downpayment on a smaller house. (Nonetheless, she is happier than before, because she derives more utility from the owned home than a rented place). The worker moves to a bigger house every period in our model because there are no transaction costs. If there were transaction costs, the worker would move infrequently, and change housing consumption by discrete amounts, rather than continuously. (The housing ladder would become a true ladder, instead of having a continual upward slope). She may even buy first a larger house than the house rented before, anticipating the future transaction cost. But the basic features remain the same.

she becomes an unconstrained home owner, using her saving partly to repay the debt (or increase the housing equity ownership). In $Figure\ 4B$, the medium productivity worker continues to accumulate her equity holding until she reaches the level of equity-holding at sm^*, the intersection of $s(s, q, ym)$ and the 45-degree line.

The behavior of the high productivity worker is similar to the medium productivity one, except that she accumulates more equities: $s' = s(s, q, yh)$ lies above $s' = s(s, q, ym)$ and her converging equity-holding sh^* is larger than that of medium productive worker sm^*. Therefore, the equity holding of all the workers is distributed in $s \in [0, sh^*]$, with mass of workers at both $s = 0$, $s = sm^*$ and $s = sh^*$. The retiree decumulates assets very slowly as the rate of return is lower than the growth-adjusted rate of time preference.

Putting together these arguments, we can draw a picture of a typical life-cycle in $Figure$ 5. The horizontal axis counts years from the beginning of work-life, and the vertical axis measures housing consumption (h) and equity-holding (s). Starting from no inheritance, he chooses to live in a rented house without saving during the young and low wage periods until the $19th$ year. When he becomes a medium productivity wage worker at the $20th$ year, he starts saving vigorously. Quickly, he buys a house subject to the collateral constraint. Then he moves up fast the housing ladder to become a unconstrained home owner at the $22nd$ year. Afterwards, he starts increasing the fraction of his own equity of the house (similar to repaying the debt). By the time of retirement, he has repaid all the mortgage and has accumulated equities higher than the value of his own house.[16] When the worker hits the wall of retirement (with the arrival of a retirement shock) at the $51st$ year, his permanent income drops, and he moves to a smaller house. He also sells all the equities to buy an annuity contract on the equities, because the annuity earns the gross rate of return which is $(1/\sigma) > 1$ times as much as straightforward equity-holding. But his effective utility discount factor shrinks by a factor σ too. Thus as the rate of return on the annuity is not sufficiently high to induce the retiree to save enough, he decumulates slowly the relative equity-holding, downsizing his consumption of goods and housing services relative to the working population as he gets older. When he dies, his assets drop to zero, according to the annuity contract

[16] Remember that the aggregate equity-holding of structures of all the households is the sum of all the houses and productive structures in equilibrium.

(which pays zero if the contract holder dies).

3.4 Comparison of Steady States

We present our results from the steady state economy in a series of tables and we compare the implications of the model with the data in the 1992 Survey of Consumer Finances (SCF, 1992). *Table* 1 reports the five quintiles of earnings and net worth implied by the model and compares them with their empirical counterparts. The earnings quintiles are matched almost exactly since the parameters of the earnings process were chosen to achieve this objective before the model is solved. Given the skewed earnings distribution, the model generates a very skewed net worth distribution as well. In fact, the distribution implied by the model is slightly more skewed to the right than the data suggest but we think this is a reasonable first step to analyze the implications of the model for the other endogenous variables of interest. The distribution of net worth for homeowners is even more heavily skewed to the right in the model than in the data reflecting the endogenous implication of the model that only very poor households remain tenants. The self-reported house value for homeowners is more evenly distributed than net worth (both in the data and in the model), even though again the model tends to generate a more skewed distribution than in the data.

Table 2 (panel A) compares mean net worth between the data and the model for different groups. The total net worth (normalized by per capita GDP) adds up to the calibration target of the model (3.29). Conditional on home owning, owners are more wealthy than tenants, both in the model and in the data. The model does approach matching the average net worth of owners (4.76 in the data versus 5.52 in the model), but completely misses the net worth of tenants (predicted by the model to be really poor, barely accumulating any wealth while in the data they do own something). This is a weakness of the quantitative predictions of the model but given the richness of other moments that we match we are going to proceed with the analysis and stop reporting statistics for tenants and leave a more explicit calibration that captures the exact wealth accumulation for this group to future work. The model also generates implications about the average size of a house and this can be compared to its empirical equivalent (conditional on home-ownership). The average house value normalized by per capita GDP is 1.93 in the data versus 2.34 in the model. One measure of indebtedness is the ratio of house value to net worth (h/s in the model). The mean number (conditional on homeownership) is 1.39 in the data versus 1.49 in the model. Moreover, we find (but do not report for compactness) that younger households are more indebted and this indebtedness decreases over the life cycle. Panel B illustrates that the model captures well the rising homeownership over the lifecycle. Panel C reports the normalized net worth and home value for the different groups over the life cycle. Household net worth and house values increase over the life cycle in the data and this feature is generated by the model.

We interpret these results as suggesting that the model generates reasonable implications relative to the information in the 1992 SCF. Given this interpretation, we now would like to understand how the endogenous variables in the model (house prices and home-ownership rates) depend upon exogenous fundamentals. In this section we compare steady states. We restrict our attention to three main changes in the fundamentals: a higher productivity growth, greater financial development and a fall in the world real interest rate, since we view these as reasonable exogenous changes to fundamentals given the US experience in the 1990s.

Table 3 reports steady state comparisons for the baseline (U.S.) calibration (panel A). In the baseline calibration in the first column, the fraction of tenants in the population is 36%, which is the US homeownership in the early 1990s. The fraction of constrained home owners is 13.9%. The fraction of houses lived in by tenants and constrained home owners is smaller than the fraction of their population, because they live in smaller houses than the unconstrained home owners, on average. The average size of a tenant's house is about 19.5% (= 7.02/35.92) of the average house size of the economy, and the average house size of constrained home owners is about 21% of the economy average. The tenants and the constrained home owners live in smaller houses than the unconstrained home owners, mainly because the former have lower permanent income. The distribution of equity-holding is even more unequal across the groups of households in different modes of housing. The fraction of total equities held by tenants is negligible (0.1%), the fraction of total equities held by constrained home owners is 2.97%, and the remainder is held by unconstrained home owners.

Turning to prices and aggregate variables, the gross rate of return on equity-holding is 1.0669 in terms of goods, and is equal to $1.0669 \div G_r^{1-\alpha} = 1.0662$ in terms of the consumption basket. The latter is smaller than the inverse of the discount factor, which, adjusted for growth effects, equals $(1/\beta)\left(G_w/G_r^{1-\alpha}\right)^{\rho} = 1.095$. This is not because people are impatient, but because people tend to save substantially during the working period to cope with idiosyncratic shocks to labor productivity and to mitigate the collateral constraint. Many general equilibrium models with uninsurable idiosyncratic risk have such a feature, including Bewley (1983) and Aiyagari (1994). Even though some aggregate variables are not the same as the numbers in *Table* 3, they are broadly consistent with the main features of the US economy. The ratio of average housing value to the average wage is 2.4 years, while the housing price to rental ratio is 8.6 years in the baseline economy. The ratio of value of total tangible assets to GDP is 3.3 years, while the share of housing in total tangible assets is 45% (compared to 41% in the post war US economy, see appendix C).

From (27) we know that the growth of rents in the balanced growth path equals $G_Y^{1-\gamma}$. This implies that the baseline economy steady state annual growth rate in rents will be 0.3% when $\gamma = 0.9$. Davis et. al. (2008) compute the annual rent for the U.S. economy since 1960 and the mean real growth rate is found to be 1.17% with a standard deviation of 1.5%. We therefore view this prediction of the model as empirically plausible. Another prediction of the model involves the long run growth in house prices which is predicted to be equal to the growth rate in rents (therefore 0.3%). Using the OFHEO average annual house price data from 1960 to 2007 we calculate a real (deflating using the US CPI) annual growth rate of 2.1% with a standard deviation of 3.3%. Given that the model features no quality adjustments that might justify a faster increase, we view this prediction also consistent with the implications of the model.

Moreover, the baseline results also allow us to impute the value of land in tangible assets. In the steady state, we can capitalize the value of land and compute its ratio to the value of total tangible assets. This share equals

$$\frac{\frac{1-\gamma}{1-(G_Y/R)}}{\frac{\gamma}{1-(\lambda/R)} + \frac{1-\gamma}{1-(G_Y/R)}} \tag{29}$$

as a function of the share of land in property income $1-\gamma$. Note that physical capital depreciates through λ, while the imputed rental income of land grows at the rate of aggregate output growth in the steady state because the ratio of land value to aggregate GDP is stable in the steady state. Thus, in the baseline economy in which $R = 1.0669$ and $G_Y = 1.029$, when the share of land income is $1 - \gamma = 10\%$, the share of land in the value of tangible assets is equal to 33%. Davis and Heathcote (2007) produce estimates of the share of land in U.S. residential tangible assets and the annual average between 1930 and 2000 is 24.7% with a standard deviation of 9.6%. Given that the housing share of total tangible assets in the baseline economy is around 0.45, the share of land in the value of residential tangible assets is then $0.45 * 33/100 = 15\%$. We therefore view this prediction as broadly consistent with the data.[17] What should also be noted is that in the transition to a new steady state (29) will vary substantially depending on the expected growth rate in output, the discount factor and γ.

