

# **WONDY V - A One-Dimensional Finite-Difference Wave Propagation Code**

**Marlin E. Kipp, R. J. Lawrence**

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WONDY V - A One-Dimensional Finite-Difference  
Wave Propagation Code

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ABSTRACT

WONDY V solves the finite difference analogs to the Lagrangian equations of motion in one spatial dimension (planar, cylindrical, or spherical). Simulations of explosive detonation, energy deposition, plate impact, and dynamic fracture are possible, using a variety of existing material models. In addition, WONDY has proven to be a powerful tool in the evaluation of new constitutive models. A preprocessor is available to allocate storage arrays commensurate with problem size, and automatic rezoning may be employed to improve resolution. This document provides a description of the equations solved, available material models, operating instructions, and sample problems.

## CONTENTS

1.	INTRODUCTION . . . . .	9
2.	BASIC FORMULATION . . . . .	13
2.1	Conservation of Momentum, Mass, and Energy . . . . .	13
2.2	Pressure and Sound Speed . . . . .	14
2.3	Stress and Strain . . . . .	16
2.4	Difference Equations . . . . .	20
2.5	Artificial Viscosity . . . . .	25
2.6	Stability . . . . .	28
2.7	Order of Computations . . . . .	30
3.	EQUATIONS OF STATE AND CONSTITUTIVE RELATIONS . . . . .	33
3.1	STAT1 - General Solids - 1 . . . . .	33
3.1.1	Hydrodynamic Fluids . . . . .	33
3.1.2	Elastic-Plastic Solids . . . . .	40
3.2	STAT2 - High Explosives . . . . .	48
3.2.1	Ideal Gas Model . . . . .	50
3.2.2	JWL Model . . . . .	52
3.2.3	BKW Model . . . . .	52
3.3	STAT3 - Ideal Gas Law . . . . .	53
3.4	STAT4 - General Solids - 2 . . . . .	54
3.4.1	Strain Hardening Solids . . . . .	55
3.4.2	Strain Rate Dependent Solids . . . . .	60
3.5	STAT6 - Distended Solids . . . . .	65
3.6	Fracture Options . . . . .	72

4.	INITIAL AND BOUNDARY CONDITIONS . . . . .	77
4.1	Initial Variable Definition . . . . .	77
4.2	Initial zoning . . . . .	80
4.3	Boundary Conditions . . . . .	84
4.4	MORSTOR (Energy Deposition) . . . . .	87
4.5	Activity Test . . . . .	89
5.	INPUT, OUTPUT AND ERROR CHECKS . . . . .	91
5.1	Input (Restarts) . . . . .	91
5.2	Output . . . . .	93
5.3	Error Checks . . . . .	96
6.	REZONING . . . . .	101
6.1	Rezone Criteria . . . . .	101
6.2	Dividing Zones . . . . .	104
6.3	Combining Zones . . . . .	106
7.	STORAGE ARRANGEMENT . . . . .	111
7.1	The Main STORE Array . . . . .	111
7.2	The Preprocessor . . . . .	117
7.3	Additional Variables . . . . .	118
	REFERENCES . . . . .	121
	APPENDIX A - Code Structure and Glossary of Variables Appearing in COMMON	123
	APPENDIX B - Research Material Studies . . . . .	139
	APPENDIX C - Non-Dimensionalization . . . . .	147
	APPENDIX D - Sample Problems . . . . .	153
	APPENDIX E - WONDY V Input Instructions . . . . .	183
	APPENDIX F - Plotting Input Instructions . . . . .	215

## PREFACE

Readers of this report and users of WONDY V should be cautioned that, although the code has been used considerably and appears to be relatively error free, some "bugs" may well remain. Needless to say, the authors cannot take responsibility for versions of the code that do not correspond exactly to the description provided in this report.

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## 1. INTRODUCTION

WONDY V is a revised computer code in the series of one-dimensional finite difference wave propagation codes originally called WAVE<sup>1-4</sup>, and more recently named WONDY<sup>5-8</sup>. The present code includes all of the capabilities of WONDY IV<sup>8</sup>, as well as refinements in material modeling. A preprocessor is now an option, providing the user with the means to match the code's common storage to the problem size, which will usually result in reduced computer storage (and cost) requirements.

In general, this code integrates the one-dimensional Lagrangian differential equations of motion by use of finite difference analogs. The geometry employed can be rectilinear, cylindrical or spherical. The first of these three is the familiar case of uniaxial strain, appropriate for a plate of infinite lateral extent, and commonly used to simulate gas gun plate impact experiments. The latter two geometry options allow for the analysis of divergent wave and material motion. The method of artificial viscosity is employed to solve problems involving the propagation of pressure discontinuities, or shock waves.

Equations of state and constitutive relations for a number of classes of materials are provided as options in the code. Solids are assumed to obey the Mie-Gruneisen equation of state and can be treated as either hydrodynamic or elastic-plastic. For the latter, plastic flow occurs on the basis of the von Mises yield criterion where the yield strength can be infinite, constant, or variable. The variable yield strength can be a function of strain, energy, or plastic work. The variation of yield strength with plastic work is derived from an isotropic strain hardening assumption. An

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\*Superscripts refer to references listed at the end of the report.



alternate constitutive relation treats rate dependent stress relaxation and anisotropic strain hardening. These models also allow the material to vaporize if the problem so requires. Equations of state for ideal gasses, detonating high explosives and initially porous or distended solids are also provided. To supplement these models the code contains an explicit treatment for dynamic fracture which allows both separation and rejoining to occur.

Additional equations of state and material response models are continually being developed for WONDY, and the code's flexibility allows for their inclusion with a minimum of effort on the part of the user. In particular, the number of variables carried in each zone may be altered by an input parameter, so that models formulated with internal state variables are easily added to the code. Areas in which WONDY has been employed as a research tool include dynamic fracture, piezoelectricity, ferroelectricity, viscoelasticity, explosive reactions, granular and porous materials and bubbly liquids. Short descriptions of these models are found in Appendix B.

Initial and boundary conditions may be specified to meet user requirements. For example, time dependent pressure loads may be applied, and energy deposition, explosive detonations, or plate impact conditions may be prescribed. The code allows for the introduction of gaps or voids between individual plates, and for offsets from the origin to establish the appropriate curvature for non-planar geometries. The user may request a dynamic rezone technique that continually adjusts the Lagrangian mesh to provide both improved resolution in shock fronts and reduced computational time.

The input and output routines in WONDY V are quite flexible. Numbered input records are employed so that their order is immaterial. In addition,

many variables are assigned standard default values if they are not included in the data. The principal output routine is the standard edit which lists all the main variables at preselected times. The variables included in the edit may be selected by the user as part of the input instructions. Three additional output routines are provided: the first generates a file suitable for subsequent plotting; the second creates a restart file from which the calculation may be continued if desired; and the third can easily be adapted to specific user needs.

It should be noted here that WONDY has been programmed without recourse to any specific physical units (with the exception of the BKW explosive model, Section 3.2.3). Hence, any self-consistent set of units may be employed, or alternately one may non-dimensionalize a given calculation with respect to a length, time, or other parameter characteristic of the problem (Appendix C). Some sets of units which have been utilized with WONDY are given in the accompanying table.

This manual is separated into sections that allow the user easy reference both for general purposes and to clarify input instructions. The basic formulation of the code is discussed in Section 2, with the conservation laws and general material model form set forth. The method of differencing these equations, and the computational scheme, including artificial viscosity and stability arguments, complete the section. The routinely available material models are outlined in Section 3, including the options for treating fracture. The remaining sections are devoted to input and boundary conditions (4), input and output specifications, (including diagnostics) (5), rezoning techniques (6), and storage arrangements of the variables (7). Six appendices are included in which are discussed code structure and COMMON variables (A), auxilliary material models (B), non-dimensionalization (C),

sample problems (D), WONDY input instructions (E), and plotting input instructions (F).

Self Consistent Units for WONDY\*\*

Quantity	c.g.s.	c.g. $\mu$ sec	S.I.	SWAP	i.p.s.
Time	sec	$10^{-6}$ sec	sec	$10^{-6}$ sec	sec
Length	cm	cm	meter	mm	inch
Mass	gm	gm	kg	$10^{-4}$ gm	lb sec <sup>2</sup> /in
Density	gm/cm <sup>3</sup>	gm/cm <sup>3</sup>	kg/m <sup>3</sup>	$10^{-1}$ gm/cm <sup>3</sup>	lb sec <sup>2</sup> /in <sup>4</sup>
Energy	erg	$10^{12}$ erg	Joule	$10^6$ erg	in lb
Energy/Unit mass	erg/gm	$10^{12}$ erg/gm	Joule/kg	$10^{10}$ erg/gm	in <sup>2</sup> /sec <sup>2</sup>
Pressure	dyne/cm <sup>2</sup> <sup>†</sup>	$10^{12}$ dyne/cm <sup>2</sup> <sup>†</sup>	Newton/m <sup>2</sup> <sup>††</sup>	$10^9$ dyne/cm <sup>2</sup> <sup>†</sup>	lb/in <sup>2</sup>

\*\*Some auxiliary programs and routines for WONDY include dimensional constants. The appropriate units must be used when these routines are employed. Note that since the acceleration due to gravity is not employed in the code, the units of mass and density must conform to a self consistent gravitational system.

† 1 kilobar =  $10^9$  dynes/cm<sup>2</sup>; 1 megabar =  $10^3$  kilobar =  $10^{12}$  dynes/cm<sup>2</sup>

†† 1N/m<sup>2</sup> = 1 Pa; 1 bar =  $10^5$ Pa; 100 MPa = 1 kilobar

## 2. BASIC FORMULATION

The conservation laws of momentum, mass, and energy, complemented by a material law, lead to a formulation that can be differenced and numerically solved. The description of material behavior in the time durations characteristic of wave motion are of interest here.

### 2.1 Conservation of Momentum, Mass, and Energy

The one-dimensional differential equation expressing conservation of momentum is

$$\rho a = - \frac{\partial \sigma}{\partial x} - \frac{\partial q}{\partial x} + (\alpha - 1) \frac{\dot{\phi}}{x} , \quad (2.1)$$

where  $x$  is the Lagrangian spatial coordinate,  $\rho$  the density,  $a$  the acceleration,  $\sigma$  the Cauchy stress in the  $x$  direction, and  $q$  the viscous stress, both taken positive in compression. The quantity  $\phi$  is the difference between the stresses in the longitudinal and transverse directions ( $\alpha = 1$  for rectangular, 2 for cylindrical, 3 for spherical one-dimensional coordinates).

The material particles will be followed in their motion, and thus the acceleration is given simply by

$$a = \frac{\partial u}{\partial t} , \quad (2.2)$$

where  $u$  is the velocity defined by

$$u = \frac{\partial x}{\partial t} . \quad (2.3)$$

Mass conservation is expressed by

$$\frac{\rho}{\rho_0} = \frac{dV}{dv} , \quad (2.4)$$

where  $dV$  is an element of volume at time  $t = 0$ , when the density is  $\rho_0$ , and  $dv$  is the current volume of the same element at time  $t$ .

The one-dimensional equation for conservation of energy equates the rate of increase of internal energy per unit mass to the rate at which work is being done by the stresses and the rate at which heat is being added,

$$\rho \frac{\partial \mathcal{E}}{\partial t} = (p + q) \frac{1}{\rho} \frac{\partial \rho}{\partial t} + P_d + \frac{\partial h}{\partial x} + (\alpha - 1) \frac{h}{x} + \rho Q \quad (2.5)$$

where  $\mathcal{E}$  is the internal energy per unit mass,  $Q$  is the heat added (say by chemical reaction or radiation) per unit mass and unit time,  $h$  is the heat flux due to heat conduction, and  $P_d$  is the rate of deviator stress work. Note that the work done by the viscous stress  $q$  has been included.

Heat addition  $Q$  may be assigned as required, while  $h$  will depend on the temperature gradient. Since the energy equation is included in the constitutive subroutine,  $Q$  and  $h$  do not appear anywhere else in the code.

## 2.2 Pressure and Sound Speed

An equation of state, or constitutive law, is required to complete the formulation, and is usually taken in the form

$$p = f(\rho, \mathcal{E}) \quad (2.6)$$

where  $p$  is the pressure,  $\rho$  the density, and  $\mathcal{E}$  the energy per unit mass. The nature of the subsequent differencing is such that a convenient form for (2.6) is

$$p = f_1(\rho) + f_2(\rho) \mathcal{E} \quad (2.7)$$

Although the sound speed is not used explicitly in the calculation of any of the state variables it is necessary to determine the artificial viscosity and the time step. The sound speed can be defined as

$$c \equiv \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \quad , \quad (2.8)$$

where  $( )_s$  indicates that the differentiation is carried out at constant entropy. Differentiating (2.7) at constant entropy yields

$$\left(\frac{\partial p}{\partial \rho}\right)_s = \frac{df_1}{d\rho} + \mathcal{E} \frac{df_2}{d\rho} + f_2 \left(\frac{\partial \mathcal{E}}{\partial \rho}\right)_s \quad . \quad (2.9)$$

Noting the thermodynamic relation

$$\left(\frac{\partial \mathcal{E}}{\partial \rho}\right)_s = \left(\frac{\partial \mathcal{E}}{\partial \mathcal{V}}\right)_s \left(\frac{\partial \mathcal{V}}{\partial \rho}\right)_s = (-p) \left(-\frac{1}{\rho^2}\right) = \frac{p}{\rho^2} \quad , \quad (2.10)$$

where

$$\mathcal{V} = \frac{1}{\rho} \quad (2.11)$$

is the specific volume, the sound speed can be easily obtained from

$$c^2 = \frac{df_1}{d\rho} + \mathcal{E} \frac{df_2}{d\rho} + \frac{pf_2}{\rho^2} \quad . \quad (2.12)$$

Since the equation of state (2.7) can also be expressed in terms of the specific volume via (2.11), (2.12) can be rewritten as a function of specific volume

$$\begin{aligned} c^2 &= -\frac{1}{\rho^2} \left(\frac{\partial p}{\partial \mathcal{V}}\right)_s \\ &= -\mathcal{V}^2 \left(\frac{df_1}{d\mathcal{V}} + \mathcal{E} \frac{df_2}{d\mathcal{V}} - pf_2\right) \quad , \end{aligned} \quad (2.13)$$

where  $\left(\frac{\partial \mathcal{E}}{\partial V}\right)_s = -p$  has been used.

The results given in this section are common to all of the equation of state subroutines in Section 3.

