Wonder, the Rainbow and the Aesthetics of Rare Experiences

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Aesthetics, the pleasure of the sensory or intellectual cognisance of fit, or grasp of beauty.

It is a rare and wonderful experience to pick up a book found accidentally to find it speaks directly to one's senses. It is rarer still to find a colleague who has also found the same book, again fortuitously, and responded similarly. This piece describes a book which sounds as if it has little to do with our field, but which turns out to have much to do with mathematics, with education, and mathematics education in particular. Both of us have used it in our work.

First, we explain aspects of our work to which this book speaks, before detailing how.

Toward wonder

I (Anne) work with new teachers in a context defined by targets, statements, learning objectives, structures for accountability, the frequent ascription of level numbers to students, testing and league tables. Oddly, within this apparent gridlock of heavily-framed practice, there is also the requirement that teachers should:

> plan opportunities to contribute to pupils' spiritual development (DfEE, 1998, Section B4d, p 12)

There is bewilderment among mathematics educators at this requirement. First, no one is sure what it means; second, being sure would mean definition, which should, in the current climate, lead to measurability which seems contradictory to the notion of spiritual development; third, the requirement seems to contradict the assumptions of measurability by suggesting there is something in education which cannot be measured

In mathematics, educators have tended to respond to the demand for attention to a spiritual dimension by appealing to the sense of awe and wonder which can be generated by experience of the infinite. At a high school level of mathematics, the equation $e^{i\pi} + 1 = 0$ is also offered as an opportunity for awe. Other candidates for the role of awe-inspirer are the identity $\sin^2 x + \cos^2 x \equiv 1$, the Pythagorean proof of the irrationality of $\sqrt{2}$ and geometrical generalities such as the co-incidence of the medians of a triangle.

An alternative track to awe and wonder might be to suggest that mathematics can describe patterns appearing in nature. But how does one generate awe in students if they are unable to appreciate the beauty which others see in these structures, or who can see beauty, but do not find it enhanced by mathematical expression? Not only do UK teachers search for ways to induct their pupils into these aesthetic perceptions, but educators also search for ways to induct new teachers similarly, so that they may be able to incorporate such opportunities into their teaching

I had a growing disaffection with this pedestrian approach to awe and wonder in mathematics, as if there were common sites for expressing awe, like scenic viewpoints seen from a tourist bus, whose position can be recorded on the curriculum as one passes by, *en route* for something else. Spontaneous appreciation of beauty and elegance in mathematics was not, for me, engendered by occasional gasps at nice results, nor by passing appeals to natural or constructed phenomena such as the patterns in sunflowers or the mathematics of tiling

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I (Nathalie) was looking for a more adequate account of motivation in mathematics education and, in particular, one that acknowledged its roots in wonder and the aesthetic I saw the growing emphases on relevance (usually taken to mean connection with 'real-life'), problem solving and interdisciplinary activities (mathematics through music, art, sports, etc.) as attempts both to justify mathematics teaching and to motivate student interest and achievement, by dwelling within their existing horizons.

These attempts seem largely to ignore what might be intrinsically satisfying for students in mathematical activity: how can their tendencies for wonder and exploration, their desires to express themselves, their need to understand themselves, be fulfilled by mathematics? Or, as Dewey (1913) might insist, what is it about mathematical activity that lies in the direction of the student's growth?

Indirectly, many mathematics educators appeal to the notion of aesthetics in their discussions of the importance and value of mathematics. (Marion Walter (2001), in her article in the previous issue of this journal, provides a more direct appeal through her integrated, aesthetic-mathematical response to geometry and geometrical art.) Yet few of them explicitly address the role of aesthetics in mathematics learning. Those who do, often focus on the aesthetic judgements of beauty and elegance made about mathematical objects (theorems, proofs, equations, definitions, etc.), and are concerned with the extent to which students can appreciate such judgements.

Very few of them mention the possibility of students' aesthetic experience in mathematics learning, one that is synoptic and integrative, that has aesthetic quality and is suffused by a distinctive emotion. Such experiences are not only familiar to mathematicians, whether amateur or professional, but they also represent one of the animating purposes of mathematical knowledge – why we do and value mathematics. Are these experiences available to students, and what makes them possible? Perhaps if they were to begin in wonder . . ?