Columns 2 and 3 of *Table* 3 report the results for a different level of financial development, keeping the world interest rate constant at its closed economy counterpart in column 1.[18] Column 2 is the case of a more advanced financial system, where the fraction of house that needs downpayment is 0.1 instead of 0.2 (the baseline number). The main difference relative to the baseline economy is that now there are more constrained home owners instead of tenants. Intuitively, because borrowing becomes easier, relatively poor households buy a house with high leverage (outside equity ownership) instead of renting. Column 3, by comparison, is the case of no housing mortgage ($\theta = 1$) so that the household must buy the house from its own net worth. In this economy, more than a half of households are tenants. Financial development affects substantially the home-ownership rate. On the other hand, financial development by itself has limited effects on prices and aggregate quantities in steady state. This result arises because the share of net worth of tenants and constrained households (who are directly influenced by the financing constraint) is a small fraction of aggregate net

[17] Thus, our assumption of a Cobb-Douglas production function for structures is generally consistent with the U.S. data. Moreover, for Japan Kiyotaki and West (2006) provide evidence that the elasticity of substitution between land and capital is not significantly larger than unity for the period 1961-1995.

[18] To conserve space we will be presenting results keeping the world interest rate constant, since we view this analysis more likely to capture trends in house prices in the data.

worth, and because the required adjustment is mostly achieved through the conversion of houses from rental to owner-occupied units.

In column 4, we consider an economy in which the growth rate of labor productivity is three percent instead of two percent. A higher growth rate of productivity, keeping the world interest rate constant, raises substantially the housing price-rental ratio from 8.6 to 9.6, a result to be expected from the intuition captured in (29) and (27). The value of housing to the average wage rises from 2.4 to 2.5, as does the value of tangible assets to GDP. In the new steady state, the percentage of tenants is much higher (50% from 36%) as housing prices have risen substantially.

In Column 5, we consider an economy where the world interest rate is reduced by one percentage point. A lower world interest rate increases the value of land in total tangible assets and increases the house price to rental ratio from 8.6 to 9.9. The value of tangible assets to GDP rises from 3.3 to 3.8 while the number of tenants increases as in the previous case to around 50% from the initial 36% in response to the higher equilibrium housing prices.

3.4.1 "UK calibration"

One of the key messages of our work is that the constraint imposed by land as a fixed factor of production can have important implications for the behavior of house prices and homeownership, both over time and over the life cycle. Nevertheless, illustrating the role of different share of land in production of tangible assets $(1-\gamma)$ across different incomplete market economies in general equilibrium models will be plagued by having to change a number of different parameters simultaneously. Therefore, and to show the effects of γ as cleanly as possible, we change 3 parameters from the previous calibration and argue that this can give useful insights to a country like the U.K..[19] Basically, we view the U.K. as having a lower γ and an integral part of the world economy (with free capital flows). We therefore find the combination (keeping everything else as in the US calibration) of $\{\beta, \gamma, \psi\}$ that in a closed economy would hit three different targets that correspond to UK data. Specifically, $\{\beta, \gamma, \psi\}$ are chosen so that the world interest rate remains at 6.69%, the ratio of tangible

[19] We say "UK" calibration because all other parameters are still calibrated with US numbers in mind.

assets to GDP rises to 4.29[20] and the homeownership rate rises to 68% (the UK number in the early 1990s). The higher tangible assets to GDP ratio is achieved using a value of γ equal to 0.783, and the remaining two parameters are $\beta = 0.9612$ and $\psi = 0.0598$.

The baseline results (column 1) in Panel B of *Table* 3 illustrate one of the messages of the paper, namely that the larger share of land in the production of tangible assets can substantially affect the price to rental ratio, the value of tangible assets to GDP and the value of housing to GDP. Specifically, the ratio of housing value to the average wage rises from 2.39 in the $\gamma = 0.9$ economy to 3.23 in the $\gamma = 0.78$ one, while the price to rental ratio rises from 8.58 to 10.96. Why is the value of tangible assets and the price to rent ratio much higher in this economy? Applying (29) for the baseline economy in which $R = 1.0669$ and $G_Y = 1.029$, we conclude that if the share of land income is $1 - \gamma = 10\%$ then the share of land in the value of tangible assets is equal to 33%. When $\gamma = 0.78$, on the other hand, we have $G_Y = 1.028$, and the share of land in the value of tangible assets is 55% for the same real rate of return. Since land neither depreciates nor accumulates through more supply, an increase in the share of land reduces the effective depreciation of tangible capital and increases the price to rent ratio.

Do our conclusions from changing the level of financial development, the labor productivity growth and the world interest rate change in this new environment? Changing the collateral constraint again only works through the homeownership rate and does not affect equilibrium prices, in the same way as before. A higher productivity growth changes substantially the house price to rental ratio (from 11.0 to 12.9), with similar large effects on the house value to wage ratio. A reduction in the world interest rate also substantially affects equilibrium prices. The main effect comes from the higher value of land which makes the price to rental ratio rise much more in the UK calibration. In this economy the price to rent ratio rises from 11.0 to 13.2 (a 21% increase), while in the US calibration ($\gamma = 0.9$) this ratio rises from 8.6 to 9.9 (a 15% increase). In equilibrium, the homeownership rate falls in response to higher prices in both of these experiments.

[20] This is the average ratio for the UK economy between 1987 and 2007, the period for which the data exist.

4 Winners and Losers in Housing Markets

We now examine how the small open economy reacts to a once-for-all change in different fundamental conditions n technology and the financial environment. We change a parameter once-and-for-all unexpectedly and solve for the path of prices and quantities that lead the economy to the new steady state. Here, we assume perfect foresight except for the initial surprise. Details of the numerical procedure can be found in Appendix A, but the basic procedure is as follows. First guess a set of rental rates over the next (say) 50 years, which converges to the new steady state; then solve backwards the household problem based on these prices; and finally update this price vector until the market for use of tangible assets clears in all periods. To highlight the importance of land, we compare the reaction of the economy with a larger share of land in the production of tangible assets ($\gamma = 0.78,$ the "UK calibration") with the baseline economy ($\gamma = 0.9$, the US calibration).

4.1 Welfare Evaluations

We are particularly interested in how an unanticipated change in fundamentals affects the welfare of various groups of households differently. Here, using the joint distribution of current productivity and equity holdings from the previous period $\Phi\left(\varepsilon_t(i), s_{-1}\left(i\right)\right)$ in the steady state before the shock hits, we define the group as the set I_g of individual households of a particular labor productivity (low, medium, high, and retired (l, m, h, r)), and a particular range of equity holdings of the previous period which corresponds to a particular home-ownership mode (tenant, constrained owner or unconstrained owner) in the old steady state. For example, the low-wage worker tenant group is a group of agents with low labor productivity who choose to be tenants under the old steady state.

One simple measure of the distribution effect is the average rate of change of net worth. Let $j\left(i\right)$ be present labor productivity of ($j(i) = h, m, l$ and r) of individual i. Then the net worth of individual i depends upon the wage rate and equity price as:

$$x(i) = w\epsilon^{j(i)} + q\widetilde{s}_{-1}(i),$$

where $\epsilon^j = (1-\tau)\varepsilon^j$ for worker of productivity j and $\epsilon^j = (b/w)$ for $j = r$, retired, $\widetilde{s}_{-1}\left(i\right) =$

$s_{-1}(i)$ if i was a worker and $\widetilde{s}_{-1}(i) = s_{-1}(i)/\sigma$ if i was a retiree in the previous period. Then, the average rate of change in net worth (non-human wealth) of group I_g is:

$$\text{average of } \left(\frac{[w_n \epsilon^{j(i)} + q_n \widetilde{s}_{-1}(i)]}{[w_o \epsilon^{j(i)} + q_o \widetilde{s}_{-1}(i)]} - 1 \right) \text{ for all } i \in I_g \tag{30}$$

where (w_o, q_o) are the wage rate and equity price in the old steady state, and (w_n, q_n) are those immediately after the shock.

To calculate welfare changes we use the value functions. Given that we have solved for the prices and value functions for all the periods in the transition, we know that the value functions at the period when the change in fundamentals takes place is a sufficient statistic for the welfare effect of the shock. Let $V_o^{j(i)}(x(i))$ be the value function at the old steady state and $V_n^{j(i)}(x(i))$ be the value function in the period of the shock's arrival as a function of net worth $x(i)$ and labor productivity.[21] We compute a measure of welfare change for the group I_g as:

$$\overline{\mu}_g = \text{average of } \left[\left(\frac{V_n^{j(i)}([w_n \epsilon^{j(i)} + q_n \widetilde{s}_{-1}(i)])}{V_o^{j(i)}([w_o \epsilon^{j(i)} + q_o \widetilde{s}_{-1}(i)])} \right)^{\frac{1}{1-\rho}} - 1 \right] \text{ for all } i \in I_g. \tag{31}$$

We call this measure as the certainty expenditure equivalent, because we convert the change of the value into the dimension of expenditure before taking the average.[22]

[21] Note that V_n is the value function that has been derived after the full perfect foresight transition has been solved for and therefore includes all this information about the transition to the new steady state.