### 2.3 Stress and Strain

In one dimensional motion, there is a coincidence of coordinate axes with principle stress and strain orientations. In this absence of shear strains, it is convenient to work in terms of the strain rate or stretching. In the direction of motion, i.e., the x direction, the stretching is defined as

$$d_x \equiv \frac{\partial u}{\partial x} .$$

In rectangular coordinates there is no motion in the y and z coordinate directions so that

$$d_x = \frac{\partial u}{\partial x} , \quad d_y = 0 , \quad d_z = 0 , \quad \text{for } \alpha = 1 . \quad (2.14)$$

In cylindrical coordinates there is no motion in the z direction, so that  $d_z = 0$ . However, motion in the x direction will induce a circumferential strain, so that (x is the radial direction)

$$d_x = \frac{\partial u}{\partial x} , \quad d_y = \frac{u}{x} , \quad d_z = 0 , \quad \text{for } \alpha = 2 . \quad (2.15)$$

In spherical coordinates there is a hoop strain induced in mutually perpendicular circumferential directions when there is motion in the x direction, so that (x is again the radial direction)

$$d_x = \frac{\partial u}{\partial x} , \quad d_y = \frac{u}{x} , \quad d_z = \frac{u}{x} , \quad \text{for } \alpha = 3 . \quad (2.16)$$

The volumetric strain rate or dilatation is defined as

$$d = d_x + d_y + d_z \quad . \quad (2.17)$$

Thus it must be related to the rate at which the density is changing by

$$d = - \frac{1}{\rho} \frac{\partial \rho}{\partial t} \quad . \quad (2.18)$$

Stretching deviators are defined as

$$d_x^d = d_x - \frac{1}{3} d = d_x + \frac{1}{3\rho} \frac{\partial \rho}{\partial t} \quad , \quad (2.19)$$

and similarly for  $d_y^d$ ,  $d_z^d$ . They are a measure of the rate of distortion independent of the volume change. From (2.17) and (2.19) it is evident that

$$d_x^d + d_y^d + d_z^d = 0 \quad . \quad (2.20)$$

Since the shear strains are zero in one dimensional motion, shear stresses are zero. The stress components in the coordinate directions are  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$ . The pressure is defined as

$$(-p) = \frac{1}{3} (\sigma_x + \sigma_y + \sigma_z) \quad , \quad (2.21)$$

the minus sign appearing in agreement with the convention that stresses are considered positive in tension, while pressure is considered positive in compression. Stress deviators are defined as

$$\sigma_x^d = \sigma_x - (-p) = \sigma_x + p \quad , \quad (2.22)$$



and similarly for  $\sigma_y^d$ ,  $\sigma_z^d$ . From (2.21) and (2.22) it is evident that

$$\sigma_x^d + \sigma_y^d + \sigma_z^d = 0 \quad . \quad (2.23)$$

The rate at which mechanical work is being done by the stresses, i.e., the stress power, is given by

$$P = \sigma_x d_x + \sigma_y d_y + \sigma_z d_z \quad . \quad (2.24)$$

Using (2.19), (2.21), (2.22), and (2.20) the stress power may be expressed as

$$P = P_s + P_d \quad , \quad (2.25)$$

where  $P_s$  is given by

$$P_s = \frac{p}{\rho} \frac{\partial \rho}{\partial t} \quad , \quad (2.26)$$

representing the rate at which work is being done by the pressure against a volume change, and  $P_d$  is given by

$$P_d = \sigma_x^d d_x^d + \sigma_y^d d_y^d + \sigma_z^d d_z^d \quad , \quad (2.27)$$

representing the rate at which work is being done by the deviator stresses against distortion. Using (2.20) and (2.23) the components in the y direction can be eliminated, yielding

$$P_d = 2\sigma_x^d d_x^d + \sigma_x^d d_z^d + \sigma_z^d d_x^d + 2\sigma_z^d d_z^d \quad . \quad (2.28)$$

The momentum equation (2.1) requires the quantity

$$\phi \equiv \sigma_x - \sigma_y \quad . \quad (2.29)$$

Note that, using the definition (2.22),  $\phi$  can be written as

$$\phi = \sigma_x^d - \sigma_y^d \quad , \quad (2.30)$$

and using (2.23), this can be put into the more convenient form,

$$\phi = 2\sigma_x^d + \sigma_z^d \quad . \quad (2.31)$$

Also the quantity  $\sigma$  in the momentum equation, from (2.22), is

$$\sigma = -\sigma_x = p - \sigma_x^d \quad , \quad (2.32)$$

where  $\sigma$  is taken positive in compression for convenience. Considerable simplification arises when  $\alpha = 1$  or 3. The symmetry inherent in rectangular and spherical one dimensional motion implies that  $\sigma_y = \sigma_z$ . Thus (2.20) and (2.23) can be written

$$d_y^d = d_z^d = -\frac{1}{2} d_x^d \quad , \quad (2.33)$$

$$\sigma_y^d = \sigma_z^d = -\frac{1}{2} \sigma_x^d \quad . \quad (2.34)$$

Therefore, for  $\alpha = 1$  or 3 (2.28) and (2.31) become

$$P_d = \frac{3}{2} \sigma_x^d d_x^d \quad , \quad (2.35)$$

and

$$\phi = \frac{3}{2} \sigma_x^d \quad . \quad (2.36)$$

## 2.4 Difference Equations

In the finite difference method all quantities are sampled at discrete material particles and at discrete times. The particles are labeled in order with an index  $j$ , and times are labeled in order with an index  $n$ . Thus the value of an arbitrary quantity  $\Psi$  at the  $j^{\text{th}}$  particle and  $n^{\text{th}}$  time is denoted  $\Psi_j^n$ . The differential equations are set into finite difference form by consistent use of simple, centered, second-order analogs,

$$\begin{aligned} \left(\frac{\partial \Psi}{\partial x}\right)_{j+1/2}^n &= \frac{\Psi_{j+1}^n - \Psi_j^n}{x_{j+1}^n - x_j^n} \quad , \\ \left(\frac{\partial \Psi}{\partial t}\right)_j^{n+1/2} &= \frac{\Psi_j^{n+1} - \Psi_j^n}{t^{n+1} - t^n} \quad , \end{aligned} \tag{2.37}$$

and linear interpolation expressions,

$$\begin{aligned} \Psi_{j+1/2}^n &= \frac{1}{2} (\Psi_{j+1}^n + \Psi_j^n) \quad , \\ \Psi_j^{n+1/2} &= \frac{1}{2} (\Psi_j^{n+1} + \Psi_j^n) \quad , \end{aligned} \tag{2.38}$$

The difference equations used, corresponding to (2.1) through (2.4) are

$$\begin{aligned} a_j^n &= 2 \left\{ \frac{(\sigma_{j-1/2}^n + \rho_{j-1/2}^n) - (\sigma_{j+1/2}^n + \rho_{j+1/2}^n)}{\rho_{j+1/2}^n (x_{j+1}^n - x_j^n) + \rho_{j-1/2}^n (x_j^n - x_{j-1}^n)} \right\} + \\ &+ 2(\alpha - 1) \left\{ \frac{(\phi_{j+1/2}^n + \phi_{j-1/2}^n)}{\rho_{j+1/2}^n (x_{j+1}^n + x_j^n) + \rho_{j-1/2}^n (x_j^n + x_{j-1}^n)} \right\} \end{aligned} \tag{2.39}$$

for the acceleration, and

$$u_j^{n+1/2} = u_j^{n-1/2} + \frac{1}{2} (\Delta t^{n+1/2} + \Delta t^{n-1/2}) a_j^n, \quad (2.40)$$

for the velocity, where  $\Delta t^{n+1/2} = t^{n+1} - t^n$ . The position is thus

$$x_j^{n+1} = x_j^n + \Delta t^{n+1/2} u_j^{n+1/2}, \quad (2.41)$$

and conservation of mass yields

$$\rho_{j-1/2}^{n+1} = \frac{m_{j-1/2}}{(x_j^{n+1})^\alpha - (x_{j-1}^{n+1})^\alpha} \quad (2.42)$$

where  $m$  is a zone constant initialized at  $t = 0$  to

$$m_{j-1/2} = \rho_{j-1/2}^0 \left\{ (x_j^0)^\alpha - (x_{j-1}^0)^\alpha \right\}. \quad (2.43)$$

Note that this is the actual mass only for rectangular ( $\alpha = 1$ ) geometry\*.

These equations are subject to excessive roundoff when  $\alpha = 2$  and 3.

The zone constant  $m$  can be written in the equivalent form,

$$m_{j-1/2} = \rho_{j-1/2}^0 (x_j^0 - x_{j-1}^0) \xi_{j-1/2}^0,$$

---

\*The actual mass  $M_{j-1/2}$  is given by

$$(\alpha = 1) \quad M_{j-1/2} = m_{j-1/2} \quad , \quad \text{mass/unit cross sectional area, in one-zone slab,}$$

$$(\alpha = 2) \quad M_{j-1/2} = \pi m_{j-1/2} \quad , \quad \text{mass/unit axial length, in one-zone cylindrical shell,}$$

$$\text{and } (\alpha = 3) \quad M_{j-1/2} = \frac{4\pi}{3} m_{j-1/2} \quad , \quad \text{mass in one-zone spherical shell.}$$

This distinction may be important for user-programmed output routines that use mass-related quantities such as momentum.

where

$$\xi_{j-1/2}^0 = \begin{cases} 1 & \text{for } \alpha = 1 \\ x_j^0 + x_{j-1}^0 & \text{for } \alpha = 2 \\ (x_j^0)^2 + x_j^0 x_{j-1}^0 + (x_{j-1}^0)^2 & \text{for } \alpha = 3 \end{cases}$$

The mass equation may be written in the alternate forms

$$\begin{aligned} m_{j-1/2} &= \rho_{j-1/2}^{n+1} \left\{ (x_j^{n+1})^\alpha - (x_{j-1}^{n+1})^\alpha \right\} \\ &= \rho_{j-1/2}^n \left\{ (x_j^n)^\alpha - (x_{j-1}^n)^\alpha \right\} \end{aligned}$$

Rearranging and subtracting leads to

$$\frac{1}{\rho_{j-1/2}^{n+1}} = \frac{1}{\rho_{j-1/2}^n} + \frac{1}{m_{j-1/2}} \left\{ (x_j^{n+1})^\alpha - (x_{j-1}^{n+1})^\alpha - (x_j^n)^\alpha + (x_{j-1}^n)^\alpha \right\}$$

and therefore the mass equation (2.42) appears in the code as

$$\rho_{j-1/2}^{n+1} = \left\{ \frac{1}{\rho_{j-1/2}^n} + \frac{\Delta t^{n+1/2}}{m_{j-1/2}} \left[ \xi_j^{n+1/2} u_j^{n+1/2} - \xi_{j-1}^{n+1/2} u_{j-1}^{n+1/2} \right] \right\}^{-1}$$

where

$$\xi_j^{n+1/2} = \begin{cases} 1 & \text{for } \alpha = 1 \\ x_j^{n+1} + x_j^n & \text{for } \alpha = 2 \\ (x_j^{n+1})^2 + x_j^{n+1} x_j^n + (x_j^n)^2 & \text{for } \alpha = 3 \end{cases}$$

Two quantities useful in later calculations are

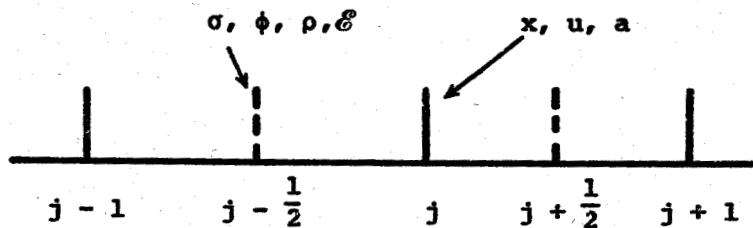
$$\left(\frac{\Delta\rho}{2\rho^2}\right) \equiv \frac{2(\rho_{j-1/2}^{n+1} - \rho_{j-1/2}^n)}{(\rho_{j-1/2}^{n+1} + \rho_{j-1/2}^n)^2} \quad (2.44)$$

and

$$\left(\frac{1}{\rho} \frac{\partial\rho}{\partial t}\right) \equiv \frac{2(\rho_{j-1/2}^{n+1} - \rho_{j-1/2}^n)}{\Delta t^{n+1/2} (\rho_{j-1/2}^{n+1} + \rho_{j-1/2}^n)} \quad (2.45)$$

In the code these quantities are called DELRHO and RHODOT respectively.

Note that  $a$ ,  $u$ , and  $x$  are centered at  $j$ , while all other quantities are centered at  $j - \frac{1}{2}$  in space. This suggests the following interpretation:



If lines are drawn on the material at the initial instant to define a material coordinate mesh, which distorts with the material as the motion proceeds, the positions, velocities, and accelerations of these lines defining the zone boundaries are found at discrete times. The same material particles are always contained in a given zone. Stresses, densities, etc., are found which may be regarded as averages over each zone between successive zone boundaries.

All quantities are centered at  $n$  in time, except the velocity  $u$ , which is centered at  $n + \frac{1}{2}$ . This presents no difficulty except at the initial instant. Velocities are usually either zero or constant prior to  $t = 0$ , so that  $u_j^{-1/2} = u_j^0$ , and starting the computation is not a problem.

In order to facilitate storage, zone quantities, e.g.,  $\sigma_{j-1/2}$ ,  $q_{j-1/2}$ ,  $\rho_{j-1/2}$ , etc., are indexed  $j$ . The velocity  $u^{n+1/2}$  is stored at  $n + 1$ .

The energy equation (2.5), using the difference analogs (2.37) becomes, in the absence of heat conduction,

$$\begin{aligned} \epsilon_{j-1/2}^{n+1} = & \epsilon_{j-1/2}^n + \left( p_{j-1/2}^{n+1} + p_{j-1/2}^n + 2q_{j-1/2}^{n+1/2} \right) \left( \frac{\Delta\rho}{2\rho^2} \right) \\ & + \Delta\epsilon_{j-1/2}^d + \Delta Q_{j-1/2} \end{aligned} \quad (2.46)$$

where  $\Delta Q_{j-1/2}$  is the heat addition during the time increment  $\Delta t^{n+1/2}$ .  $\Delta Q_{j-1/2}$  is ordinarily initialized to zero when there are no energy sources. To treat external energy sources a routine should be included which initializes  $\Delta Q_{j-1/2}$  appropriately (see Section 4.1). The term  $\frac{\Delta\rho}{2\rho^2}$  has been defined in (2.44), and

$$\Delta\epsilon^d = \frac{2\Delta t^{n+1/2} p_d}{\rho_{j-1/2}^{n+1} + \rho_{j-1/2}^n} \quad (2.47)$$

The latter term is non-zero only when material strength is included as in Section 3.1.