In wonder

In his book, *Wonder, the Rainbow and the Aesthetics of Rare Experiences*, Philip Fisher (1998) sets out to investigate the role of wonder in discovery, in learning, and in what he terms the "ongoing fragile project of making sense" (p 8) Through two paintings by the contemporary artist Cy Twombly (*Il Parnasso* and *Untitled*), through the historical study of rainbows, through Socrates' discussion with the untaught slaveboy about doubling the square, Fisher explores the inter-relationship of aesthetic and scientific responses Whether in mathematics, in the sciences or in the arts, he wants to convince us that surprise and novelty arouse curiosity and a desire to know His central claim is that wonder is the 'poetics of thought': that is, wonder is the core energy which takes us along the pathway from the unexpected moment of aesthetic delight through to the experience of intrinsic satisfaction and intelligibility.

In the first chapter, he ranges Shakespeare, Le Corbusier, Eliot, Descartes and Wittgenstein (among others) against 'the ordinary': Shakespeare because he wrote *The Tempest*, Le Corbusier because of the shock of his architecture, Eliot because he broke grammatical rules, Descartes because of his rainbow studies [1] and Wittgenstein because he declared that there is no feeling of the ordinary. To be ordinary, Fisher claims, is to be unnoticeable, incapable of being experienced. Only that which we see as if for the first time can evoke wonder - and that, in turn, can provoke learning

> The tie between wonder and learning is clear in the moment when after long confusion and study you suddenly say, "Now I get it!" Plato [...] uses mathematics for this because the moment of 'getting it' is extremely clear in mathematics (p 21)

Fisher is, as an outsider, treading here in the footsteps of mathematical insiders Poincaré (1908/1955) and Hadamard (1945), unaware that sometimes 'getting it' can turn out to be misleading or, worse still, circular. Yet, those moments of insight can reveal our *logics of feeling*, as Gattegno (1974) put it, those intuitive and aesthetic modes of thinking that allow us to formulate conjectures and ideas. To pursue a link between wonder and learning, we need to know more about what Fisher means by these terms.

Building on Descartes' view, Fisher argues that wonder is primarily characterised by surprise, by its unexpectedness and suddenness It is also fundamentally visual; he claims that what he terms the 'arts of time' - narration, dance, and music - cannot evoke wonder, since they cannot be *seen* all at once. Yet surprise by itself is not enough; there also has to be simultaneous certainty and a sense of realisation which joins the surprise. The realisation is the learning, and it is marked, recorded, by a feeling of pleasure Without the certainty, there is no sense of wonder Fisher claims:

> the mind says 'Aha!' in the aesthetic moment when the spirit says 'Ah!' (p. 31)

The gasp is accompanied by the relief of resolution. From a philosophical perspective, it is as if one is suddenly tempted by transcendentalism, but in the same moment brings reason to bear and rationalises the experience, though perhaps one can only articulate it later, after reflection

As Hawkins (1984) so eloquently writes, it is:

> the experience in which our grasp of things suddenly outruns our imagination. More properly: we catch ourselves in the act of going - of knowing - beyond imagination, and thus we first know ourselves as knowers. (p. 127)

Fisher's view of learning has it proceeding from sequences of small steps of wonder, from sequences of surprising moments when prior experience is momentarily upset, but which that same prior experience prepares us to understand.

This brings to mind a paper by Movshovits-Hadar (1988), in which she suggests structuring students' experiences of mathematics so that results which might otherwise seem commonplace emerge as surprising special cases. An example would be arranging for the emergence of the Pythagorean relationship as a special case of comparisons between squares of sides on a variety of kinds of triangle Fisher would call this a *staging*, when the teacher (as is the case with Socrates' demonstration to Meno) - who has already learned and knows the mathematical idea - can show the idea in a dramatic way, can present it in wonder.

But the capacity for wonder is also an attitude toward experience - Socrates stated this clearly when he wrote that all philosophy begins in wonder If certainty is a necessary ingredient of wonder, as well as surprise, then what are its precursors: what makes us capable of wondering? Green (1971) argues that one such precursor is the sense of contingency, the awareness that things need not be as they are:

> Wonder is the product, not of ignorance, but of the knowledge that facts are problematic. (p. 197)

Mathematics is a world in which the struggle between the dependable and the contingent crystallise, in which dependable facts abound as a result of strict assumptions. But what if we *could* divide by 0, what if we *were* to throw away the parallel postulate, what if the irrational numbers *were* our counting blocks, what if we really *could* have staircases like those drawn by Escher, what if we *could* redefine differentiability to cope with some kinds of discontinuity? The capacity to wonder also involves a confession of limitation or ignorance To wonder is to acknowledge one's ignorance, not in a state of despair or passivity, but in the pleasurable pursuit of further knowledge.