[22] We also computed the net worth equivalent that would make a household indifferent between the period before and after the shock as the value of $\lambda(i)$ such that

$$V_o^{j(i)}([w_o \epsilon^{j(i)} + q_o \widetilde{s}_{-1}(i)]) = V_n^{j(i)}(\lambda(i)[w_n \epsilon^{j(i)} + q_n \widetilde{s}_{-1}(i)])$$

The value of $\lambda(i)$ measures how much the initial net worth must be multiplied immediately after the shock in order to maintain the same level of the expected discounted utility as the old steady state. We can find the net worth equivalent uniquely, because the value functions are monotonically increasing. We can then compute the average of individual $\lambda(i) - 1$ for a particular group g of agents as $\widetilde{\mu}_g$. This welfare measure suffers from the drawback that net worth does not include the value of human capital. Thus, if two groups have different ratios of net worth (liquid wealth) to human capital, a difference in $\widetilde{\mu}_g$ may reflect the difference of the ratio of human to non-human wealth rather than the difference in the welfare effect.

4.2 Transition of Small Open Economy following a Change in Fundamentals

Figure 6 shows the responses to a once-for-all increase in the growth rate of labor productivity from 2% to 3%. Because the economy is growing, all the following figures show the percentage difference from the steady state growth path of the baseline economy. In both economies the housing price increases substantially initially and continues to increase afterwards. In the economy with a larger share of land ($\gamma = 0.78$), the increase in house prices is larger, and real house price inflation afterwards is higher. The housing price-rental ratio is going to be higher, anticipating the increase in the rental price in the future. The home-ownership rate gradually declines because young workers take a longer time to accumulate a sufficient downpayment to buy a house. Consumption of goods and housing services increase initially as well as afterwards, reflecting higher permanent income. The share of productive tangible assets (Z_{Yt}/Z_t) falls initially, to accommodate a larger demand for residential tangible assets by converting productive to residential tangible assets.

Table 4 reports the average rate of change of welfare (31) in Panel A and the average rate of change of current net worth (30) in Panel B for each group against changes in the fundamentals, for the baseline economy ($\gamma = 0.9$) and the economy with a larger share of land ($\gamma = 0.78$). The first and second columns report the average rate of changes from an increase in the growth rate of labor productivity from 2% to 3%. Given the higher productivity growth, households are on average better off with a higher permanent income. (Remember the retiree's benefit is proportional to the wage rate of present workers). The higher housing price, however, affects the welfare of different groups of households differently. Those who buy (or expand) houses in the future gain less from the housing price hike, while those who sell houses in the future gain more. Specifically, unconstrained homeowners as a group gain more than tenants and constrained homeowners. The gap in welfare effects between unconstrained homeowners and the other groups is particularly large for the retirees. Overall, one main message from this analysis is that the redistribution effect is larger in the economy with the larger share of land since the house price hike is bigger in this economy.

We can observe the change in current net worth in Panel B. The net worth of unconstrained homeowners increases by a much larger amount than tenants' net worth because the former own much more non-human wealth. Thus, those with larger holdings of shares experience a bigger increase in net worth with the house price rise, and the increase is more pronounced where land is more important.

Figure 7 shows how these two economies react to a once-for-all fall in the world real interest rate by 1%. In both economies, housing prices and output increase with large inflows of capital, and the adjustment of housing prices is fast. In the economy with a larger share of land, the swing of net exports and consumption is larger, output takes a longer time to increase despite the large increase in the capital stock, because a large amount of tangible assets gets allocated to housing in the early stages of the transition. The home-ownership rate declines gradually because the lower real interest rate discourages saving, delaying the age of switching from renting to owning a house over the life cycle.

The third and fourth columns of *Table* 4 report the reaction of welfare to this decrease in the world real interest rate for the two economies with different shares of land. Looking at the value of net worth in Panel B, all groups have a larger net worth from a higher house price, and the net worth increase is larger group-by-group in the economy with a larger share of land ($\gamma = 0.78$). As we discussed in the Introduction (especially in footnote 4), however, the increase in housing price per se does not have an aggregate wealth effect on consumption nor welfare, but mainly redistributes wealth between net sellers and net buyers of houses. Unconstrained homeowner retirees gain most from the house price hike due to a lower interest rate. Although workers gain from a higher wage rate due to capital inflow, workers as a whole are savers who suffer from a lower interest rate, particularly high income workers. Thus despite the capital gains on housing, the high income workers and unconstrained homeowner workers lose from a lower interest rate in our calibration, and the loss is larger when the share of land is small ($\gamma = 0.9$), that is, when the capital gains on the house is small.

These two experiments illustrate the idea that the relationship between housing price changes and welfare depends upon the underlying cause of the house price change. House prices are higher by a similar magnitude after either a higher productivity shock or a lower world interest rate, but in our calibrations workers as a whole gain from the productivity improvement but lose as a whole from the interest rate decrease[23].

We have also done the experiment of lowering the downpayment requirement from 20% to 10% permanently. This provides extra liquidity for households, especially for constrained home owners, and encourages consumption initially. At the same time, with a less stringent collateral constraint, some low wage workers and tenants from the previous period buy houses. Overall, however, relaxing the financing constraint has a very limited effect on housing price and aggregate production in the transition, a result similar to the comparisons of the steady states, because the necessary adjustment is mostly achieved by the modest conversion of rented to owned units rather than by the housing price. This contrasts Ortalo-Magne and Rady (2006), who show that relaxing the collateral constraint increases the housing price substantially by increasing the housing demand of credit constrained households. In their model, the net worth of the home-owners with outstanding mortgage is sensitive to the housing price due to the leverage effect, which magnifies the effect of any shock to fundamentals, while there is no leverage effect in our equity financing economy. Also the supply of houses and flats is inelastic in their model. Thus, relaxing the collateral constraint will generate a large inflow of new owners of flats and houses, which is not offset by an increase in the supply, through conversion from rented to owned units, conversion from productive to residential tangible assets and capital accumulation. A comprehensive analysis of the leverage effect and the portfolio decision in the presence of uninsurable earnings and aggregate risk is a topic for future research.

4.3 A Scenario for House Price Changes?

Putting together the simulation results from these experiments, we can conclude that, if we were to explain the large increase in housing prices in many developed countries in the last decades, we could look for increases in the expected growth rate of labor productivity

[23] Attanasio et. al. (2009) make a similar point empirically. They find that renters' consumption is positively correlated with house price increases, contradicting the convenctional wealth channel. They attribute this finding to common factors driving both consumption demand and house prices, namely better longer-run income prospects. Thus, the shock causing higher house prices can be key in determining the effect on consumption (and, therefore, welfare).

and for decreases in the real interest rate. Moreover, to generate a positive correlation between homeownership rates and house price rises since the early 1990s, we will also need to simultaneously improve access to credit. An empirically plausible calibration will be to simultaneously increase the expected growth rate of labor productivity from 2% to 3%, decrease the world interest rate by one percent and reduce the collateral constraint from 20% to 10%.

The implications for house prices and homeownership rates are given in figures 8 and 9 respectively for the US experience, and figures 10 and 11 for the UK. For the US calibration figure 8 illustrates that the model can explain a substantial component of the recent house price increases. Moreover, the model captures well the increase in home-ownership rates, even though this increase is much faster in the model than in the data given the perfect foresight/information assumptions of the model. Interestingly the model does predict a fall in the homeownership rate after the initial increase as house prices begin to rise. The wealth changes and the welfare effects from this simultaneous shock for the US economy are given in column 5 of table 4. Households are both richer and better off in response to this combination of shocks, with the unconstrained home owner retirees gaining the most in both wealth and welfare.

The responses of the calibration for the "UK" economy are given in figures 10 and 11. The model captures a lower fraction of the recent runup in housing prices in the UK, but it also predicts a slight increase in homeownership rates with a decrease predicted in the future as housing prices reach a higher level. The wealth and welfare calculations in the last column of table 4 illustrate the both variables increase by more in this economy rather than in the $\gamma = 0.9$ one and that the effect is biggest for the unconstrained retirees.

5 Conclusions

This paper develops an aggregate life-cycle model to investigate the interaction between housing prices, aggregate production, and household behavior over a lifetime. We take into account land as a fixed factor for producing residential and commercial tangible assets in order to analyze the implications for the aggregate time series and the cross section of household choices. Comparing two small open economies with different shares of land in the production of tangible assets, the economy with a larger share of land has a higher housing price-rental ratio and a lower homeownership rate in the steady state. The transitions of the small open economy along the perfect foresight path illustrate that, where the share of land is larger, once-for-all shocks to the growth rate of labor productivity or the world interest rate generate a greater movement in housing prices. A permanent change in the collateral constraint, however, has a limited impact on housing prices and aggregate production, even though it affects the home-ownership rate substantially.

We also find that the permanent increase in the growth rate of labor productivity and the decrease in the world real interest rate substantially redistribute wealth from the net buyers of houses (relatively poor tenants) to the net sellers (relatively rich unconstrained homeowners) with the house price hike. On average, households gain from the increase in the growth rate of labor productivity and do not gain from the decrease in the world interest rate. Because the gap in welfare effects between winners and losers in the housing market is substantial, especially where land is more important for production of tangible assets compared to capital, we think that a credible welfare evaluation should take into account household heterogeneity and contract enforcement limitations in housing and credit markets that generate realistic life-cycles of consumption and homeownership.