The constitutive law (2.7) is now recalled, and centered at  $n + 1$  in time and  $j - \frac{1}{2}$  in space:

$$p_{j-1/2}^{n+1} = f_{1j-1/2}^{n+1} = f_{2j-1/2}^{n+1} \epsilon_{j-1/2}^{n+1} \quad (2.48)$$

It may be seen that equations (2.46) and (2.48) are two simultaneous equations for the two unknowns  $p_{j-1/2}^{n+1}$  and  $\epsilon_{j-1/2}^{n+1}$ . Equations (2.46) and (2.48) may be solved explicitly yielding

$$e_{j-1/2}^{n+1} = \frac{e_{j-1/2}^n + \left( f_{1j-1/2}^{n+1} + p_{j-1/2}^n + 2q_{j-1/2}^{n+1/2} \right) \left( \frac{\Delta\rho}{2\rho^2} \right) + \Delta e^d + \Delta Q}{1 - f_{2j-1/2}^{n+1} \left( \frac{\Delta\rho}{2\rho^2} \right)} \quad (2.49)$$

where  $q$  is the viscosity,  $\left( \frac{\Delta\rho}{2\rho^2} \right)$  is given by (2.44),  $\Delta e^d$  is given by (2.47) and  $\Delta Q$  is the energy added during the current time increment  $\Delta t^{n+1/2}$ . Once  $e_{j-1/2}^{n+1}$  is obtained the hydrodynamic pressure can be calculated using equation (2.48). The total stress can then be obtained by adding to this result any deviatoric stress component which may be present. Note that by restricting the basic equation of state to the form (2.7), the use of iteration to solve for the pressure and energy has been avoided. In WONDY, equations (2.48) and (2.49) are evaluated in the "equation of state" routines discussed in Section 3.

### 2.5 Artificial Viscosity

For many materials, the stiffness, and therefore the wave velocity, increases with compression, causing a propagating wave front to steepen into a shock wave, or discontinuity. The presence of such a front violates the continuity assumptions inherent in the differential equations. Within the finite difference analogs, the techniques to effectively deal with these discontinuities occur at two levels: (1) track the shock front explicitly, or (2) smooth the shock over several zones via artificial viscosity. The former method is used in characteristic codes such as CONCHAS<sup>9</sup>, in shock-fitting codes such as SKTWO<sup>10</sup>, and in the SWAP<sup>11</sup> code.

The alternative is to include viscosity, which spreads the shock fronts and prevents formation of discontinuities. Thus shock waves are recognized as very steep but finite gradients in the solution. It is clear that a shock



wave must occupy several zone widths in order to satisfy the requirement that differences in quantities across a zone remain small.

Natural viscosity can be used. However, for most materials natural viscosity is so small that shocks would be extremely narrow and very difficult to resolve numerically. To insure that a shock occupies several zones, it would then be necessary to use such small zones, that for the usual physical problems, an extremely large number of zones would be required.

For this reason an artificially large viscosity is introduced. Care is necessary so that the viscous term does not affect the solution anywhere except near shocks. At shocks the solution is intentionally distorted to insure that gradients are much lower than in nature, so that a reasonable number of zones can be used in a given problem. In effect, use of artificial viscosity broadens or smears shock waves. Thus, for normal materials, the tendency for pressure waves to steepen is balanced by the spreading caused by the artificial viscosity.

The exact choice of form for the artificial viscosity is somewhat arbitrary. WONDY uses a quadratic viscosity<sup>12</sup> in the form

$$q = \rho b_1^2 \left( \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right)^2, \quad (2.50)$$

where  $b_1$  is a constant with dimensions of length. Since  $\left( \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right)$  represents the volumetric strain rate,  $q$  is essentially a bulk viscosity.

The quadratic form is chosen so that the viscosity is very small except when rates become large, at which time the viscosity becomes very large. The quadratic form is therefore most effective in controlling gradients at shocks while introducing minimal disturbances elsewhere.

A linear viscosity<sup>13</sup> is also used in the form

$$q = b_2 c \left( \frac{\partial \rho}{\partial t} \right) , \quad (2.51)$$

where  $c$  is the sound speed and  $b_2$  is a constant with dimensions of length. The linear viscosity is effective in controlling small spurious oscillations in which gradients are insufficient to make the quadratic viscosity effective. Great care is necessary in the use of linear viscosity, as there is a much greater chance of distorting the solution in areas away from shocks.

The constants  $b_1$  and  $b_2$  determine the shock width<sup>12</sup>. Since it is desirable that the shock encompass a given number of zones, independent of the choice of zone size,  $b_1$  and  $b_2$  are non-dimensionalized by use of the zone size,

$$b_1 = B_1 \Delta x , \quad b_2 = B_2 \Delta x . \quad (2.52)$$

In finite difference form, the artificial viscosity thus becomes

$$q_{j-1/2}^{n+1/2} = \frac{\rho^{n+1} + \rho^n}{2} \left\{ B_2 \left( \frac{x_j^{n+1} - x_{j-1}^{n+1} + x_j^n - x_{j-1}^n}{2} \right) c_{j-1/2}^n \left( \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right) + B_1 \left( \frac{x_j^{n+1} - x_{j-1}^{n+1} + x_j^n - x_{j-1}^n}{2} \right)^2 \left( \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right) \left| \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right| \right\} , \quad (2.53)$$

where  $\left( \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right)$  is given by (2.45). Since rarefactions do not normally steepen into shocks, the viscosity is set to zero when  $\frac{\partial \rho}{\partial t} < 0$ <sup>14</sup>. It should be noted that in some instances<sup>15</sup>, it has been found necessary to retain the artificial viscosity contribution during expansion in order to obtain acceptable solutions.

## 2.6 Stability

The computation is advanced each cycle by a time increment

$$\Delta t^{n+1/2} = t^{n+1} - t^n \quad (2.54)$$

This characteristic time is dependent upon the zone size and the local sound speed, and is the key to the stability of the explicit numerical method. If the time increment becomes too large, instabilities result that may appear as oscillations which grow very rapidly with time. The criterion for stability of the difference equations used here is<sup>16</sup>

$$\Delta t < \frac{\Delta x}{B_2 c + 2B_1 \left| \frac{\dot{\rho}}{\rho} \right| \Delta x + \sqrt{(B_2 c + 2B_1 \left| \frac{\dot{\rho}}{\rho} \right| \Delta x)^2 + c^2}} \quad (2.55)$$

where  $\Delta x = x_j^{n+1} - x_{j-1}^{n+1}$ , and  $B_1$  and  $B_2$  are the artificial viscosity coefficients defined in Section 2.5. Note that if a foam (Section 3.6) is being used,  $B_1$  and  $B_2$  must be replaced with  $B_1 + B_3$  and  $B_2 + B_4$  respectively.

The criterion (2.55) is applied to each zone, the minimum value over all zones being used to advance the calculation. The criterion (2.55) is actually computed at the conclusion of each zone computation, the minimum value first being used in (2.40) and (2.41) on the next cycle. Thus (2.55) is written

$$\Delta t_{j-1/2}^{n+3/2} = \frac{K_{t1} \Delta x_{j-1/2}^{n+1}}{B_2 c_{j-1/2}^{n+1} + 2B_1 \left| \frac{\dot{\rho}}{\rho} \right| \Delta x + \sqrt{(B_2 c_{j-1/2}^{n+1} + 2B_1 \left| \frac{\dot{\rho}}{\rho} \right| \Delta x)^2 + (c_{j-1/2}^{n+1})^2}}, \quad (2.56)$$

where  $\frac{\dot{\rho}}{\rho}$  is given by (2.45). The timestep calculated by (2.55) is based upon a linearized stability analysis, and presumed to be applicable to the more general case. To ensure stability of the calculation, a factor  $K_{t1}$  appears

in (2.56), ( $0 < K_{t1} < 1$ ), to allow the timestep to be artificially reduced. If  $K_{t1}$  is left unspecified, the code defaults it to 0.95. The appearance of unstable oscillations in a calculation suggests the possibility that (2.55) may not adequately represent a stable timestep, and that  $K_{t1}$  should be reduced to recover a stable calculation.

In order to limit the rate of increase of  $\Delta t$ , the value actually used on the next cycle is

$$\Delta t^{n+3/2} = \text{Min} \left( \Delta t_{j-1/2}^{n+3/2}, K_{t2} \Delta t^{n+1/2} \right) \quad (2.57)$$

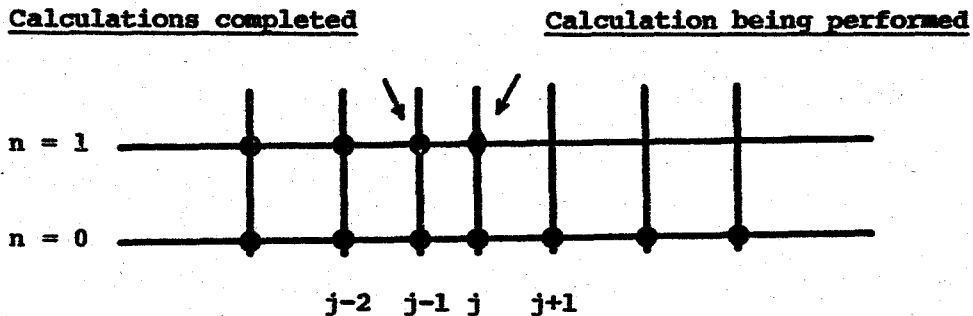
The factor  $K_{t2}$  is an input variable and is normally chosen to be a little greater than 1.0. If this feature is not desired,  $K_{t2}$  may be made a very large number, say 100. If it is left unspecified in the input the code defaults it to 1.05. In any case, the product of  $K_{t1}$  and  $K_{t2}$  should be kept less than 1.0.

Occasionally, in order to maintain a physically acceptable solution, it is necessary to start a calculation with a smaller  $\Delta t$  than normally required for computational stability. Such a case arises, for instance, if there is a large pressure or velocity discontinuity in the initial conditions, and the normally stable timestep could lead to zone inversions on the first cycle. The desired initial time increment may be read in as input in DELT(4). Then  $K_{t2}$  may be used to control the rate at which  $\Delta t$  increases until it is controlled entirely by stability. If this feature is not desired, DELT(4) may be left blank. The program then automatically assigns a value of  $10^5$ .

When energy sources are included, the energy added each cycle as  $\Delta Q$  in (2.46) must be small. Thus the time step is further restricted to be less than one hundredth the deposition time TDEP while the energy is being added.

## 2.7 Order of Computation

The calculation proceeds as follows: at  $t = 0$ , all quantities are defined for all zones by the initial data (via the initializing routines described later). The computation is performed successively at each zone starting with the left-hand boundary. At the  $j^{\text{th}}$  zone the momentum equation (2.39) is used to compute the acceleration at the  $j^{\text{th}}$  zone boundary. Velocity at a time  $\frac{1}{2}\Delta t^{n+1/2}$  after the initial instant at the  $j^{\text{th}}$  zone boundary follows from (2.40). Position at time  $\Delta t^{n+1/2}$  after the initial instant at the  $j^{\text{th}}$  zone boundary follows from (2.41).



The new position of the  $(j - 1)^{\text{st}}$  zone has already been found at this stage of the calculation. The mass equation (2.42) can therefore be used to determine the density in the zone between  $j - 1$  and  $j$ . For elastic-plastic materials the velocities at  $j - 1$  and  $j$  can be used to determine strain rates at  $j - \frac{1}{2}$ . The energy equation (2.46), equation of state (2.48), and deviatoric stress relationships are then used to determine the energy and stresses at  $j - \frac{1}{2}$ . These calculations are accomplished in the equation of state subroutines (Section 3).

At this point the various tests for rezoning the Lagrangian mesh can be made (see Chapter 6). If the zones are too small the variables associated with zones  $j - 1$  and  $j - 2$  are appropriately combined, or if the zones are too large two zones are created from zone  $j - 1$ .

The computation at the  $j^{\text{th}}$  zone is now complete, including the rezoning which may or may not be performed on previous zones, and the next zone in sequence can be treated in the same way. When all the zones have been treated, the solution for  $\Delta t^{n+1/2}$  after the initial time has been constructed. The procedure can be repeated for the next time increment. Further repetition allows construction of the solution for the entire time of interest.

### 3. EQUATIONS OF STATE AND CONSTITUTIVE RELATIONS

There are several options for equations of state and constitutive relations in the code. In the present version six different models can be accommodated. Alternate equations of state are most commonly included by replacing an unused model. Each model is programmed in a separate subroutine and an input parameter for each material layer determines the appropriate subroutine required. These models treat 1) general solids (including vaporization), 2) detonating high explosives, 3) perfect gases, 4) rate-dependent and strain hardening solids, and 5) foams or initially distended solids. Other models are available, some of which are described in Appendix B. The function of the "equation of state" routines is to use the newly calculated density, in conjunction with the values of the state variables (e.g. pressure, energy, etc.) from the previous timestep, to advance the state variables to the new time.

#### 3.1 STAT1 - General Solids - 1

##### 3.1.1 Hydrodynamic Fluids

The constitutive model generally used for solids has a basic equation of state of the Mie-Grüneisen form, which can be written

$$p - p_H = \Gamma \rho (\mathcal{E} - \mathcal{E}_H) \quad , \quad (3.1)$$

where  $p_H(\rho)$  and  $\mathcal{E}_H(\rho)$  are the pressure and energy along some reference path and are functions of density only, and where  $\Gamma(\rho)$  is the Gruneisen ratio and is also a function of density only. The reference pressure  $p_H(\rho)$  and energy  $\mathcal{E}_H(\rho)$  are generally taken from experimental data along the Hugoniot. Two forms are common for  $p_H$ . In the first,

$$P_H = \frac{\rho_0 c_0^2 \eta^2}{(1 - s\eta)^2} , \quad (3.2)$$

where  $\rho_0$  is the initial density at zero pressure and ambient temperature,  $c_0$  and  $s$  are material constants, and

$$\eta \equiv 1 - \frac{\rho_0}{\rho} . \quad (3.3)$$

This form follows from the observation that for many materials, the shock velocity  $U$  may be expressed as a linear function of particle velocity  $u$

$$U = c_0 + su , \quad (3.4)$$

where  $c_0$  and  $s$  are given constants. Using the relationships

$$P = u \rho_0 U ,$$

and

$$\eta = u/U ,$$

along with (3.4) leads to the form (3.2). Note that, assuming  $s > 1$ , the form of the denominator in (3.2) yields a limiting compression given by

$$\eta < \eta_{lim} = \frac{1}{s} ,$$

or equivalently, from (3.3),

$$\rho < \frac{s\rho_0}{s-1} .$$

It should also be noted that at  $\eta = -\frac{1}{s}$ , there is a tensile minimum, and thereafter in tension, negative sound speeds are calculated. (Since the



form (3.4) applies only to compressive states, caution is advised when tensions are expected to form.) Alternately,  $p_H$  is given as a power series expansion in  $\eta$ ,

$$p_H = K_0 \eta (1 + k_1 \eta + k_2 \eta^2 + k_3 \eta^3 + \dots) \quad (3.5)$$

where the  $k$ 's are non-dimensional constants. In order to match  $\frac{dp_H}{d\eta}$  at  $\eta = 0$ , it is necessary to assume that

$$K_0 = \rho_0 c_0^2 \quad (3.6)$$

where  $c_0$  corresponds to the bulk sound speed and  $K_0$  is the adiabatic bulk modulus at zero pressure and room temperature.