How would Fisher talk to those (including the poet John Keats) for whom explanation of the science of the rainbow appears to spoil their delight? He uses the emergence of curiosity after an experience of wonder as a bridge between wonder and thought; the process of wondering. Perhaps those who have had an explanation of the rainbow thrust upon them, before they had wondered about it independently, before realising their understanding was wanting, before *wanting* an explanation, would find scientific description an anti-climax ... unless they find scientific explanation wonderful in its own terms.

In a sense, this suggests a distinction between different types of wonder. When I wonder *why* the rainbow appears, or *how* it appears, my wonder will cease when I find the answer - my curiosity will be satisfied. Yet I can also (continue to) wonder *at* the rainbow How can it be that there is a rainbow? This question will not be resolved by an investigation; it simply shows that I stand astonished before the contingency of the rainbow, even though I know *how* it works.

Similarly, I can continue to wonder *at* the fact that the centres of the three equilateral triangles constructed on the sides of an arbitrary triangle always form an equilateral triangle (Napoleon's theorem), even if I have seen and understood three different proofs. There is just something there that is marvellous - it seems that it did not really have to be so and yet it is, in its simple splendour.

But if I only wonder *at*, I may miss other wonderings, such as supposing I construct triangles similar to the original one instead, what is possible then? How does wondering *at* relate to learning? Fisher does not say, but perhaps wondering *at* is closely tied to motivation, since, as Green (1971) argues, it is in principle never sated and because it always contains the seeds of what he terms a 'temperate' rather than a mere flippant curiosity.

But Fisher is not concerned here with formal instruction and offers no help in this direction. Instead, he comments (with Descartes) that wonder declines with familiarity and that one has to work to keep alert to notice extraordinary things, and to avoid becoming "addicted to even trivial differences" (p. 56) without reflective pursuit of knowledge.

The rooting of wonder in difference, in the new and unexpected, reminds us of the work of Ference Marton (e.g. Marton and Booth, 1997), who understands learning as extension of awareness of the range of possible variations of whatever is the focus of study. Marton, unlike Fisher, *is* concerned with formal instruction and claims that part of the teacher's role is to arrange for students to experience variation, in our case *differences*, in various dimensions of mathematical concepts and structures.

However, in our view, such variation does not need the introduction to new or extraordinary experiences that Fisher insists upon; it can be achieved by a simple shift in perception, a new way to look at what is close at hand - looking at a triangle on a sphere - using, for example, the 'What-If-Nots?' of Brown and Walter (1983) or the transformations suggested by Watson and Mason (1998).

In Fisher's chapter entitled 'Wonder and the steps of thought', he starts with an overview of how Descartes' analytical geometry allowed situations to be seen visually, all at once, so that complex problems could be broken down into small 'seeings', each of which might intuitively rise from moments of wonder. He then moves to Plato's *Meno* and takes the reader through this Socratic dialogue, showing the relationship between the visual representation, the boy's responses (each making good sense and contributing to the final solution, even when they appeared to lead nowhere) and Socrates' prompts. Through this analysis, he links wonder and learning as follows:

> The proof is plodding, dogged, and mechanical while being, at the same time, ingenious, imaginative and wonderful. The interconnection between these two seemingly opposite series of qualities describes the relation of science to wonder, and it makes clear the passionate energies that are inextricable from exacting thought. (p. 73)

For teacher education, therefore, Fisher's attention to wonder provides a pathway to seeing something spiritual in learning about mathematical structure, in seeing mathematical results as surprises, in following up first instances of wonder with small steps of further wonder, in exploiting the visual and the intuitive, both fuelling and being fuelled by curiosity.