Appendix A: Solving the model

Solving the household's decision problem

We discretize net worth (x_t^i) using 400 grid points, with denser grids closer to zero to take into account the higher curvature of the value function in this region. The grid range for the continuous state variable is verified ex-post by comparing it with the values obtained in the simulations. For points which do not lie on the state space grid, we evaluate the value function using cubic spline interpolation along net worth. We simulate the idiosyncratic exogenous productivity shock from its three-point distribution. The realizations of these exogenous random variables are held constant when searching for the market clearing prices (p and r). We use the policy functions to simulate the behavior of 10000 agents over 600 (the exact number depends on the probability of exiting working life and the survival probability) periods and aggregate the individual housing and equity demands to determine the market clearing rental and housing price and the equilibrium household allocations.

Solving the perfect foresight model

We guess a sequence of tangible asset rental rates $\{r_t\}_{t=1}^T$ such that the rental rate has converged to the new steady state. For an exogenous real interest rate R in the small open economy, use (22) to calculate a sequence of capital stocks $\{K_t\}_{t=1}^T$ and then use (2) to compute the sequence of $\{Z_t\}$. Then we get tangible asset prices $\{q_t, p_t\}_{t=1}^T$ from (25) and $V_t^F = q_t Z_{t-1} = p_t Z_t - I_t$ (which follows from the firm flow-of-funds and the zero profit condition). Given these guessed prices, we solve the household's problem backwards from period T when the economy is assumed to have converged to the new steady state. Households are assumed to know the realization of the entire path of tangible asset prices and rental rates. The value function in period T is the value function for the new steady state. Then the value function in period T-1 is computed as follows:

$$V_{T-1}\left(x_{T-1}|r_{T-1}, p_{T-1}\right) = \max_{c_T, h_T}\left[u\left(c_{T-1}, h_{T-1}\right) + \beta V_T\left(x_T|r_T, p_T\right)\right]$$

We simulate the model forward, starting from the capital stock and the joint distribution of labor productivity and equity of the original steady state. In each period, we simulate a cross-section of 10000 agents over 600 periods and aggregate their individual housing choices, computing the excess demand for tangible assets in each period. We increase the rental rate in periods with an excess demand in the market for tangible assets use, and decrease the rental rate in periods with an excess supply, generating a new path $\{r_t\}_{t=1}^T$ of the rental rate. We repeat this until successive paths of the rental rate are less than 0.0001% from each other.

Appendix B: Stationary Representation of Value Functions

The stationary representation of the household's problem

Using the property of the steady state equilibrium of Section 2.4, we normalize the quantities and prices using the power function of labor in efficiency units $M_t \equiv A_t N_t$ and population N_t. Both variables are exogenous state variables, and there can be a jump or a kink in the trend if labor productivity experiences a once-for-all change in its level or growth rate. Let us denote the normalized variable X_t as $\widetilde{X}_t$. Then we have:

$$
\begin{aligned}
\widetilde{K}_t &= K_t/M_t^{\ \frac{1-\eta}{1-\gamma\eta}}, \quad \widetilde{S}_t^* = S_t^*/M_t^{\ \gamma\frac{1-\eta}{1-\gamma\eta}} \\
(\widetilde{w}_t, \widetilde{x}_t) &= (w_t, x_t)/(M_t^{\ \frac{1-\eta}{1-\gamma\eta}}/N_t) \\
(\widetilde{h}_t, \widetilde{s}_t) &= (h_t, s_t)/(M_t^{\ \gamma\frac{1-\eta}{1-\gamma\eta}}/N_t) \\
(\widetilde{r}_t, \widetilde{p}_t, \widetilde{q}_t) &= (r_t, p_t, q_t)/M_t^{\ (1-\gamma)\frac{1-\eta}{1-\gamma\eta}} \\
\widetilde{V}_t^i &= V_t^i/\left[\frac{M_t^{\ \frac{1-\eta}{1-\gamma\eta}}/N_t}{M_t^{\ (1-\alpha)(1-\gamma)\frac{1-\eta}{1-\gamma\eta}}}\right]^{1-\rho}, \text{ for } i = l, m, h, \text{ or } r
\end{aligned}
$$

We also define the normalized discount factor as:

$$
\widetilde{\beta} = \beta\left(\frac{G_w}{G_r^{1-\alpha}}\right)^{1-\rho}.
$$

Let us assume population grows along the steady state path. Let $\widetilde{A}_t$ be deviation of labor productivity from the trend. Then the vector of normalized state variables adjusted by the productivity change are:

$$
\overline{\widetilde{A}}_t = \left(\widetilde{A}_t, \widetilde{K}_{t-1}, \widetilde{S}_{t-1}^*, \widetilde{\Phi}_t\left(\varepsilon_t, \widetilde{s}_{t-1}(i)\right)\right)'.
$$

Using these normalized variables, we can define the normalized value function. For an example, the stationary representation of the retiree's problem is (noting that prices and quantities grow at different rates, explaining the use of (28) in the normalizations:

$$
\widetilde{V}^r\left(\widetilde{x}_t, \overline{\widetilde{A}}_t\right) = Max(
$$

$$
\begin{gathered}
\max_{\widetilde{s}}\left\{\begin{array}{c}\frac{1}{1-\rho}\left[\frac{\widetilde{x}_t-(\widetilde{p}_t-\widetilde{r}_t)\widetilde{s}_t}{[\widetilde{r}_t/(1-\psi)]^{1-\alpha}}\right]^{1-\rho} \\ +\widetilde{\beta}\sigma\widetilde{V}^r\left(\widetilde{b}_{t+1}+\frac{\widetilde{q}_{t+1}}{\sigma}\frac{\widetilde{s}_t}{G_w}, \overline{\widetilde{A}}_{t+1}\right)\end{array}\right\}, \\
\max_{\widetilde{s}}\left\{\begin{array}{c}\left\{\left[\frac{\widetilde{x}_t-\left(\widetilde{p}_t-\widetilde{r}_t+\frac{\widetilde{r}_t}{\theta}\right)\widetilde{s}_t}{\alpha}\right]^{\alpha}\left[\frac{\widetilde{s}_t\widetilde{r}_t/\theta}{1-\alpha}\right]^{1-\alpha}\right\}^{1-\rho}/(1-\rho) \\ +\widetilde{\beta}\sigma\widetilde{V}^r\left(\widetilde{b}_{t+1}+\frac{\widetilde{q}_{t+1}}{\sigma}\frac{\widetilde{s}_t}{G_w}, \overline{\widetilde{A}}_{t+1}\right)\end{array}\right\}, . \\
\max_{\widetilde{s}}\left\{\begin{array}{c}\frac{1}{1-\rho}\left[\frac{\widetilde{x}_t-(\widetilde{p}_t-\widetilde{r}_t)\widetilde{s}_t}{\widetilde{r}_t^{\ 1-\alpha}}\right]^{1-\rho} \\ +\widetilde{\beta}\sigma\widetilde{V}^r\left(\widetilde{b}_{t+1}+\frac{\widetilde{q}_{t+1}}{\sigma}\frac{\widetilde{s}_t}{G_w}, \overline{\widetilde{A}}_{t+1}\right)\end{array}\right\})
\end{gathered}
$$

Appendix C: Data sources and definitions

To compute the share of income of productive tangible assets (η), we use quarterly data from the US Flow of Funds accounts and from the NIPA for the period of 1952 Q1 - 2005Q4. We follow Cooley and Prescott (1995). We define unambiguous capital income as the sum of corporate profits (π), net interest (i), non-housing rental income (r) from the NIPA (table 1.12)[24]. We also measure the depreciation of capital (DEP) by the consumption of fixed capital (NIPA, table 1.14). We allocate η fraction of proprietors' income (Y_P, NIPA, Table 1.12) to the income from productive tangible assets. Then, the income from productive tangible assets, Y_{ZP}, can be computed as the sum of unambiguous capital income, depreciation, and η fraction of proprietors' income:

$$Y_{ZP} = \pi + i + r + DEP + \eta Y_P = \eta Y$$

where Y is GDP excluding explicit and implicit rents from housing. Solving this for η, we have

$$\eta = \frac{\pi + i + r + DEP}{Y - Y_P}$$

This is a similar expression for the share of capital in output found in Cooley and Prescott (1995, p.19).

Averaging the quarterly data for the U.S. from 1952 to 2005, we obtain a value of η equal to 0.26. This is lower than the share of capital in output in the real business cycle literature (estimates there range between 0.3 and 0.4) because our η excludes the capital intensive production of housing services. We can decompose economy-wide tangible assets between the household and the firm. The exact definitions in the data and their counterparts in the theoretical model are given in the following table:

[24] We use the average share of residential to total structures to compute non-housing rental income from the total rental payments of all persons reported in NIPA table 1.12.