The energy  $\mathcal{E}_H$  is related to  $p_H$  by

$$\mathcal{E}_H = \frac{p_H \eta}{2\rho_0} \quad (3.7)$$

In addition the internal energy  $\mathcal{E}$  is equal to zero at  $p = 0$  and  $\rho = \rho_0$ .

The Grüneisen ratio is usually expressed as

$$\Gamma = \Gamma_0 (1 + h_1 \eta + h_2 \eta^2 + \dots) \quad (3.8)$$

where the  $h$ 's are non-dimensional constants. Thus, rearranging (3.1) yields

$$p = p_H \left\{ 1 - \frac{\Gamma}{2} \left( \frac{\rho}{\rho_0} - 1 \right) \right\} + \Gamma \rho \mathcal{E} \quad (3.9)$$

so that the  $f$ 's in the basic equation of state (2.7)

$$p = f_1(\rho) + f_2(\rho)\mathcal{E}$$

become

$$f_1 = P_H \left( 1 - \frac{\Gamma \mu}{2} \right) , \quad (3.10)$$

and

$$f_2 = \Gamma \rho , \quad (3.11)$$

where

$$\mu = \frac{\rho}{\rho_0} - 1 , \quad (3.12)$$

and  $p_H$  is given by either (3.2) or (3.5) and  $\Gamma$  is given by (3.8).

Note that in (3.8) if the only non-zero term is  $h_1 = -1.0$ ,  $\Gamma \rho$  becomes a constant, i.e.,

$$\Gamma = \Gamma_0 \frac{\rho_0}{\rho} , \quad (3.13)$$

which is a commonly assumed form for the Grüneisen parameter.

The sound speed (2.12) can be computed from (3.10) and (3.11), where the required derivatives of  $f_1$  and  $f_2$  are

$$\begin{aligned} \frac{df_1}{d\rho} &= \frac{dP_H}{d\eta} \frac{d\eta}{d\rho} \left( 1 - \frac{\Gamma \mu}{2} \right) - P_H \left( \frac{\Gamma}{2} \frac{d\mu}{d\rho} + \frac{\mu}{2} \frac{d\Gamma}{d\eta} \frac{d\eta}{d\rho} \right) \\ &= \frac{\rho_0}{\rho^2} \left\{ \frac{dP_H}{d\eta} \left( 1 - \frac{\Gamma \mu}{2} \right) - \frac{P_H}{2} \left( \Gamma (\mu + 1)^2 + \frac{d\Gamma}{d\eta} \mu \right) \right\} , \end{aligned} \quad (3.14)$$

and

$$\frac{df_2}{d\rho} = \Gamma + \frac{d\Gamma}{d\eta} (1 - \eta) . \quad (3.15)$$

The Grüneisen ratio (3.8) can be differentiated as

$$\frac{d\Gamma}{d\eta} = \Gamma_0 (h_1 + 2h_2\eta + 3h_3\eta^2 + \dots) , \quad (3.16)$$

and the derivative of  $p_H$  determined using (3.2)

$$\frac{dp_H}{d\eta} = \frac{\rho_0 c_0^2 (1 + s\eta)}{(1 - s\eta)^3} \quad , \quad (3.17)$$

or (3.5)

$$\frac{dp_H}{d\eta} = K_0 (1 + 2k_1\eta + 3k_2\eta^2 + \dots) \quad . \quad (3.18)$$

Under certain circumstances, especially when the material is heated by external energy sources, the material may vaporize. To take this behavior into account a vapor or modified expansion equation of state must be utilized. Note that the vapor equation of state is used for distended materials ( $\rho < \rho_0$ ), independent of the internal energy of the material.

It is taken in the form

$$p = \rho \left\{ H + (\Gamma_0 - H) \sqrt{\mu + 1} \right\} \left\{ \mathcal{E} - \mathcal{E}_g \left[ 1 - \exp(M\eta(1 - \eta)) \right] \right\} \quad , \quad (3.19)$$

where the Grüneisen ratio is always assumed constant. This form is chosen for the following reasons. When  $\frac{\rho}{\rho_0} \ll 1$ , i.e., for very expanded materials, the equation essentially reduces to

$$p = H\rho(\mathcal{E} - \mathcal{E}_g) \quad , \quad (3.20)$$

where the material constant  $\mathcal{E}_g$  represents the sublimation energy of the material. Equation (3.20) is therefore equivalent to the perfect gas law,

$$p = (\gamma - 1) \rho(\mathcal{E} - \mathcal{E}_g) \quad , \quad (3.21)$$

if  $H = \gamma - 1$ , where  $\gamma$  is the ratio of specific heats of the perfect gas, and the sublimation energy is subtracted from the internal energy.

When  $\rho = \rho_0$ , the equation reduces to

$$p = \Gamma_0 \rho_0 \mathcal{E} \quad , \quad (3.22)$$

and is therefore continuous with the Mie-Grüneisen equation (3.1) at this density. Differentiating the vapor equation (3.19) and setting  $\rho = \rho_0$  leads to

$$\frac{\partial p}{\partial \rho} = \Gamma_0 \mathcal{E} + \frac{1}{2}(\Gamma_0 - H) \mathcal{E} + \Gamma_0 \rho_0 \frac{\partial \mathcal{E}}{\partial \rho} + N \Gamma_0 \mathcal{E}_S \quad . \quad (3.23)$$

Differentiating the Mie-Grüneisen equation in the form (3.9) and setting  $\rho = \rho_0$  leads to

$$\frac{\partial p}{\partial \rho} = c_0^2 + \rho_0 \frac{d\Gamma}{d\rho} \mathcal{E} + \Gamma_0 \mathcal{E} + \Gamma_0 \rho_0 \frac{\partial \mathcal{E}}{\partial \rho} \quad , \quad (3.24)$$

for  $p_H$  given by either (3.2) or (3.5) providing that (3.6) is satisfied. In order that the slopes match, (3.23) and (3.24) are equated, yielding the condition

$$N = \frac{c_0^2}{\Gamma_0 \mathcal{E}_S} + \frac{1}{2} \left( 2h_1 - 1 + \frac{H}{\Gamma_0} \right) \frac{\mathcal{E}}{\mathcal{E}_S} \quad ,$$

where the derivative of (3.8) has been used. The second term is much smaller than the first when  $\mathcal{E} \ll \mathcal{E}_S$ , and  $N$  is usually chosen as

$$N = \frac{c_0^2}{\Gamma_0 \mathcal{E}_S} \quad , \quad (3.25)$$

so that the two equations (3.9) and (3.19) are equal in value and approximately continuous in slope at  $\rho = \rho_0$  and for low energies.

Equation (3.19) can be put into the form of (2.7),

$$p = f_1(\rho) + f_2(\rho)\mathcal{E} \quad ,$$

by expressing the f's as

$$f_1 = A(\exp B - 1)\rho\mathcal{E}_s \quad , \quad (3.26)$$

$$f_2 = A\rho \quad . \quad (3.27)$$

The sound speed will be given by (2.12). The required derivatives are

$$\frac{df_1}{d\rho} = \left( A + \rho \frac{dA}{d\rho} \right) (\exp B - 1) \mathcal{E}_s + \rho A \exp B \frac{dB}{d\rho} \mathcal{E}_s \quad , \quad (3.28)$$

and

$$\frac{df_2}{d\rho} = A + \rho \frac{dA}{d\rho} \quad , \quad (3.29)$$

where

$$A = \left\{ H + (\Gamma_0 - H) \sqrt{\mu + 1} \right\} \quad ,$$

$$\frac{dA}{d\rho} = \frac{\Gamma_0 - H}{2\rho_0 \sqrt{\mu + 1}}$$

and

$$B = N\eta(1 - \eta) \quad ,$$

$$\frac{dB}{d\rho} = N \frac{\rho_0}{\rho^2} (1 - 2\eta) \quad .$$

(3.30)

The vapor equation (3.19) represents materials reasonably well for high energies ( $\mathcal{E} > \mathcal{E}_s$ ) and low densities but can yield fairly large errors for

energies in the range  $\mathcal{E}_s > \mathcal{E} > \mathcal{E}_m$  where  $\mathcal{E}_m$  is the melt energy. To help correct this problem any stress calculated via (3.19) is limited by

$$\sigma_{j-1/2}^{n+1} > \sigma_{\min} \quad , \quad (3.31)$$

where

$$\sigma_{\min} = \sigma_{\min}^0 \left( 1 - \frac{\mathcal{E}}{\mathcal{E}_m} \right) < 0 \quad . \quad (3.32)$$

$\mathcal{E}_m$  and  $\sigma_{\min}^0$  are input constants with the former generally representing the melt energy and the latter (a negative input quantity) representing a maximum tensile stress that the material can sustain at low energies. Note that the latter quantity should be substantially larger in magnitude than any fracture stress associated with the material or fracture may be unintentionally suppressed. Note also that this provides only an approximation of vaporization, not an explicit treatment of phase changes.

Since  $H = \gamma - 1$  is a constant associated only with the equation (3.19), it is used as an indicator to bypass the vapor equation of state. If  $H$  is input to the code as  $H < 0$ , the routine selects the Mie-Gruneisen expression (3.9) as it extrapolates to negative strains ( $\rho < \rho_0$ ). This latter choice may be the most appropriate one for problems involving very low energy densities.

### 3.1.2 Elastic-Plastic Solids

The prior description is complete for materials with no strength, i.e., when materials behave hydrodynamically. For lower stress levels or where stress deviators cannot be assumed negligible, the elastic-plastic portion of this material model must be considered.

For such a material, which can support shear stress, the constitutive relation is of the general form,

$$\sigma_x = f(d_x, d_y, d_z, \theta) \quad , \quad (3.33)$$

with similar equations for  $\sigma_y$  and  $\sigma_z$ . However, (3.33) may be decomposed into separate equations for pressure and the deviators through the use of (2.19) and (2.22). Thus for the deviator stresses

$$\sigma_x^d = f(d_x^d, \rho, \theta) \quad , \quad (3.34)$$

and similar equations for  $\sigma_y^d$  and  $\sigma_z^d$ , where the pressure is obtained as described above. The deviator relations are specifically,

$$\frac{\partial \sigma_x^d}{\partial t} = 2G d_x^d \quad , \quad (3.35)$$

where  $G(\rho, \theta)$  is the shear modulus and is taken as a function of the thermodynamic state. If the material exhibits plasticity, the deviator stresses have an upper limit determined by the yield condition. The von Mises yield condition is

$$f_y = (\sigma_x^d)^2 + (\sigma_y^d)^2 + (\sigma_z^d)^2 < \frac{2}{3} Y^2 \quad , \quad (3.36)$$

where  $Y(\rho, \theta)$  is a material constant labelled the flow stress. It is more convenient to eliminate the y component by the use of (2.23), obtaining

$$f_y = 2 \left\{ (\sigma_x^d)^2 + \sigma_x^d \sigma_z^d + (\sigma_z^d)^2 \right\} < \frac{2}{3} Y^2 \quad . \quad (3.37)$$

When  $\alpha = 1$  or  $3$ , the symmetry condition (2.34) reduces (3.37) to

$$f_y = \frac{3}{2} (\sigma_x^d)^2 < \frac{2}{3} Y^2 \quad . \quad (3.38)$$

Thus, putting these relations into finite difference form,  $d_x^d$  is given by (2.19) and (2.14) as

$$d_x^{n+1/2} = \frac{2 \left( \begin{matrix} n+1/2 & n+1/2 \\ u_j & -u_{j-1} \end{matrix} \right)}{\left( \begin{matrix} n+1 & n \\ x_j & +x_j \end{matrix} \right) - \left( \begin{matrix} n+1 & n \\ x_{j-1} & +x_{j-1} \end{matrix} \right)} + \frac{1}{3} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right), \quad (3.39)$$

where  $\left( \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right)$  is given by (2.45). Then  $\sigma_x^d$  is given as if the material were entirely elastic, by

$$\sigma_x^d = \sigma_x^{dn} + 2\Delta t \begin{matrix} n+1/2 & n+1/2 \\ G_{j-1/2} & d_x^{n+1/2} \end{matrix}. \quad (3.40)$$

This value is limited by the yield condition<sup>17</sup>. For  $\alpha = 1$  or 3

$$f_y = \frac{3}{2} (\sigma_x^d)^2. \quad (3.41)$$

Thus, if  $f_y < \frac{2}{3} (y_{j-1/2}^{n+1})^2$ , then  $\sigma_x^{dn+1/2} = \sigma_x^d$ . However, if

$f_y > \frac{2}{3} (y_{j-1/2}^{n+1})^2$ , then

$$\sigma_x^{dn+1/2} = \left( \text{Sgn } \sigma_x^d \right) \frac{2}{3} y_{j-1/2}^{n+1}. \quad (3.42)$$

The deviator stress work is, from (2.35) and (2.47),

$$\Delta \mathcal{E}^d = \frac{3}{2} \Delta t \begin{matrix} n+1/2 & d & n+1/2 \\ d_x & & \end{matrix} \frac{\sigma_x^{dn+1/2} + \sigma_x^{dn}}{\rho_{j-1/2}^{n+1} + \rho_{j-1/2}^n} \quad (3.43)$$

and from (2.36)

$$\phi_{j-1/2}^{n+1} = \frac{3}{2} \sigma_x^{dn+1/2}. \quad (3.44)$$



However, when  $\alpha = 2$ , it is first necessary to compute  $d_z^d$  given by (2.15) and (2.19) as

$$d_z^{n+1/2} = \frac{1}{3} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right) , \quad (3.45)$$

where  $\left( \frac{1}{\rho} \frac{\partial \rho}{\partial t} \right)$  is again given by (2.45). Then  $\sigma_z^d$  is given as if the material were entirely elastic, by

$$\sigma_z^d = \sigma_z^{n+1/2} + 2 \Delta t \quad G_{j-1/2}^{n+1/2} \quad d_z^{n+1/2} . \quad (3.46)$$

The yield condition is therefore, from (3.37),

$$f_y = 2 \left\{ (\sigma_x^d)^2 + \sigma_x^d \sigma_z^d + (\sigma_z^d)^2 \right\} . \quad (3.47)$$