It would be possible to dismiss this approach as too Platonic or Kantian for modern palates, depending, as it seems to, on the assumption that what is wonderful is outside ourselves and has that quality in and of itself. This suggests it is up to learners to discover the wonder or to notice it when conditions are favourable. On the one hand, we have the possibility that learners respond to things differently and teachers could think of finding out what they wonder about, what their habits, powers, abilities are and then work with those. On the other, we can assume that 'wonder' is a common-enough experience for us to offer situations which we and others find wonderful and learn, from our students, about the universality (or not) of wonder.

As co-authors of this piece, working together by e-mail but almost strangers otherwise, we do not even know if we respond similarly, but nevertheless are able to talk about wonder as a common experience. As educators, we have to offer undreamed-of possibilities, taking students to the opera, reading them hard poetry, showing them some 'wonderful' geometry, and working with our perceptions of their response.

Fisher assumes a universality of wonder - that the boy will find wonder in that which Socrates finds wonder. Some might argue that, as an aesthetic category that is socio-historically dependent, boys, girls and teachers will differ in their judgements of wonder. Many a teacher finds wonder in the Fundamental Theorem of Arithmetic, yet it leaves many students numb. However, the very common experience of teachers that students are amazed and intrigued by the way 'infinity' crops up in new contexts, by patterns generated when using dynamic geometry software or by 'strange' properties of the number 9 in a base-ten numeration system, suggests that there are stimuli which commonly trigger excited response in learners.

Moreover, learners may need to be inducted into the wonder of mathematics, to experience wonder vicariously through the teacher (including the stages of pleasure and frustration that sense-making requires) and, more urgently, to set aside the illusion of mathematics as systematic knowledge so complete that there is nothing more to expect.

Fisher also hints at another common basis for wonder: our own human powers of insight, pointing out, using the example of Simon Stevin's discovery about forces on an inclined plane, that we are amazed by the human powers of breakthrough, of seeing a simple and elegant way out of a complex situation in an instant in time. Similarly, students marvel at the famous story about Gauss adding the integers from 1 to 100 in such a simple and elegant way.

After the example from mathematics and the one from the science of everyday experiences (the rainbow), Fisher offers a third example from the visual arts. Where the first two examples revealed wonder in the poetics of thought, Fisher wants to show that we can also start with the aesthetic pleasure afforded by a work of art and:

find the exactness and sequential pleasures of thought and its drive toward intelligibility (p 138)

Using a pair of abstract and unfamiliar paintings, either of which might first strike a viewer as a random set of meaningless details, Fisher describes the path of thought through which the feeling of 'getting it' is achieved.

With this third example, Fisher is attempting to achieve two goals The first is to emphasise that this path of thought, though distinct in many ways from that in other domains of mathematical problem solving, shares a common 'poetic of wonder' that guides us to a feeling of intelligibility and pleasure This claim is surely worthy of further consideration by educators who are interested in the interdisciplinary possibilities of school domains such as 'mathematics and poetry' or 'mathematics and art'. For Fisher is intimating that their commonality - and hence the basis for the learning potential of combining them - is not that they both involve rhythm or notation, but that they share a *method*

Fisher also wants to underline the way in which Cartesian wonder precludes memory and recognition. These two paintings evoke wonder precisely because there are no codes, no icons, no styles that we can recognise: their newness, unexpectedness and unplaceability thereby guide the viewer to attention, curiosity and the process of creating intelligibility. Educators might feel discomfitted with the underlying claim that memory is one of the antagonists for the aesthetics of wonder (are we not always wondering why students cannot remember their multiplication tables, their trigonometry rules, their factoring algorithms?)

The explanation is that if we are constantly being reminded of something, we are (in effect) distracted from complete, undivided, intellectual attention. But, more importantly, and here we might find some educational resonance, reliance on memory - or on a constant capacity for recognition and association - betrays an intellectual attitude that we (or someone) already know(s) everything knowable. In contrast, by confronting situations where there is no recognition or memory - illustrated by these two paintings - we invite ourselves to new delights, new curiosities and new paths for intelligibility.

Note

[1] There have been vicious attacks by contemporary philosophers and critics on Descartes' agenda of rationalism Three hundred years later, Fisher offers redemption through Descartes' writing on wonder as set out in *The Passions of the Soul*, a book that has been quite overshadowed by the famous *Discourse on Method*. In his theory of passions, Descartes places wonder (as opposed to anger, desire and fear as the Ancients had done) as the primary human passion.

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