Economic concept	Flow of Funds concept
pZ_y	Non-farm, non-financial tangible assets (Non-residential tangible assets+Equipment+software+Inventories) Flow of funds, Tables B.102 and B.103 FL102010005.Q+FL112010005.Q-FL115035023.Q
$p\int h(i)di = pH$	Household tangible assets (Residential tangible assets+Equipment+software+Consumer durables) Flow of funds, Table B.100 FL152010005.Q+FL115035023.Q

Non-corporate tangible assets include residential properties occupied by renters. Therefore, this series (FL115035023.Q) is subtracted from pZ_y and added to household tangible assets. Using these definitions, we compute the average numbers of $Z_Y/(Z_Y+H) = 0.59$ between 1952:Q1 and 2005:Q4. The ratio of total tangible assets to GDP $(p(Z_y+H)/Y)$ is 3.3, giving an average value of residential tangible assets to GDP of around 1.94. If farm corporate and non-corporate tangible assets (FL132010005.Q in the Flow of Funds)[25] are added to the non-farm tangible assets, then the ratio of household tangible assets to total tangible assets falls from 0.59 to 0.55, while the ratio of total tangible assets to GDP rises from 3.3 to 3.6.

Appendix D: Survey of Consumer Finances

We use primarily the 1992 SCF to calibrate our parameters. The labor income process is intended to use entrepreneurial income on top of wages and salaries. Following Castaneda et. al. (2003) we add to wages and salaries and proportion of proprietors' income that can be attributed to self-employment. Thus, total labor income is wages and salaries plus 0.93 of business income where the 0.93 comes from the average ratio of (wages_sal/(wages_sal+bus_inc)). Net worth is total assets minus total debt for each household, corresponding to variable s in the model. The house value is the self-reported value of the primary residence conditional

[25] Thanks to Michael Palumbo (Board of Governors) of kindly sending us this series in private correspondence.

on owning a house. The SCF homeownership rate matches the Census one in 1992 exactly (64%).

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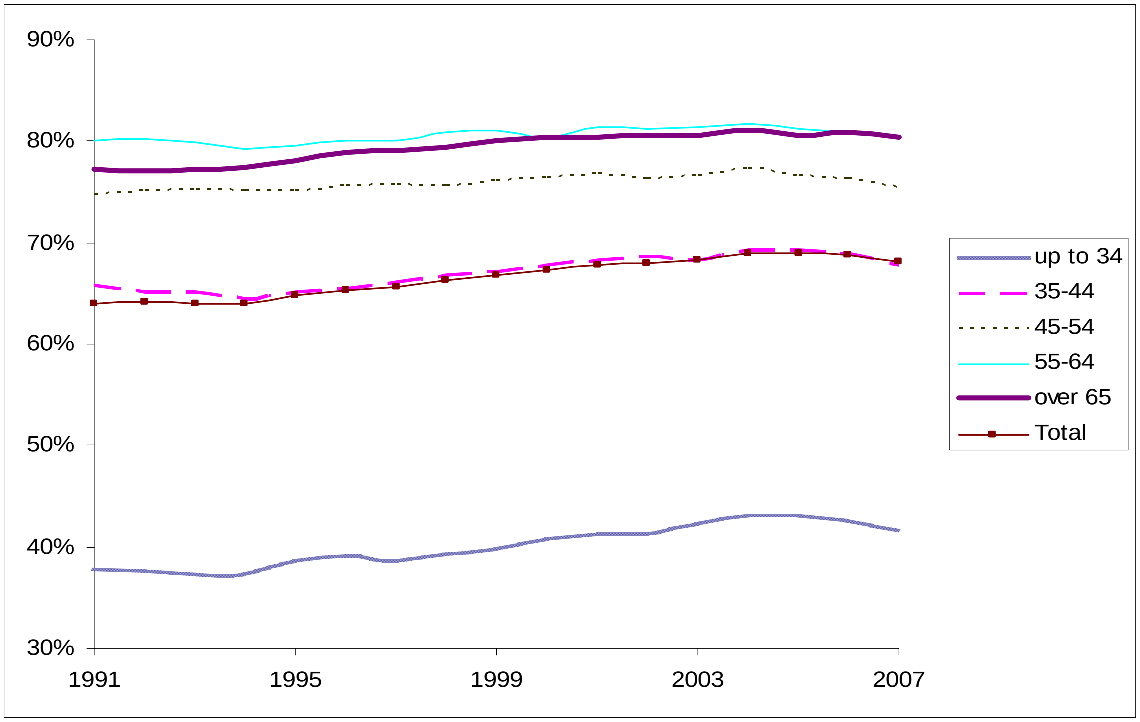
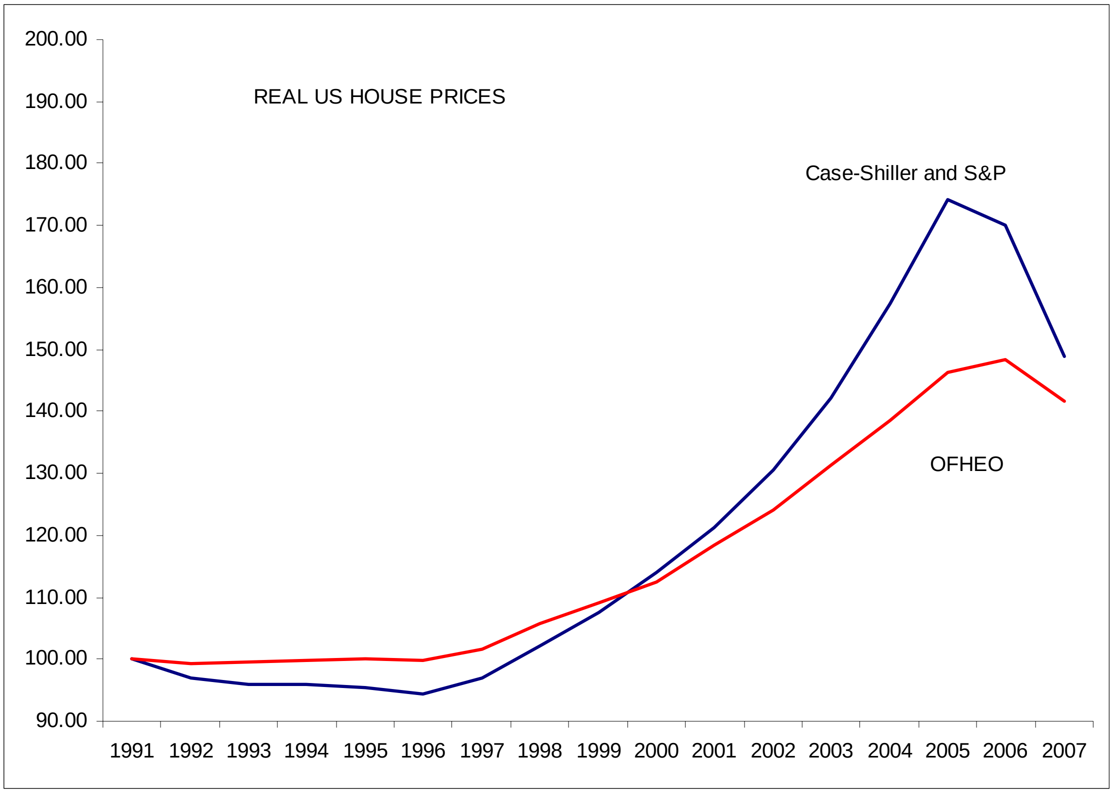
Figure 1: US home-ownership rates (total and by age group) for 1991 – 2007



Source: http://www.census.gov/hhes/www/housing/hvs/historic/index.html

Figure 2: US real housing price index: 1991 = 100



Notes: OFHEO is the purchase-only equally weighted house price index produced by the Office of Federal Housing Enterprise Oversight. Case-Shiller is the value-weighted house price index jointly produced by S&P and Case-Shiller. Both indices are deflated by the US CPI for urban consumers.