Then if  $f_y < \frac{2}{3} \left( Y_{j-1/2}^{n+1} \right)^2$ ,  $\sigma_x^{d, n+1} = \sigma_x^d$ , and  $\sigma_z^{d, n+1} = \sigma_z^d$ . However,

if  $f_y > \frac{2}{3} \left( Y_{j-1/2}^{n+1} \right)^2$ , then

$$\left( \sigma_x^d \right)_{j-1/2}^{n+1} = \sqrt{\frac{\frac{2}{3} \left( Y_{j-1/2}^{n+1} \right)^2}{f_y}} \sigma_x^d , \quad (3.48)$$

$$\left( \sigma_z^d \right)_{j-1/2}^{n+1} = \sqrt{\frac{\frac{2}{3} \left( Y_{j-1/2}^{n+1} \right)^2}{f_y}} \sigma_z^d \quad (3.49)$$

and the deviator stress work is, from (2.28) and (2.47),

$$\Delta \epsilon^d = \frac{\Delta t^{n+1/2}}{\rho_{j-1/2}^{n+1} + \rho_{j-1/2}^n} \left[ \left( \sigma_x^{d, n+1} + \sigma_x^{d, n} \right) \left( 2d_x^{n+1/2} + d_x^{n+1/2} \right) \right. \\ \left. + \left( \sigma_z^{n+1} + \sigma_z^{d, n} \right) \left( 2d_z^{n+1/2} + d_x^{n+1/2} \right) \right] \quad (3.50)$$

and from (2.31),

$$\phi_{j-1/2}^{n+1} = 2\sigma_x^{d, n+1} + \sigma_z^{d, n+1} \quad (3.51)$$

The energy  $\epsilon_{j-1/2}^{n+1}$  and pressure  $p_{j-1/2}^{n+1}$  are then found from (2.49) and (2.48) as before. Then from (2.32)

$$p_{j-1/2}^{n+1} = p_{j-1/2}^{n+1} - \sigma_x^{d, n+1} \quad (3.52)$$

The sound speed appropriate to the elastic-plastic case is

$$\left( c_{j-1/2}^{n+1} \right)^2 = \frac{3(1-\nu)}{(1+\nu)} c^2 \quad (3.53)$$

where  $c^2$  on the right is given by (2.12) and  $\nu$  is Poisson's Ratio, usually taken to be constant.

It now remains to specify the functions  $G(\rho, \epsilon)$  and  $Y(\rho, \epsilon)$  which appear on preceding pages. The most common assumption is that the shear modulus  $G$  is related to the bulk modulus  $K$  by

$$G = \frac{3(1-2\nu)}{2(1+\nu)} K \quad (3.54)$$

where  $\nu$  is Poisson's Ratio taken to be a constant. The bulk modulus is related to  $c^2$  (2.12) by

$$K = \rho c^2 \quad . \quad (3.55)$$

Thus in finite difference form, in terms of (3.53) and (3.55),

$$G_{j-1/2}^{n+1/2} = \left\{ \frac{(1-2\nu)}{2(1-\nu)} \right\}^n \rho_{j-1/2}^n \left( c_{j-1/2}^n \right)^2 \quad . \quad (3.56)$$

where  $c_{j-1/2}^n$  is the sound speed calculated by (3.53) on the previous cycle, and thus (3.56) is not precisely centered in time.

An approximation that is used for  $G$  is to assume  $G$  is a function of  $\rho$  only, written as a power series in  $\eta_1$ ,

$$G = G_0 (1 + g_1 \eta_1 + g_2 \eta_1^2 + \dots) \quad , \quad (3.57)$$

where

$$\eta_1 = 1 - \frac{2\rho_0}{\rho_{j-1/2}^n + \rho_{j-1/2}^{n+1}} \quad , \quad (3.58)$$

and

$$G_0 = \frac{3(1-2\nu)}{2(1+\nu)} K_0 \quad . \quad (3.59)$$

The yield or flow stress  $Y$  can be calculated in six different ways, selected by an input constant NOY:

If NOY = 0 the material is assumed to behave hydrodynamically and the equations of state (3.9) and (3.19), if desired, are selected to calculate the stress directly; no deviatoric stress components are used.

This is equivalent to taking  $Y = 0$ .

If NOY = 1,  $Y$  is taken to be a constant  $Y_0$ . This case corresponds to an elastic perfectly plastic material.

Setting NOY = 2 yields a perfectly elastic material. This is equivalent to an infinite yield strength or  $Y = \infty$ .

NOY = 3 selects the following expression for Y:

$$Y = Y_0(1 + y_1\eta)(1 - \mathcal{E}/y_2) > 0 \quad , \quad (3.60)$$

where  $Y_0$ ,  $y_1$  and  $y_2$  are input constants. The parameter  $y_2$  is generally taken to be the melt or sublimation energy, and this term yields a decrease in flow stress with increasing energy. The term in the constant  $y_1$  yields an increase in the yield strength with compression and a corresponding decrease with expansion. The value for Y is set to zero whenever either of the two terms becomes negative. Note that (3.60) is not intended to represent strain hardening.

For NOY = 4 and 5\* the yield stress is obtained from an isotropic strain hardening model. (See Section 3.4 for additional comments on strain hardening.) By assuming that the volumetric strain is entirely elastic, the amount of plastic work per unit initial volume done in the current time step,  $\Delta p_{j-1/2}^{d,n+1/2}$  is given by <sup>17</sup>

$$\Delta p_{j-1/2}^{d,n+1/2} = \frac{\rho_0 \sigma_y}{G_{j-1/2} \left( \frac{n}{\rho_{j-1/2}} + \frac{n+1}{\rho_{j-1/2}} \right)} \left( \sqrt{\frac{2(\gamma^{n+1})^2}{3/y}} - \frac{2(\gamma^{n+1})^2}{3/y} \right) \quad (3.61)$$

and the total plastic work per unit initial volume done on the material in the zone is then

$$p_{j-1/2}^{d,n+1} = p_{j-1/2}^{d,n} + \Delta p_{j-1/2}^{d,n+1/2} \quad . \quad (3.62)$$

---

\* Set NVAR > 13 to use either of these options.

From the isotropic hardening assumption, the yield surface grows in the same manner for any stress combinations causing plastic flow. Therefore, by expressing the way the yield stress depends on the plastic work per unit initial volume for a simple tensile test, the yield stress for any plastic loading condition can be obtained by substituting the corresponding plastic work into this expression.

For  $NOY = 4$  the uniaxial-stress engineering stress-strain curve is assumed to be bi-linear. Therefore, this case corresponds to linear isotropic strain hardening. Here, the yield stress for the simple tensile test is obtained by substituting the plastic work (3.62) in

$$Y_{j-1/2}^{n+1} = \sqrt{\frac{2Y_1Y_2}{Y_1 - Y_2} \frac{d^n}{P^{j-1/2}} + Y_0^2}, \quad (3.63)$$

where the  $y$ 's are input constants and  $Y_0$  is the initial yield stress.  $y_1$  is Young's modulus and  $y_2$  is the tangent modulus. Thus, any loading resulting in the plastic work (3.62) gives, with the bi-linear stress-strain assumption, the yield stress (3.63).

$NOY = 5$  is identical to  $NOY = 4$  except that the stress-strain curve is described by a power law hardening model instead of the bi-linear relation. For this case, the stress-strain curve is given by

$$Y = \begin{cases} Y_0 + \beta [\epsilon^\alpha - \epsilon_0^\alpha], & Y > Y_0 \\ \epsilon E, & Y < Y_0 \end{cases} \quad (3.64)$$

where  $Y_0$  is the initial yield point,  $\epsilon_0$  is the strain in one-dimensional stress at  $Y = Y_0$ ,  $\alpha$  and  $\beta$  are the hardening coefficients and  $E$  is Young's modulus. All of these are input constants with  $E$  given by  $y_3$ ,  $\alpha$  given by

$y_1$ , and  $\beta$  given by  $y_2$ . The value of the yield stress for any loading resulting in the plastic work (3.62) is obtained from an iterative solution of

$$\begin{aligned}
 \frac{d_n}{p_{j-1/2}^{\epsilon}} = & Y \left[ \frac{Y - Y_0}{\beta} + \epsilon_0^\alpha \right]^{1/\alpha} - \frac{Y^2 + Y_0^2}{2E} \\
 & - \frac{\alpha\beta}{\alpha + 1} \left\{ \left[ \frac{Y - Y_0}{\beta} + \epsilon_0^\alpha \right]^{\frac{\alpha+1}{\alpha}} - \epsilon_0^{\alpha+1} \right\},
 \end{aligned}
 \tag{3.65}$$

where the yield stress  $Y$  is at time  $n + 1$  and position  $j - 1/2$ . In this model the value of  $Y_0$  is adjusted, if required, so that it is equal to  $\epsilon_0 E$ .

Both of these hardening models employ Young's modulus, input as either  $y_1$  or  $y_3$ . Note that this parameter should be calculated from the standard elastic relation,

$$E = 3K_0(1 - 2\nu) \quad ,$$

where  $\nu$  is Poisson's ratio and  $K_0$  is given by (3.6).

### 3.2 STAT2 - High Explosives

High explosives are treated by a "programmed burn" method, where no pressures or motion appear in the undetonated explosive. Rather, the detonation wave is artificially forced to propagate at the Chapman-Jouguet velocity from the detonation point,  $x_D$ . This technique essentially treats explosives as a source of prescribed energy release. An alternate method is considered in Appendix B.

For a given  $x_D$ , the position of the point of initiation, the time at which the detonation wave will reach a particular zone is given by

$$t_{j-1/2}^b = \frac{\left| \frac{1}{2} x_j^o + x_{j-1}^o - x_D \right|}{D} + DT \quad (3.66)$$

where  $D$  is the Chapman-Jouguet detonation velocity and  $DT$  is the delay time for the initiation of detonation. Note that this allows the initiation of a detonation wave for a given high explosive layer to occur at times after the start of a problem.

The equation of state of the high explosive detonation products, using the general form (2.7), is written,

$$P_{j-1/2}^{n+1} = F \left( f_1 + \mathcal{E}_{j-1/2}^{n+1} f_2 \right) \quad (3.67)$$

where  $F$  is a burn fraction given by,

$$F = 0, \text{ if } t^{n+1} < t_{j-1/2}^b \quad (3.68)$$

$$F = \frac{D \left( t^{n+1} - t_{j-1/2}^b \right)}{B_5 \left( x_j^{n+1} - x_{j-1}^{n+1} \right)} \text{ if } t^{n+1} > t_{j-1/2}^b \quad ,$$

with the restriction  $F < 1$ . The constant  $B_5$  is a factor, generally 2.5, which spreads the detonation front over several zones.

Solving (3.67) and the energy equation (2.49) for the internal energy leads to

$$\mathcal{E}_{j-1/2}^{n+1} = \frac{\mathcal{E}_{j-1/2}^n + \left( F f_1 + P_{j-1/2}^n + 2q_{j-1/2}^{n+1/2} \right) \left( \frac{\Delta \rho}{2\rho^2} \right)}{1 - F f_2 \frac{\Delta \rho}{2\rho^2}} \quad (3.69)$$

the pressure being found from (3.67). Note that the external energy source term  $\Delta Q_{j-1/2}$  is not included in this routine.

Since explosive gasses cannot support a shear stress, it is unnecessary to distinguish between the pressure and the total stress. Thus the pressure  $p$  is stored directly as the stress  $\sigma$ . The burn time  $t^b$ , which is computed at the beginning of the problem, is stored in the location normally reserved for the pressure.

Function  $f_1$  and  $f_2$  appropriate to the explosion product gasses must be supplied. Three forms are currently available, making computations possible with almost any explosive: perfect gas, JW<sup>18</sup>, and BK<sup>19</sup>.

### 3.2.1 Ideal Gas Model

For a perfect gas, the equation of state is

$$p = (\gamma - 1)\rho\epsilon \quad , \quad (3.70)$$

where  $\gamma$  is the ratio of specific heats. Writing this in the form (2.7), the functions  $f_1$  and  $f_2$  for perfect gas explosion products are

$$\begin{aligned} f_1 &= 0 \quad , \\ f_2 &= (\gamma - 1)\rho_{j-1/2}^{n+1} \quad . \end{aligned} \quad (3.71)$$

Also

$$\left. \begin{aligned} \frac{df_1}{d\rho} &= 0 \quad , \\ \frac{df_2}{d\rho} &= (\gamma - 1) \quad , \end{aligned} \right\} \quad (3.72)$$



so that if these are inserted into (2.12), the expression for sound speed in a perfect gas is obtained, i.e.,

$$c^2 = \frac{\gamma p}{\rho} \quad . \quad (3.73)$$

If the equations for mass, momentum, and energy conservation across a shock, i.e.,

$$\begin{aligned} \rho(D - u) &= \rho_0(D - u_0) \quad , \\ p + \rho(D - u)^2 &= p_0 + \rho_0(D - u_0)^2 \quad , \\ \mathcal{E} - \mathcal{E}_0 - Q &= \frac{1}{2} (p + p_0)(\mathcal{V}_0 - \mathcal{V}) \quad , \end{aligned} \quad (3.74)$$

where  $Q$  is the chemical energy added in the detonation, are combined with the Chapman-Jouguet condition,

$$D = c + u \quad , \quad (3.75)$$

along with the expression for sound speed (3.73) and the equation of state (3.70), then the pressure, density, and energy at the Chapman-Jouguet point immediately behind the detonation wave moving into undisturbed solid explosive ( $p_0 = u_0 = 0$ ) are,

$$\begin{aligned} p_{cj} &= \frac{1}{\gamma+1} \rho_0 D^2 \quad , \\ \rho_{cj} &= \frac{\gamma+1}{\gamma} \rho_0 \quad , \\ \mathcal{E}_{cj} &= \frac{\gamma}{(\gamma^2 - 1)(\gamma + 1)} D^2 \quad . \end{aligned} \quad (3.76)$$

The chemical energy added in the detonation is

$$Q = \frac{D^2}{2(\gamma^2 - 1)} \quad (3.77)$$

To use this subroutine the density is initialized to  $\rho_0$  and all other state variables (i.e.,  $p$ ,  $\sigma$ ) are set to zero. The program initializes the energy to the value given by (3.77).