Figure 3A: Policy functions for a low productivity household

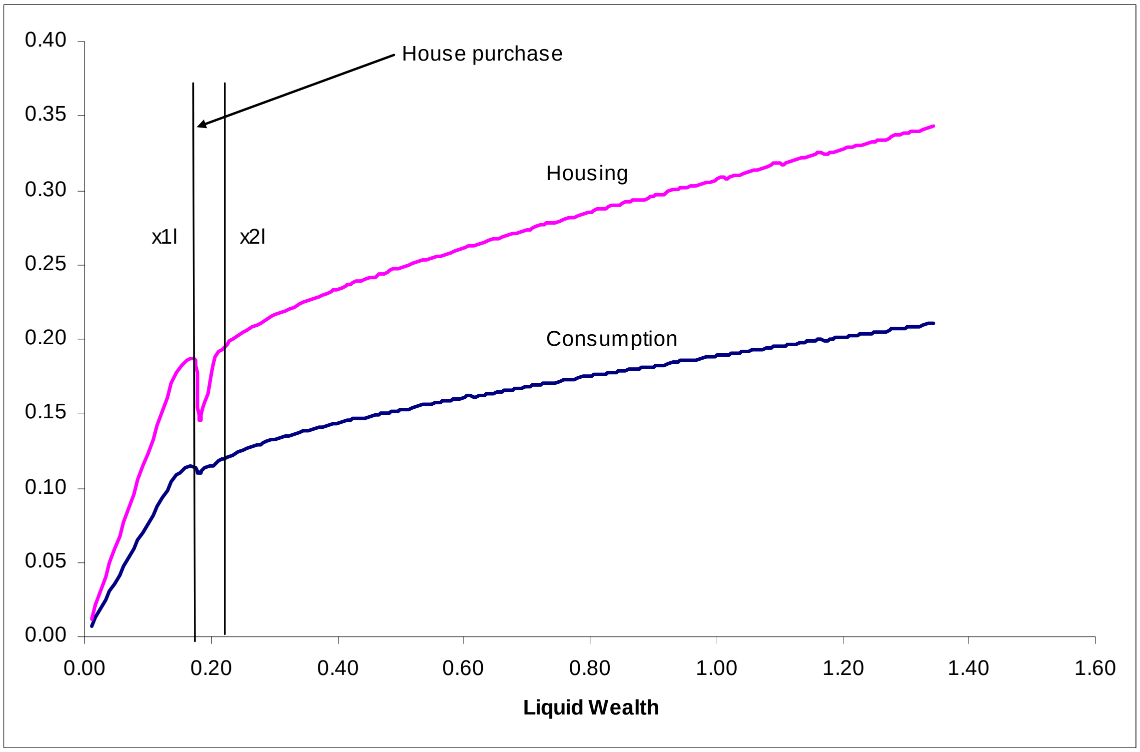


Figure 3B: Evolution of saving for a low productivity household

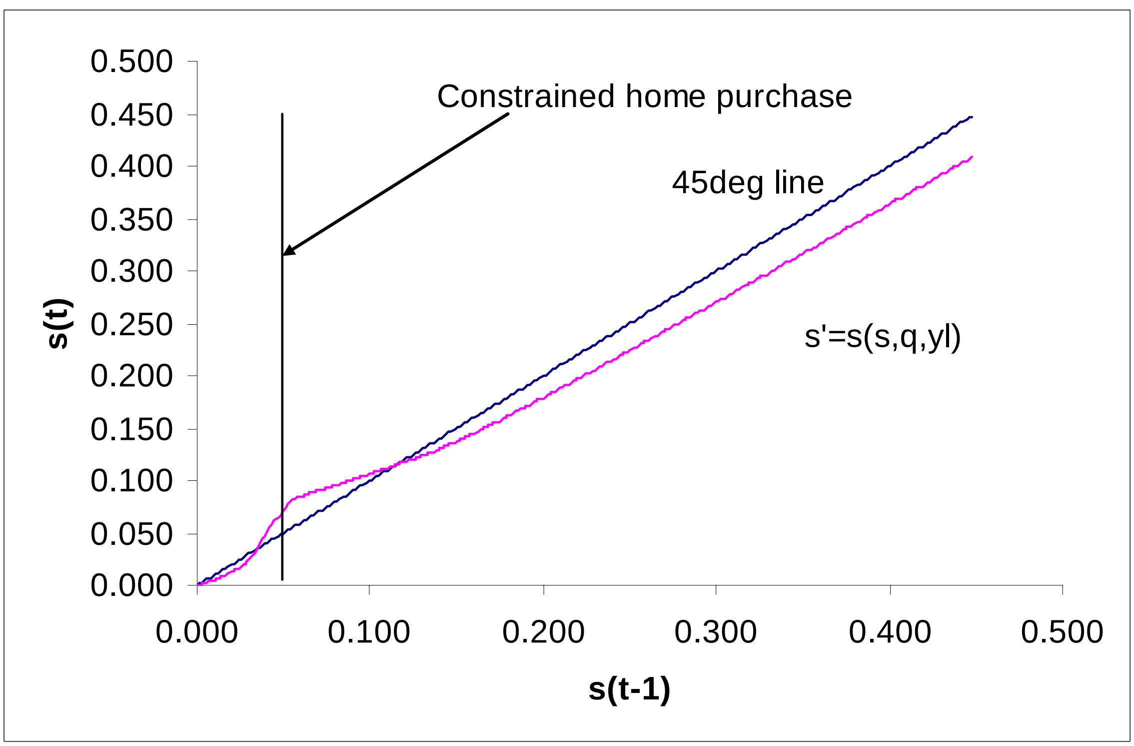


Figure 4A: Policy functions for a medium productivity household

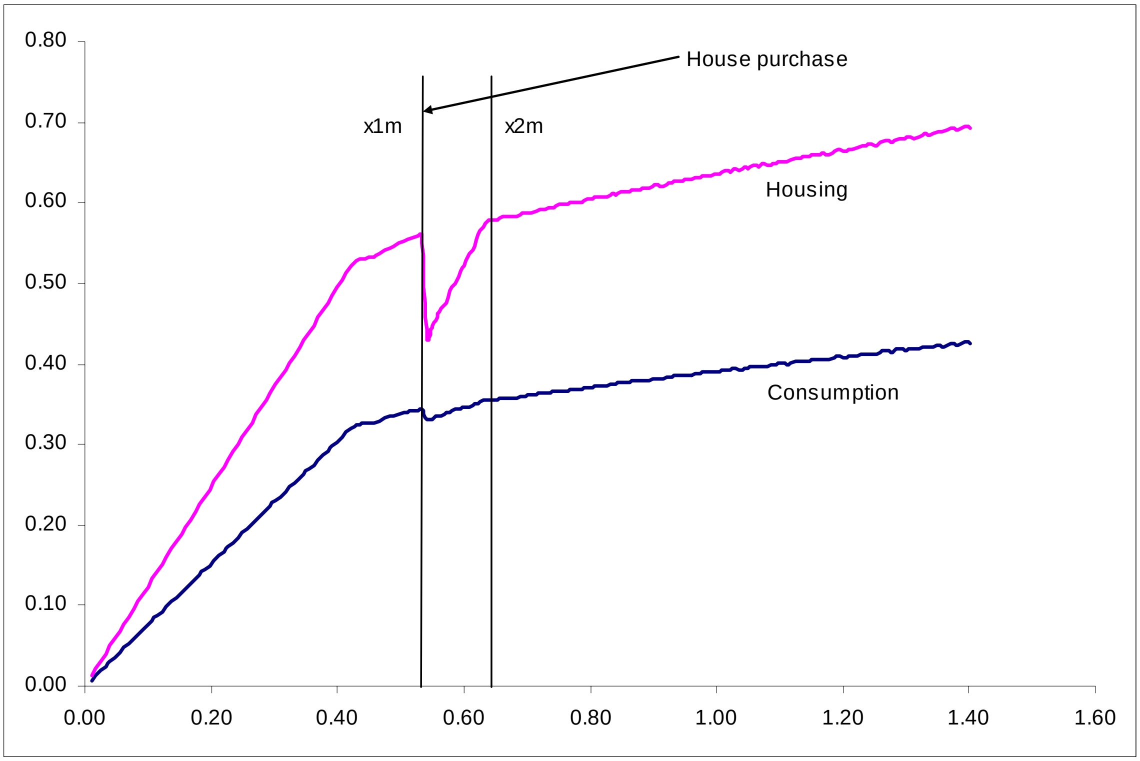


Figure 4B: Evolution of saving for a medium productivity household

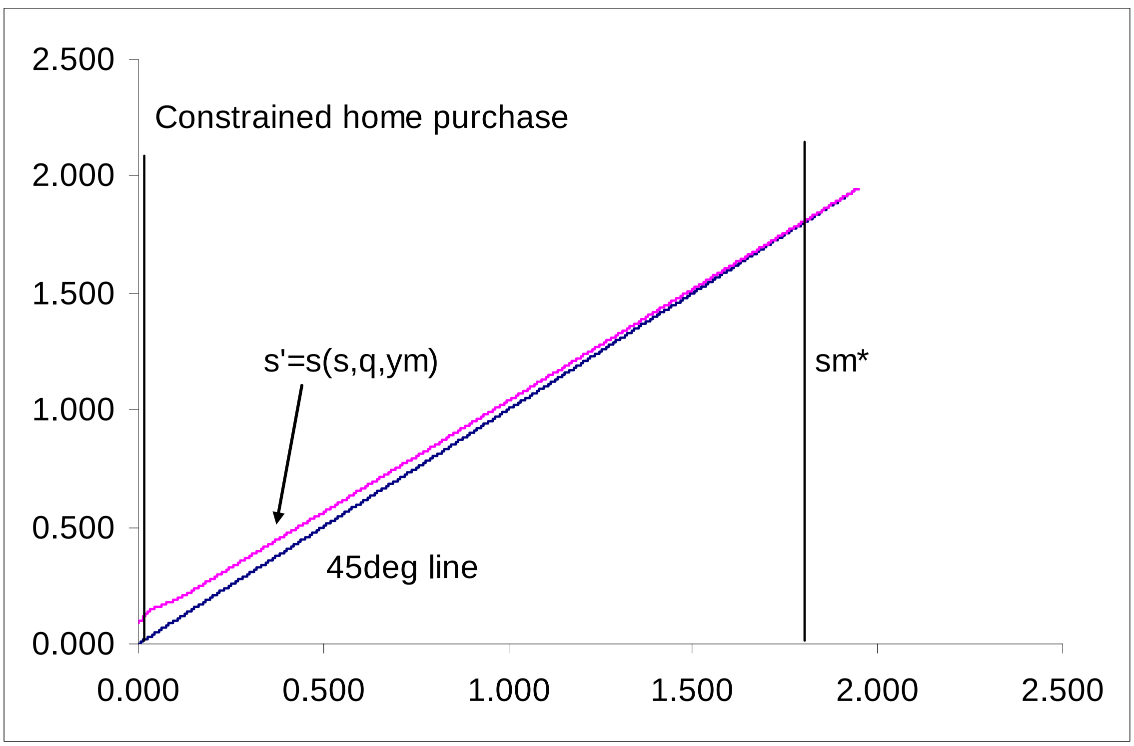


Figure 5: An example life time

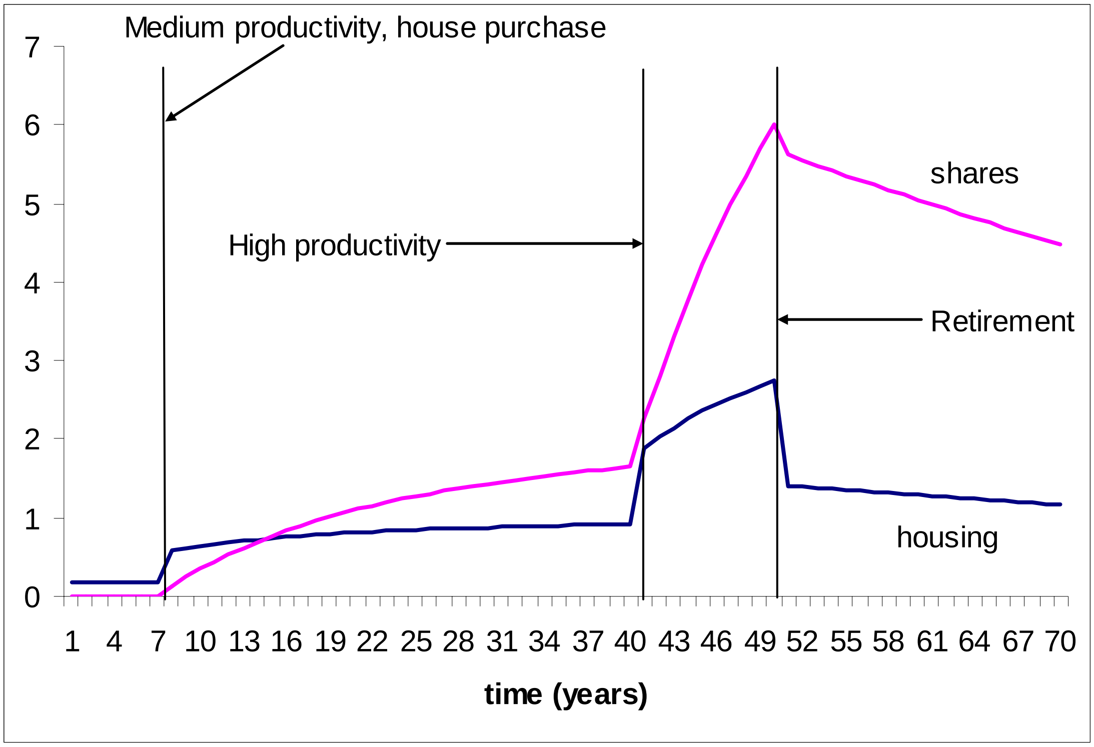


Figure 6
Transition Dynamics from a 1% increase in labor productivity growth
(solid line: γ=0.9, dotted line: γ=0.78)

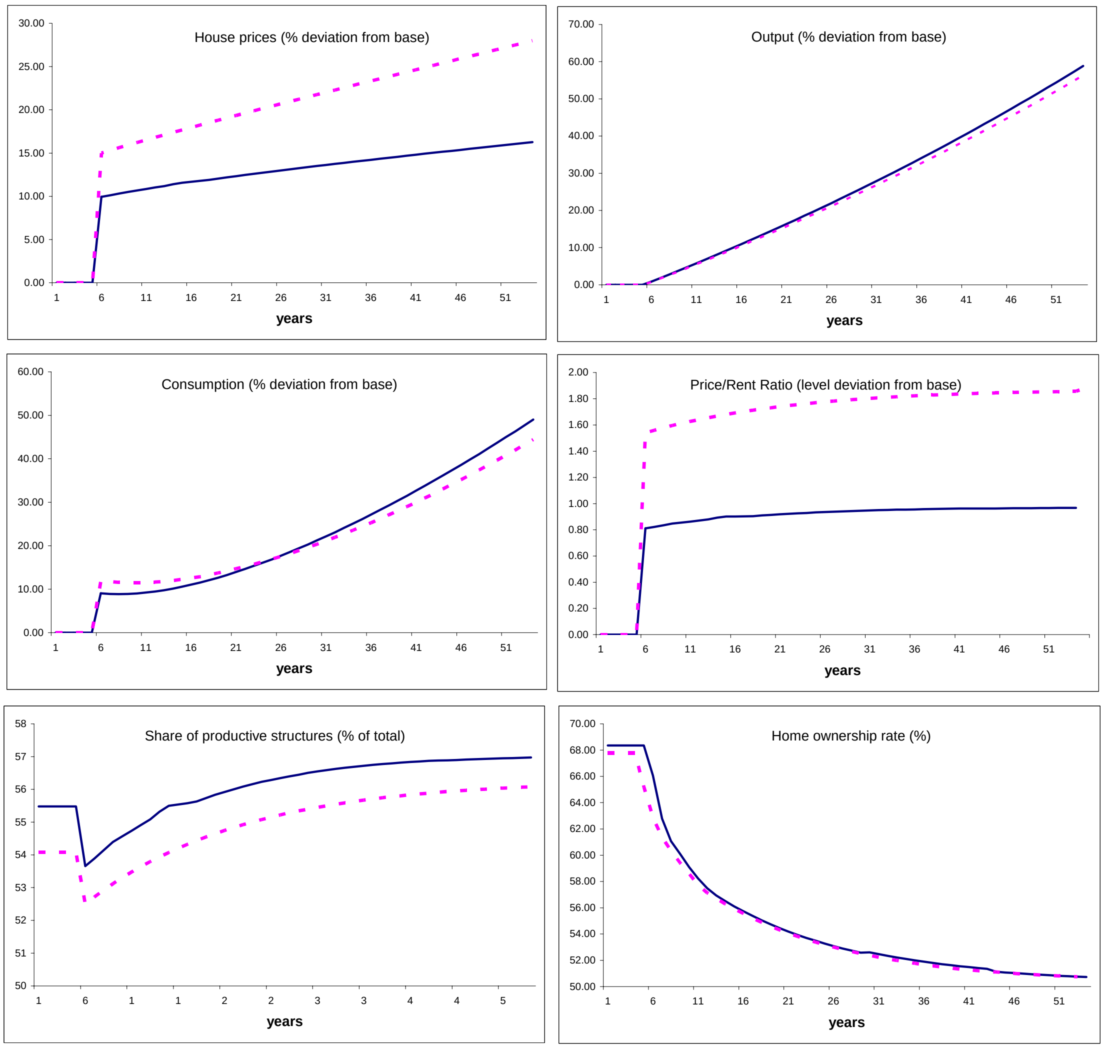


Figure 7

Transition Dynamics from a 1% decrease in the world real interest rate

(solid line: γ=0.9, dotted line: γ=0.78)