### 3.2.2 JWL Model

The perfect gas model for detonating explosives in the previous section may not represent the explosive behavior well at large expansions. The Jones-Wilkins-Lee<sup>18</sup>, or JWL, equation of state provides a better fit to explosive products behavior, and takes the form

$$p = A \left( 1 - \frac{\omega \rho}{R_1 \rho_0} \right) e^{-R_1 \rho_0 / \rho} + B \left( 1 - \frac{\omega \rho}{R_2 \rho_0} \right) e^{-R_2 \rho_0 / \rho} + \frac{\omega \rho^2}{\rho_0} e^{\rho} \quad (3.78)$$

so that in (3.67)

$$f_1(\rho) = A \left( 1 - \frac{\omega \rho}{R_1 \rho_0} \right) e^{-R_1 \rho_0 / \rho} + B \left( 1 - \frac{\omega \rho}{R_2 \rho_0} \right) e^{-R_2 \rho_0 / \rho} \quad (3.79)$$

and

$$f_2(\rho) = \omega \rho^2 / \rho_0 \quad .$$

Appropriate derivatives of equations (3.79) are required to compute the sound speed (2.12). The initial specific energy for the explosive must be entered as an input parameter.

### 3.2.3 BKW Model

An alternative to the JWL form which also provides a good fit to explosive products data is the Becker-Kistiakowsky-Wilson<sup>19</sup>, or BKW model.

The functions  $f_1(\rho)$  and  $f_2(\rho)$  for the BKW model take the form

$$f_1(\rho) = P_r(\mathcal{V}) - \frac{1}{\beta \mathcal{V}} \mathcal{E}_r(\mathcal{V}) \quad ,$$

and

(3.80)

$$f_2(\rho) = \frac{1}{\beta \mathcal{V}} \quad , \quad (\mathcal{V} = 1/\rho)$$

where

$$\ln P_r(\mathcal{V}) = A + B \ln \mathcal{V} + C \ln^2 \mathcal{V} + D \ln^3 \mathcal{V} + E \ln^4 \mathcal{V} \quad ,$$

$$\ln(\mathcal{E}_r(\mathcal{V}) + \mathcal{E}_0) = K + L \ln P_r + M \ln^2 P_r + N \ln^3 P_r + O \ln^4 P_r \quad , \quad (3.81)$$

and

$$-1/\beta = R + 2S \ln \mathcal{V} + 3T \ln^2 \mathcal{V} + 4U \ln^3 \mathcal{V} \quad .$$

In this formulation,  $P_r(\mathcal{V})$  is an isentrope through the C-J point, so the sound speed may be written as

$$c^2 = -\mathcal{V}^2 \frac{dP_r}{d\mathcal{V}} \quad , \quad (3.82)$$

or

$$c^2 = -\mathcal{V} P_r(\mathcal{V}) [B + 2C \ln \mathcal{V} + 3D \ln^2 \mathcal{V} + 4E \ln^3 \mathcal{V}] \quad . \quad (3.83)$$

The specified units for the BKW model are P in Mbar,  $\mathcal{V}$  in  $\text{cm}^3/\text{gm}$ , and  $\mathcal{E}$  in  $\text{Mbar cm}^3/\text{gm}$ ; in the code, the pressures will appear as dynes/cm<sup>2</sup>.

### 3.3 STAT3 - Ideal Gas Law

When  $F = 1$  in Eq. (3.67), the equations of Section 3.2.1 are appropriate for a gas. irrespective of whether the gases are detonation products or not.

Thus, equations (3.69) and (3.67) with  $F = 1$  can be used. Functions for  $f_1$ ,  $f_2$ ,  $\frac{\partial f_1}{\partial \rho}$  and  $\frac{\partial f_2}{\partial \rho}$  appropriate to the particular real gas being used must be supplied. If the gas is a perfect gas, then equations (3.71) and (3.72) are used, and subroutine STAT3 is supplied using this option.

In this subroutine the pressure is stored directly as the stress; the variable pressure is not used. The pressure, density, and internal energy must be initialized to appropriate values. Note that the pressure (stored as stress) must never be initialized to zero; neither should the energy. The initial values of pressure, density, and energy must exactly satisfy the equation of state. (For a perfect gas  $p$ ,  $\rho$  and  $\mathcal{E}$  must exactly satisfy (3.70), and must be inserted as initial conditions.) In addition this routine does not allow for external energy sources, although such a modification could easily be made.

#### 3.4 STAT4 - General Solids - 2

There is much experimental evidence <sup>20,21</sup> that many materials do not behave according to an elastic perfectly plastic model. Even varying the yield strength or allowing for isotropic strain hardening (c.f. STAT1, Section 3.1) appears to be inadequate for some materials. Strain rate dependence is evident in the stress relaxation observed in some metals and crystals, and a Bauschinger effect, which can be treated with anisotropic hardening, has been seen in a number of ductile alloys.

The subroutine STAT4\* included here was written to treat these two additional phenomena, while at the same time keeping most of the options

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\*This routine has been coded for rectangular ( $\alpha = 1$ ) and spherical ( $\alpha = 3$ ) geometries only; cylindrical ( $\alpha = 2$ ) geometry is not allowed.

available in STAT1 (Section 3.1). A more complete description of the present routine is given in Reference 20, along with several examples.

### 3.4.1 Strain Hardening Solids

Strain hardening is incorporated in a manner similar to that described at the end of Section 3.1, i.e., the yield strength  $Y$  is a monotonically increasing function of the plastic work (3.63). Since it is assumed that the equilibrium yield strength is rate independent, any experimental technique (e.g., a static uniaxial-stress test) that provides a relation between the plastic work and the yield strength will suffice to determine the material parameters necessary to use the model.

Two isotropic strain hardening models are included in this routine. The first assumes a bilinear uniaxial-stress engineering stress-strain curve, the same as described in Section 3.1. Thus the yield strength  $Y$  is given by (3.63) using the plastic work (3.62). These equations are repeated here for convenience. The plastic work  $d_p^e$  is

$$d_p^{e, n+1} = d_p^{e, n} + \Delta_p^{e, n+1/2} \quad , \quad (3.84)$$

where

$$\Delta_p^{e, n+1} = \frac{\rho_0 / \gamma}{G_{j-1/2} \left( \rho_{j-1/2} + \rho_{j-1/2} \right)} \left( \sqrt{\frac{2(\gamma^{n+1})^2}{3\gamma}} - \frac{2(\gamma^{n+1})^2}{3\gamma} \right)$$

and  $f_\gamma$  is the yield function given by (3.38) and  $G$  is the shear modulus.

The yield stress is

$$Y_{j-1/2}^{n+1} = \sqrt{\frac{2EE^*}{E - E^*} \frac{d_p^{e, n}}{p^{e, j-1/2}} + Y_0^2} \quad , \quad (3.85)$$

where  $E$  is Young's modulus and  $E^*$  is the tangent modulus. The second isotropic hardening model employs a power law uniaxial-stress stress-strain curve of the form

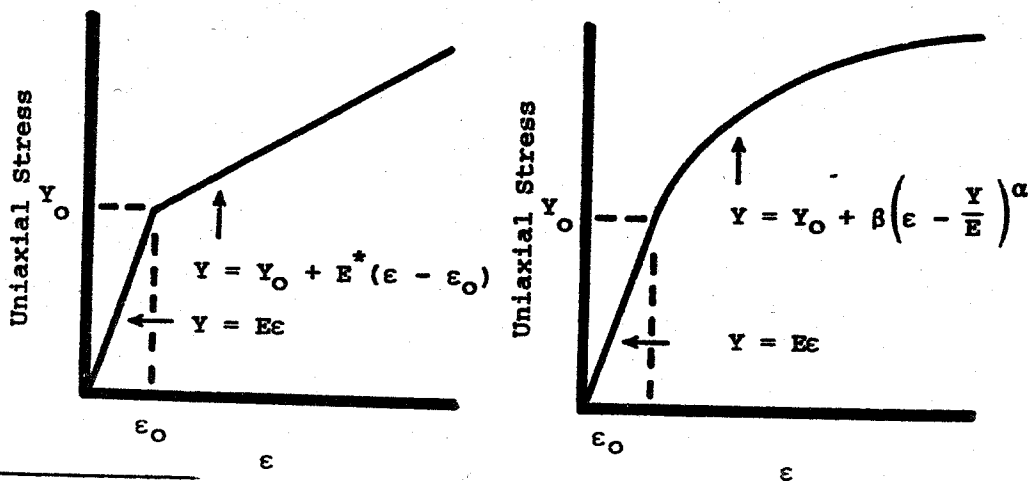
$$Y = \begin{cases} E\epsilon & Y < Y_0 \\ Y_0 + \beta \left( \epsilon - \frac{Y}{E} \right)^\alpha, & Y > Y_0 \end{cases}, \quad (3.86)$$

where  $\epsilon$  is the strain in uniaxial stress,  $E$  is Young's modulus, and the material constants  $\alpha$  and  $\beta$  are the hardening coefficients\*. Note that this equation is different from (3.64) in Section 3.1. This expression yields

$$\frac{d}{d\epsilon} \rho_j^{n-1/2} = \left( \frac{Y - Y_0}{\beta} \right)^{1/\alpha} \left( \frac{Y + \alpha Y_0}{\alpha + 1} \right), \quad (3.87)$$

which, although it is not explicit in  $Y_j^{n+1}$ , can be solved by iteration, using the value of  $\frac{d}{d\epsilon} \rho_j^{n-1/2}$  obtained from (3.84), via Newton's method, to obtain the required yield stress.

These two isotropic hardening models have been derived from stress-strain curves of the form sketched below.



\*Note that for  $Y < Y_0$ ,  $\frac{dY}{d\epsilon} = \frac{E\alpha\beta}{E \left( \epsilon - \frac{Y}{E} \right)^{1-\alpha} + \alpha\beta}$ , and thus  $\alpha < 1.0$  is required.

To treat anisotropic strain hardening and thus provide an explicit treatment for the Bauschinger effect, the stress deviator  $\sigma_x^d$  is assumed to be made up of N elements,

$$\sigma_x^d = \sum_{i=1}^N a_i \sigma_{xi}^d \quad , \quad (3.88)$$

with weighting factors  $a_i$  such that

$$\sum_{i=1}^N a_i = 1 \quad . \quad (3.89)$$

Note that, since cylindrical geometry is explicitly excluded from this routine, consideration need not be given to other components of the stress deviator. Each of the elemental deviators  $\sigma_{xi}^d$  is treated in a manner identical with that described in Section 3.1. Thus each deviator element, in analogy with (3.35), is governed by the elastic stress relations

$$\frac{\partial \sigma_{xi}^d}{\partial t} = 2G d_x^c, \quad i = 1, 2, \dots, N, \quad (3.90)$$

where  $G = G(\rho)$  is the shear modulus and  $d_x^d$  is the stretching deviator from (2.19). In analogy with (3.40) these expressions are differenced as

$$\sigma_{xi}^{d \ n+1} = \sigma_{xi}^d + 2G \frac{d_x^{d \ n+1/2}}{dt} \quad , \quad (3.91)$$

$$i = 1, 2, \dots, N,$$

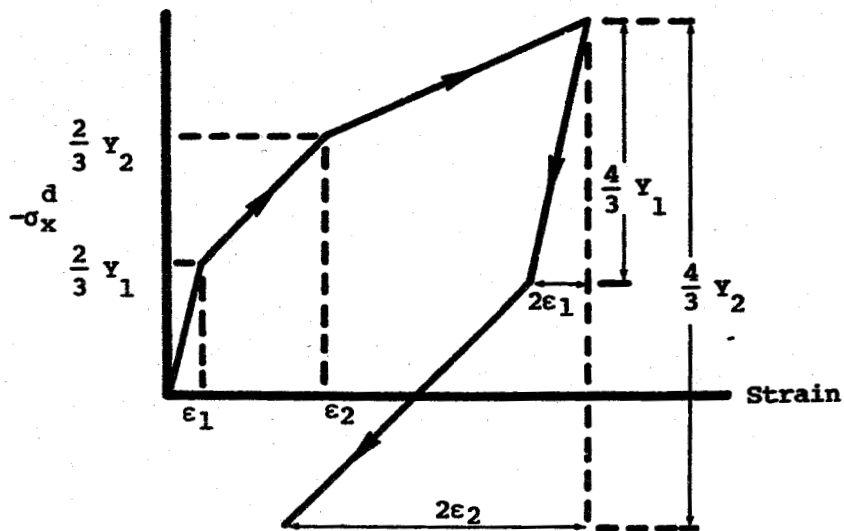
where for clarity, the additional subscript  $j - 1/2$  has been dropped. The stretching deviator  $d_x^{d \ n+1/2}$  is obtained directly from (3.39). In analogy

with (3.38) the elemental deviators are in addition subject to  $N$  yield conditions of the form

$$\frac{3}{2} (\sigma_{xi}^d)^2 < \frac{2}{3} Y_i^2 \quad . \quad (3.92)$$

Thus to obtain the total stress deviator  $\sigma_x^d$ , (3.91), subjected to (3.92), is used to calculate each of the elements. The individual elements are then summed according to (3.88). Note that to use this model, storage must be provided for each of the deviator elements at each zone, so that the integration in (3.91) can be accomplished.

For a typical loading and unloading cycle this anisotropic hardening model gives rise to the behavior sketched below. Note that the stress deviator, as plotted in the figure, is added to the pressure, as obtained from either (3.9) or (3.19), to obtain the total stress [equation (3.52), Section 3.1].



The input parameters required by this model,  $a_i$  and  $Y_i$ , can be obtained from a uniaxial-stress stress-strain curve. The curve is broken into straight line segments, and the points thus generated are tabulated sequentially as



$\sigma_i$  and  $\epsilon_i$ ,  $i = 1, 2, \dots, N$ , ( $N$  is limited to 7 or less in the present routine). The modulus for each line segment is calculated via

$$E_i = \frac{\sigma_i - \sigma_{i-1}}{\epsilon_i - \epsilon_{i-1}} \quad . \quad (3.93)$$

$E_1 = \sigma_1/\epsilon_1$  is Young's modulus and is considered constant. The elemental yield stresses  $Y_i$  are then

$$Y_i = E_1 \epsilon_i \quad , \quad (3.94)$$

and the weighting factors  $a_i$  are

$$a_i = \frac{E_i - E_{i+1}}{E_1} \quad , \quad (3.95)$$

where  $E_{N+1} = 0$ .

It is reasonable to suppose that material strength decreases with increasing temperature or internal energy. To qualitatively take this effect into account a function  $k(\mathcal{E})$  is provided. It takes the form

$$k(\mathcal{E}) = 1 - (k_0 + 2) \left( \frac{\mathcal{E}}{\mathcal{E}_m} \right) + (k_0 + 1) \left( \frac{\mathcal{E}}{\mathcal{E}_m} \right)^2 \quad , \quad (3.96)$$

where  $k_0$  and  $\mathcal{E}_m$  (generally the melt energy) are material constants. This results in

$$k(\mathcal{E}) = 1 \quad , \quad \frac{\partial k(\mathcal{E})}{\partial \mathcal{E}} = - \frac{(k_0 + 2)}{\mathcal{E}_m} \quad , \quad \text{at } \mathcal{E} = 0 \quad ,$$

$$k(\mathcal{E}) = 0 \quad , \quad \frac{\partial k(\mathcal{E})}{\partial \mathcal{E}} = \frac{k_0}{\mathcal{E}_m} \quad , \quad \text{at } \mathcal{E} = \mathcal{E}_m \quad .$$

Generally,  $k_0$  falls in the range  $-2 < k_0 < -1$ . In the anisotropic hardening model the yield stresses  $Y_i$  in (3.92) are multiplied by  $k(\mathcal{E})$  whenever the vapor equation of state (3.19) is employed for a given material.

An additional yield model, analogous to (3.60) in Section 3.1, is available in this routine. In this case the yield stress  $Y$  is given by

$$Y = Y_0(1 + y_1\eta)k(\mathcal{E}) > 0 \quad (3.97)$$

and is thus a direct function of the strain  $\eta$  and the internal energy  $\mathcal{E}$ .

In the code, all of the preceding results in the calculation of an equilibrium stress deviator by one of seven different models. The desired model is indicated by the input parameter NOY. The possible choices are summarized below.

- NOY = 0: hydrodynamic;  $Y = 0$
- NOY = 1: constant yield stress;  $Y = Y_0$ ; NVAR must be at least 13
- NOY = 2: infinite yield stress;  $Y = \infty$ ; subroutine STAT1 should be used for this option
- NOY = 3: variable yield stress (3.97); NVAR must be at least 13
- NOY = 4: bilinear isotropic strain hardening (3.85); NVAR must be at least 14
- NOY = 5: power law isotropic strain hardening (3.86); NVAR must be at least 14
- NOY < 0: anisotropic strain hardening (3.88) - (3.92) with  $|NOY|$  components (maximum of 7); NVAR must be at least  $13 + |NOY|$

#### 3.4.2 Strain Rate Dependent Solids

Strain rate dependent stress relaxation can be treated by considering only the deviatoric part of the stress. In the subsequent discussion the

previously calculated equilibrium stress deviator  $\sigma_x^d$  will be denoted by  $\sigma_e^d$  where the x has been dropped since only one component is being considered.

To treat this problem it is assumed that the final stress deviator  $\sigma^d$  can be obtained via a relaxation from some instantaneous value to the equilibrium value  $\sigma_e^d$ . This yields a constitutive relation, analogous to (3.35) for the rate independent case, of the form<sup>20</sup>

$$\frac{\partial \sigma^d}{\partial t} = 2G[d_x^d - g(\sigma^d - \sigma_e^d)] \quad , \quad (3.98)$$

where, as before, G is the shear modulus and  $d_x^d$  is the stretching deviator from (2.19). The function  $g(\sigma^d - \sigma_e^d)$  is generally called the relaxation function and will be discussed later.

The expression (3.98) could be differenced directly and inserted into the routine. However, to maintain accuracy consistent with the rest of the code, a second order accurate analog would have to be used. To avoid this difficulty a scheme involving the subcycling of (3.98) within a given zone-cycle calculation is employed. It can be shown<sup>20</sup> that using a first order difference analog for (3.98) results in an approximate error in  $\sigma^d$  given by

$$e = G\Delta t(g^n - g^f) \frac{d_x^d - g^n}{d_x^d} \quad ,$$

where  $d_x^d$  and G are considered constant over the entire time step. Thus to limit the error  $e < \delta\sigma^d$ , the normal time step  $\Delta t^{n+1/2}$  must be divided into M subcycles with time step  $\Delta t^m = \Delta t^{n+1/2}/M$  each, where M is given by

$$M = \frac{G\Delta t^{n+1/2}(g^n - g^f)(d_x^d - g^n)}{\delta\sigma^d \quad d_x^d} \quad , \quad (3.99)$$

and then rounded to the next higher integer. The needed relaxation functions are obtained from

$$g^n = g(\sigma^d_n - \sigma_e^d_n) \quad , \quad (3.100)$$

and

$$g^f = g(\sigma^d_n + \Delta\sigma_i - \sigma_e^d_{n+1}) \quad , \quad (3.101)$$

where

$$\Delta\sigma_i = 2G\Delta t^{n+1/2} d_x^d \quad . \quad (3.102)$$

Hence the finite difference scheme proceeds as follows. After the new equilibrium deviator  $\sigma_e^d_{n+1}$  is calculated with one of the methods given in the earlier part of this section,  $\Delta\sigma_i$  is computed with (3.102). The two relaxation functions  $g^n$  and  $g^f$  can then be determined from (3.100) and (3.101). Then (3.99) is used to compute  $M$ . If  $M$  is one or less no sub-cycling is required and the final stress deviator  $\sigma^d_{n+1}$  is given directly by the first order difference analog to (3.98),

$$\sigma^d_{n+1} = \sigma^d_n + \Delta\sigma_i - \Delta\sigma_r \quad (3.103)$$

where

$$\Delta\sigma_r = 2G\Delta t^{n+1/2} g(\sigma^d_n - \sigma_e^d_n) \quad . \quad (3.104)$$

However, if  $M$  is greater than one, it is necessary to subcycle the computation  $M$  times, with the time step  $\Delta t^m = \Delta t^{n+1/2}/M$ , using equations (3.102), (3.100), (3.104), and (3.103) with  $n$  replaced by  $m$ . Note that since the equilibrium deviator  $\sigma_e^d$  depends on the time step, it will also have to be appropriately subcycled. Note also that since  $d_x^d$  and  $G$  are assumed constant

over the entire zone-cycle, they do not have to be recomputed for the subcycling.

From (3.104) it can be seen that the relaxation function  $g$  must have units of  $\text{time}^{-1}$ . The present routine contains the simple linear form for  $g$ ,

$$g = \frac{\sigma^d - \sigma_e^d}{Gt_r} \quad , \quad (3.105)$$

where  $t_r$  is the material relaxation time (input to the code as A1), and, as before,  $G$  is the shear modulus. Since the expression for  $g$  appears at only one place in the routine, more complicated functions of the overstress  $(\sigma^d - \sigma_e^d)$  can easily be incorporated. The input constants A1 through A4 are provided for this purpose. It is also possible to introduce other relaxation functions dependent separately on the stress deviator  $\sigma^d$ , the equilibrium stress deviator  $\sigma_e^d$ , or even the total plastic strain,

$$\epsilon_p = \sum_n \Delta t^{n+1/2} g^n \quad .$$

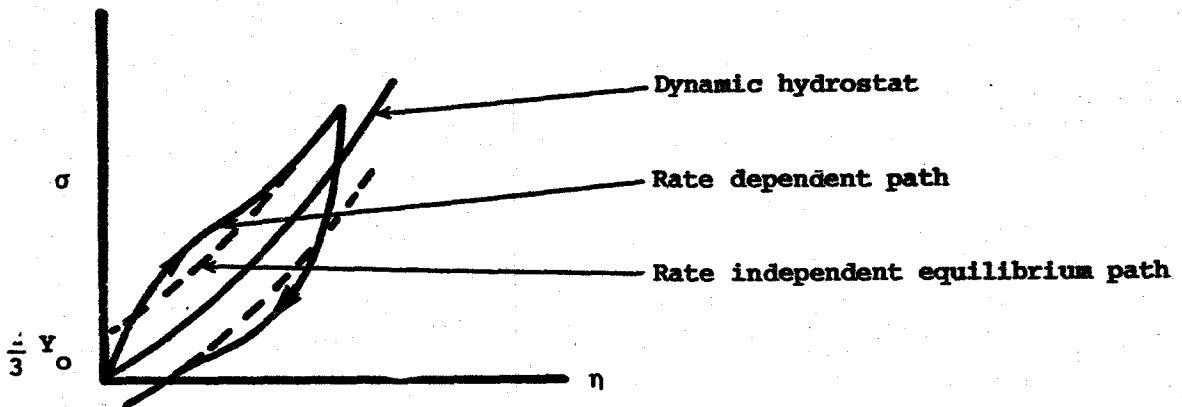
Due to the complications introduced by the subcycling, these latter types of changes are best attempted only by someone familiar with the program. Examples of the derivation and implementation of more complex relaxation functions can be found elsewhere.<sup>20,21</sup>

As with the equilibrium yield stress  $Y$ , it is probably reasonable to assume that the overstress  $(\sigma^d - \sigma_e^d)$ , or equivalently, the real viscosity, should be reduced with increasing temperature or internal energy. Accordingly, whenever the vapor equation of state (3.19) is used for a given material layer, (3.103) for the stress deviator is modified to read

$$\sigma^d{}^{n+1} = \sigma_e^d{}^{n+1} + k(\theta) [\sigma^d{}^n + \Delta\sigma_i - \Delta\sigma_r - \sigma_e^d{}^{n+1}] \quad , \quad (3.106)$$

where  $k(\mathcal{E}) > 0$  and is given by (3.96). This expression reduces to (3.103) when the internal energy density  $\mathcal{E} = 0$ , and results in a zero overstress  $(\sigma^d - \sigma_e^d)$  when  $\mathcal{E} > \mathcal{E}_m$ . Note that the equilibrium deviator  $\sigma_e^d$  may well also be a function of internal energy through (3.97) or the modified version of (3.92). It should be emphasized that the use of  $k(\mathcal{E})$  in the form (3.96) is in no way meant to infer that real materials behave in this manner; the form is included simply to introduce a qualitative temperature dependence into the constitutive relation.

To illustrate how this rate dependent stress relaxation model behaves, a typical loading and unloading cycle is qualitatively sketched below in the total stress-strain plane. Here the equilibrium behavior is characterized by an elastic-perfectly plastic model (NOY=1).



In this subroutine, several of the options available in STAT1 (Section 3.1) have been eliminated. In this context the Hugoniot can be represented only by

$$P_H = \frac{\rho_0 c_0^2 \eta^2}{(1 - s\eta)^2} ,$$

the Gruneisen parameter is restricted to

$$\Gamma = \Gamma_0 \quad \text{or} \quad \Gamma = \frac{\Gamma_0 \rho_0}{\rho} ,$$

and the shear modulus is always calculated internally as

$$G = \frac{3(1 - 2\nu)}{2(1 + \nu)} K \quad ,$$

where  $K = \rho c^2$  and is the bulk modulus. The basic equation of state is given by either (3.9) or (3.9) and (3.19). For zone-cycles on which the vapor equation (3.19) is used, the tensile stress is limited by

$$\sigma > \sigma_{\min} = \sigma_{\min}^0 k(\mathcal{E}) < 0$$

where  $k(\mathcal{E})$  is given by (3.96). The latter is analogous to equations (3.31) and (3.32) in Section 3.1.

### 3.5 STAT6 - Distended Solids

Foams or initially distended solid materials are treated according to the  $p$ - $\alpha$  model of Herrmann<sup>22,23,24</sup>. This model is hydrodynamic, where the needed reference states are described by the Hugoniot of the non-distended solid material.

A new parameter,  $\alpha$ , describing the distention of the foam as a function of pressure is defined as

$$\alpha = \frac{\mathcal{V}}{\mathcal{V}_S} = \frac{\rho_S}{\rho} \quad , \quad (3.107)$$

where  $\rho$  and  $\mathcal{V}$  are the density and specific volume of the distended material, and  $\rho_S$  and  $\mathcal{V}_S$  are the same properties of the corresponding solid material at the same pressure and energy density. Thus since the equation of state of the non-distended solid material can be written (for hydrodynamic behavior) as

$$p = f(\mathcal{V}_S, \mathcal{E}) \quad ,$$

the equation of state of the distended material becomes

$$p = f\left(\frac{\gamma}{\alpha}, \mathcal{E}\right) \quad , \quad (3.108)$$

where  $f$  is the same function in both expressions. Writing (3.108) in the form (2.7) one obtains

$$p_{j-1/2}^{n+1} = f_1\left(\frac{\gamma}{\alpha}\right)_{j-1/2}^{n+1} + f_2\left(\frac{\gamma}{\alpha}\right)_{j-1/2}^{n+1} \cdot \mathcal{E}_{j-1/2}^{n+1} \quad . \quad (3.109)$$

The distention parameter  $\alpha$  is assumed to be a function of pressure and energy density,

$$\alpha_{j-1/2}^{n+1} = g\left(p_{j-1/2}^{n+1}, \mathcal{E}_{j-1/2}^{n+1}\right) \quad . \quad (3.110)$$

The energy equation (2.46) can be rewritten as

$$\mathcal{E}_{j-1/2}^{n+1} = D p_{j-1/2}^{n+1} + B \quad , \quad (3.111)$$

where  $D$  and  $B$  are constants for a given zone at a given time cycle. These constants are  $D = \Delta\rho/2\rho^2$  computed previously via (2.44) and

$$B = \left(p_{j-1/2}^n + 2q_{j-1/2}^{n+1/2}\right) D + \Delta Q_{j-1/2}^{n+1/2} + \mathcal{E}_{j-1/2}^n \quad .$$

The desired solution is obtained by solving the three simultaneous equations (3.109), (3.110), and (3.111) for  $p_{j-1/2}^{n+1}$ ,  $\alpha_{j-1/2}^{n+1}$ , and  $\mathcal{E}_{j-1/2}^{n+1}$ . This is most easily accomplished by inserting (3.111) into (3.109) obtaining

$$p_{j-1/2}^{n+1} = \frac{f_1\left(\frac{\gamma}{\alpha}\right)_{j-1/2}^{n+1} + f_2\left(\frac{\gamma}{\alpha}\right)_{j-1/2}^{n+1} \cdot B}{1 - f_2\left(\frac{\gamma}{\alpha}\right)_{j-1/2}^{n+1} \cdot D} \quad . \quad (3.112)$$

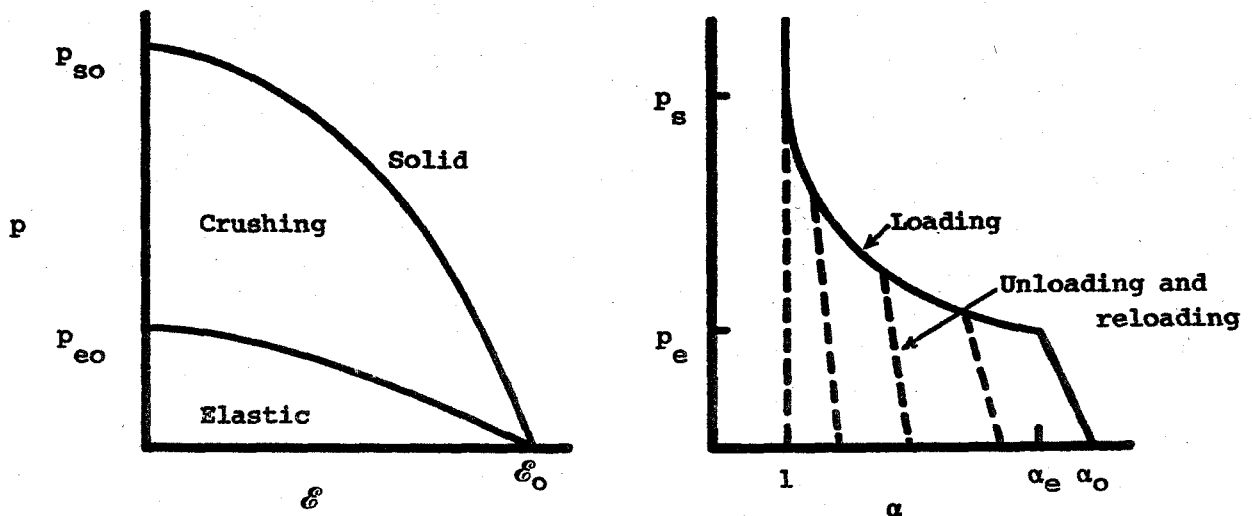


Inserting (3.111) into (3.110) yields

$$\alpha_{j-1/2}^{n+1} = g_1(p_{j-1/2}) \quad (3.113)$$

Thus (3.112) and (3.113) are two implicit expressions for  $p$  and  $\alpha$  that can be solved by iteration.<sup>22</sup> Equation (3.111) can then be employed directly to obtain the new energy.