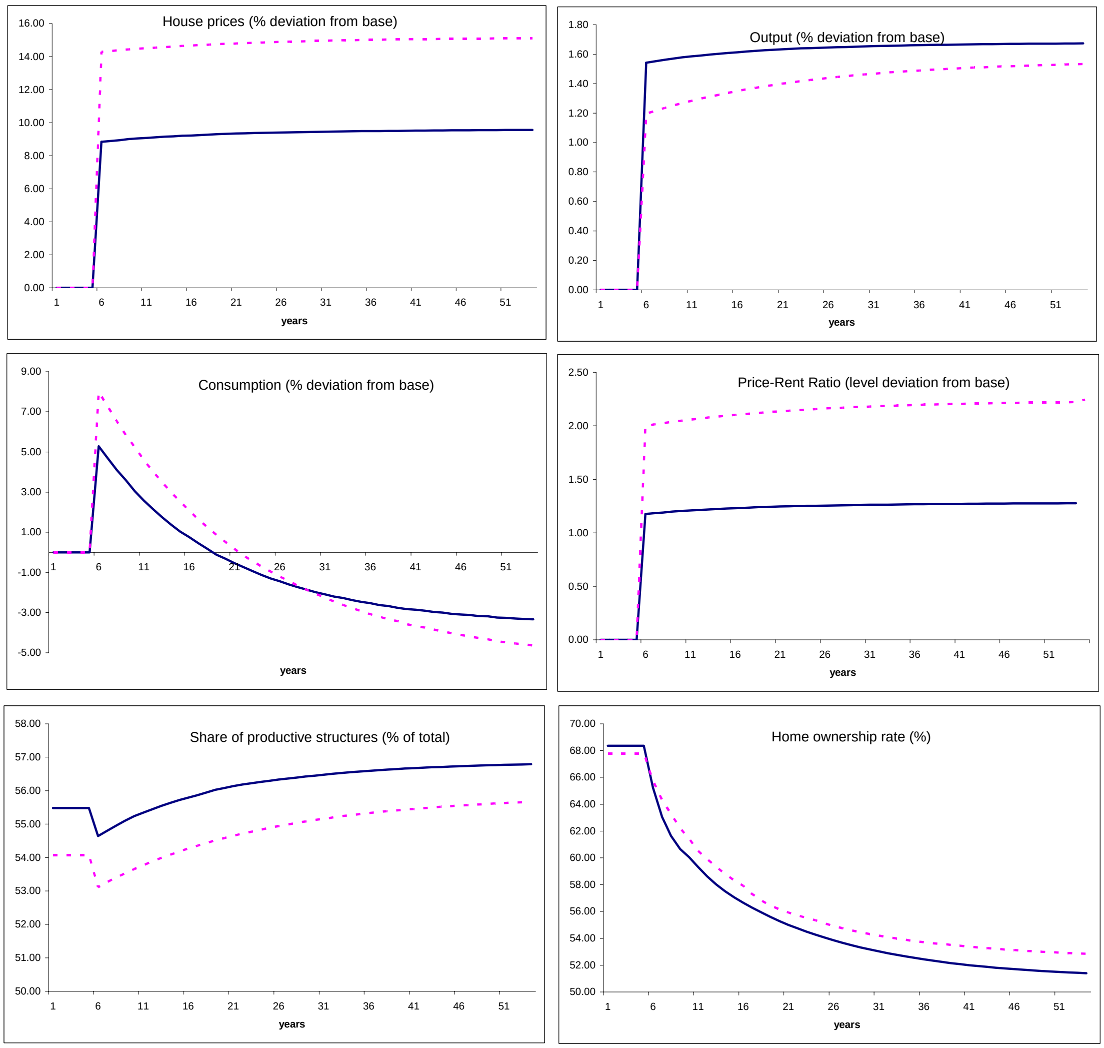


Figure 8: US - model versus data since 1991

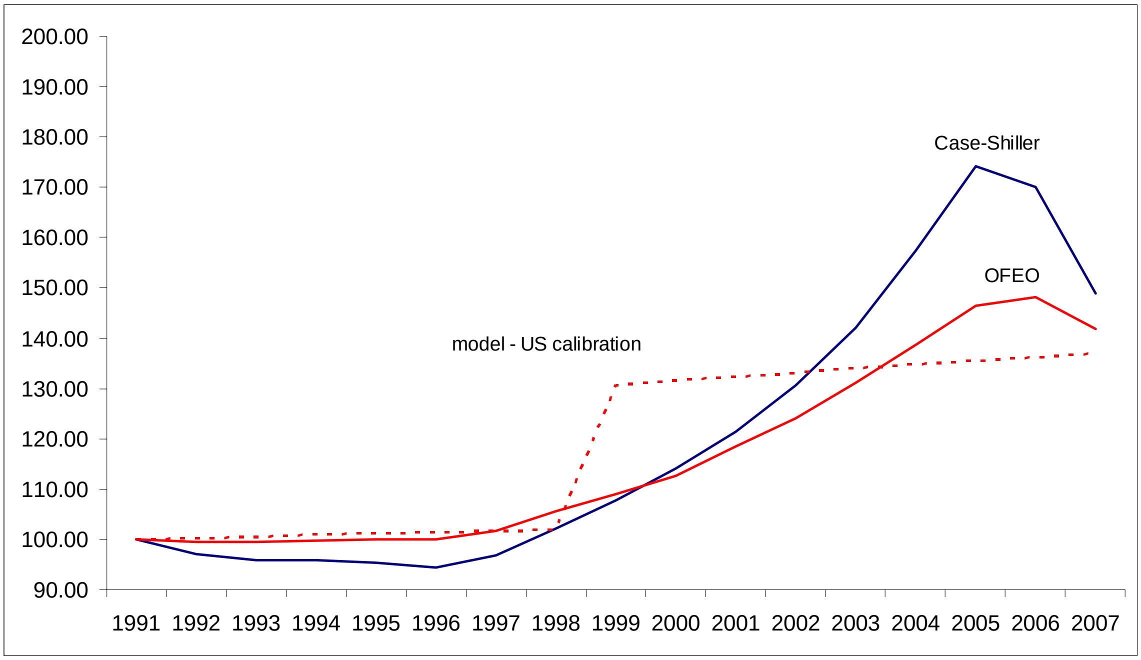


Figure 9: Aggregate home ownership rates since 1991: model versus data

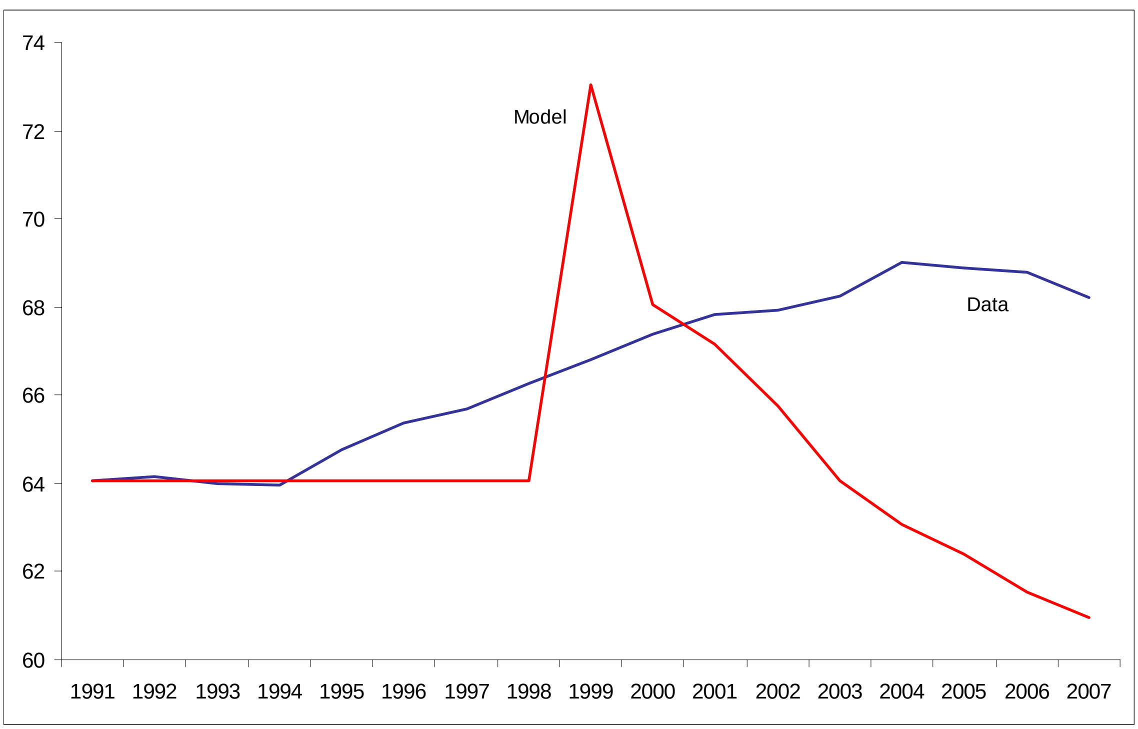


Figure 10: UK – model versus data since 1991

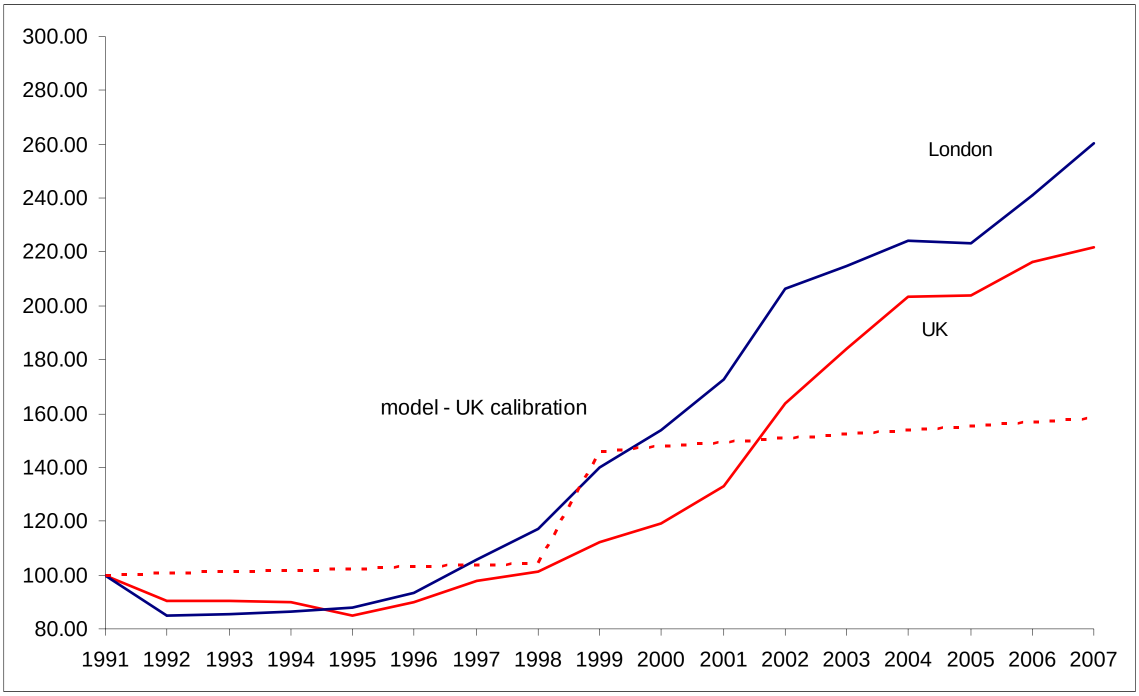


Figure 11: UK: Aggregate home ownership rates since 1991: model versus data