The distention relation (3.110) is described differently in three different regions. Above some compaction pressure  $p_s(\mathcal{E})$ , all the voids are assumed to be closed, i.e. the material is solid and  $\alpha = 1$ . Below some elastic limit pressure  $p_e(\mathcal{E})$  on loading, and for all unloading and reloading to the previous peak pressure, changes in  $\alpha$  are assumed to be due to elastic deformation of the foam structure. In this region an elastic distention relation,  $\alpha = g_e(p, \mathcal{E})$ , is used. For loading between  $p_e(\mathcal{E})$  and  $p_s(\mathcal{E})$  the voids are assumed to collapse irreversibly and a crushing distention relation,  $\alpha = g_p(p, \mathcal{E})$ , is employed. Certain continuity and smoothness conditions are imposed on the three  $p$ - $\alpha$  relations at the boundaries of these regions. The qualitative behavior is diagrammed below.



The elastic distention relation used in the program given here is chosen so that the sound speed varies smoothly between  $c_e$ , the ambient value in the virgin foam at the initial distention  $\alpha_0$ , and the bulk sound speed of the reference solid material at ambient conditions,  $c_0$ . This results in the expression 23

$$\left(\frac{d\alpha}{dp}\right)_e = \frac{\alpha}{K_0} \left(1 - \frac{\alpha}{h^2(\alpha)}\right) \quad (3.114)$$

where

$$h(\alpha) = a \alpha + (1 - a) \quad (3.115)$$

Here  $a$  is a constant, evaluated from  $c_e$  and  $c_0$  by

$$a = \frac{c_e - c_0}{c_0(\alpha_0 - 1)} \quad .$$

Also,  $K_0$  in (3.114) is the ambient bulk modulus of the reference solid and is given by (3.6). Equation (3.114) is set into finite difference form directly by writing

$$\alpha^{n+1} = \alpha^n \left\{ 1 + \frac{p^{n+1} - p^n}{K_0} \left(1 - \frac{\alpha^n}{h^2(\alpha^n)}\right) \right\} \quad (3.116)$$

Note that this expression is forward differenced. Also note that in this description, the energy density does not enter the elastic distention relation explicitly. However the boundary of the elastic region  $p_e$  is taken to be energy dependent, as will be seen.

The crushing distention relation, i.e. for pressures between  $p_e$  and  $p_g$ , is the simplest form which provides the correct intercepts at the elastic and solid boundaries and also provides continuity in the first derivative at the solid boundary.<sup>22,24</sup> The expression is

$$\alpha = 1 + (\alpha_e - 1) \left\{ \frac{p_s(\mathcal{E}) - p}{p_s(\mathcal{E}) - p_e(\mathcal{E})} \right\}^2, \quad (3.117)$$

where  $\alpha_e$  is the distention ratio at  $p_e$ , but can be very closely approximated with  $\alpha_0$ . More complex functions are easily coded if required.\* On the basis that the compaction pressure  $p_s$  and the elastic limit  $p_e$  should be decreasing functions of energy, and hence of temperature,  $p_e(\mathcal{E})$  and  $p_s(\mathcal{E})$  are formulated as

$$p_e = k(\mathcal{E})p_{e0} \quad , \quad (3.118)$$

$$p_s = k(\mathcal{E})p_{s0} \quad ,$$

where the function  $k(\mathcal{E})$  decreases monotonically from 1 at  $\mathcal{E} = 0$  to 0 at some reference energy  $\mathcal{E}_0$  where the material strength is assumed to become negligible. A simple quadratic is provided,

$$k(\mathcal{E}) = 1 - (k_0 + 2) \left( \frac{\mathcal{E}}{\mathcal{E}_0} \right) + (k_0 + 1) \left( \frac{\mathcal{E}}{\mathcal{E}_0} \right)^2 \quad . \quad (3.119)$$

Here  $k_0$  is an input constant related to the rate of change of material strength with energy at  $\mathcal{E} = \mathcal{E}_0$ . Generally  $k_0$  should lie in the range  $-2 < k_0 < -1$ . This energy dependence can be defeated by setting  $k(\mathcal{E}) = \text{const.} = 1$ . This is accomplished in the routine by inputting  $\mathcal{E}_0 = 0$ .

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\*The following alternate forms for (3.117), without energy dependence, are available in a routine described in Reference 23:

Quadratic:  $\alpha = \alpha_0 + \alpha_1 p + \alpha_2 p^2$

Cubic:  $\alpha = \alpha_0 + \alpha_1 p + \alpha_2 p^2 + \alpha_3 p^3$

Exponential:  $\alpha = 1 + (\alpha_0 - 1) \exp[a(p - p_e)]$

Only the functions  $f_1$  and  $f_2$  remain to be specified. Remembering from (3.109) that these functions are expressed in terms of the volume of the reference solid  $\mathcal{V}_s = \mathcal{V}/\alpha$ , it is appropriate to use the same description as is employed in Section 3.1. Accordingly, whenever  $\mathcal{V}_s < \mathcal{V}_{s0}$ , where  $\mathcal{V}_{s0}$  is the ambient specific volume of the reference solid,  $f_1$  and  $f_2$  are given by (3.10) and (3.11) respectively. They can be rewritten in terms of  $\mathcal{V}_s$  as

$$f_1 = p_H \left( 1 - \frac{\Gamma}{2\mathcal{V}_s} (\mathcal{V}_{s0} - \mathcal{V}_s) \right) , \quad (3.120)$$

$$f_2 = \frac{\Gamma}{\mathcal{V}_s} ,$$

where  $p_H$  is expressed, as before, as either (3.2) or (3.5) with  $\rho$  replaced by  $\rho_s$ , and  $\Gamma$  given by (3.8). When  $\mathcal{V}_s > \mathcal{V}_{s0}$ , the vapor equation of state (3.19) can be used in an analogous manner, if desired. Choosing this option results in the limitation of any calculated tensions in a manner identical with that described in conjunction with equations (3.31) and (3.32). If the form (3.19) is not chosen, (3.120) is extrapolated into this region<sup>†</sup>. Note that the material constants relating to the equation of state of the reference solid are input to the code in a manner identical with those for normal solids.

The sound speed is obtained from (2.13). Since  $\alpha$ , given by (3.110) is an implicit function of volume, care is required in carrying out the differentiation of  $f_1$  and  $f_2$ . After some manipulation the result is

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<sup>†</sup>For problems involving high internal energies, this latter approach can lead to gross errors.

$$c^2 = -\gamma^2 \left( \frac{\partial p}{\partial \gamma} \right)_s = - \frac{\gamma^2 \left( \frac{df_1}{d\gamma_s} + \mathcal{E} \frac{df_2}{d\gamma_s} - pf_2 \right)}{\left( \frac{df_1}{d\gamma_s} + \mathcal{E} \frac{df_2}{d\gamma_s} - pf_2 \right) \frac{\gamma}{\alpha} \frac{d\alpha}{dp} + \alpha} \quad (3.121)$$

where the appropriate value for  $\frac{d\alpha}{dp}$  is obtained from (3.114).

The numerical iteration technique, as well as the logic required to determine which region the material is in, are described elsewhere 22-24. Since the model is hydrodynamic, only one stress variable needs to be carried by the program. Thus the pressure is stored directly as  $\sigma$ , and  $\alpha$  is stored in the location normally reserved for pressure.

One additional point should be mentioned in regard to this constitutive model. Due to the large total volume change which occurs when a porous material is subjected to relatively modest pressures, it is necessary to include extra artificial viscosity to maintain calculational stability. The form used is the same as that described in Section 2.5. The coefficients controlling this viscosity,  $B_3$  and  $B_4$ , are analogous to  $B_1$  and  $B_2$ , and are input as constants for this material model. In the absence of a reasonable theoretical treatment of artificial viscosity in foams, values of  $B_3$  and  $B_4$  about an order of magnitude greater than the normal viscosity coefficients seem to be appropriate. Since this large compression occurs only during the crushing of the foam, the extra viscosity is added only when  $\alpha > 1$  and the material is actually compressing. Note that when this viscosity is used the time step for stability must be suitably modified (Section 2.6).

### 3.6 Fracture Options

The formation of tensile stresses, due to interacting relief waves in planar geometries, or divergent waves in cylindrical or spherical geometries, often leads to fracture of the material. This material fracture is treated in WONDY by three options, two of which result in the appearance of explicit gaps, either internally, or between layers. Alternatives, in the form of continuum damage failure models have been successfully implemented in WONDY, and are noted in Appendix B. In the current version of WONDY, each material layer and the interface between each layer utilize independent sets of failure criteria.

The first available criterion (FCRIT = 1) involves suppressing fracture and allowing the material to stretch at some energy dependent maximum tensile stress in a manner identical with that described in Section 3.1 in conjunction with the vapor equation of state. Thus the stress is limited by

$$\sigma_{j-1/2}^{n+1} > \sigma_{\text{MIN}} \quad , \quad (3.122)$$

where

$$\sigma_{\text{MIN}} = \sigma_{\text{MIN}}^0 (1 - \mathcal{E}/\mathcal{E}_{\text{MELT}}) < 0 \quad , \quad (3.123)$$

and  $\sigma_{\text{MIN}}^0$  and  $\mathcal{E}_{\text{MELT}}$  are input quantities. Here  $\sigma$  is negative in tension thus  $\sigma_{\text{MIN}}^0$  should be input to the code as a negative quantity. Since this option actually adjusts the stress existing in an interior zone, applying this criterion to a material interface has no physical significance other than to suppress fracture at this point. So that the energy balance can be maintained, this adjustment in stress, if required, is performed in the appropriate equation of state subroutine. (All the other fracture criteria

and calculations pertaining to them are located in subroutine JLOOP.) Note that this option is used for all zones in the plate for which it is called, where the stress limitation described in Section 3.1 is employed only for zones utilizing the vapor equation of state.

The second fracture criterion (FCRIT = 2) allows two zones to separate whenever the interpolated stress at their common boundary is less than the input quantity  $\sigma_{\text{FRACT}}$ . This parameter should be negative and represents the spall strength of the material or the strength of the bond at a material interface.

The third available option\* (FCRIT = 3) allows fractures when a time-integrated quantity exceeds some critical level.<sup>25</sup> Whenever the tension, interpolated to a zone boundary, exceeds a negative input value  $\sigma_0$  (possibly the tensile elastic limit for the material in question), the integral quantity

$$K = \int_0^t (\sigma - \sigma_0)^\lambda dt \quad , \quad \text{for } \sigma < \sigma_0 < 0 \quad , \quad (3.124)$$

is calculated via

$$K_j^n = K_j^{n-1} + \left( -1 \left( \frac{\sigma_{j-1/2}^n + \sigma_{j+1/2}^n}{2} - \sigma_0 \right) \right)^\lambda \Delta t^{n-1/2} \quad , \quad (3.125)$$

for

$$\frac{\sigma_{j-1/2}^n + \sigma_{j+1/2}^n}{2} < \sigma_0 < 0 \quad .$$

---

\* The input quantity NVAR must be set equal to or greater than 12 to use this option.

If the inequality in (3.125) is not met,

$$K_j^n = K_j^{n-1} \quad .$$

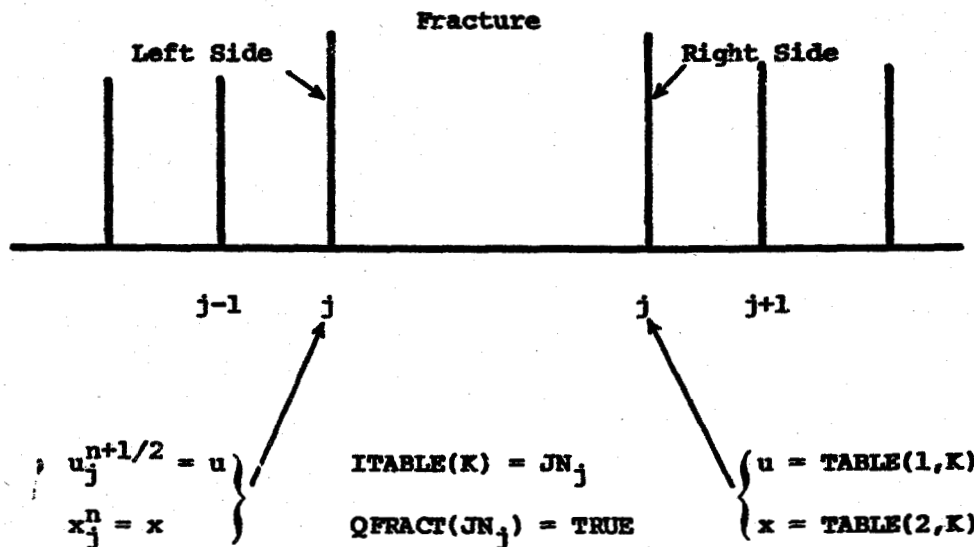
Whenever  $K_j^n$  for any zone boundary for which this option is being used exceeds  $K_{MAX}$ , two free surfaces are formed at this position, thus creating a fracture or spall plane. This option requires the input quantities  $K_{MAX}$ ,  $\sigma_0$ , and  $\lambda$ .

When employing either of the latter two criteria, and a fracture occurs, the calculation proceeds as if there were two free surfaces at this point. If subsequently these surfaces collide, then the material is considered to rejoin, and the ordinary equations appropriate for an interior zone are used. Subsequent fractures at a zone boundary that has thus fractured and rejoined are treated according to the second criterion above, irrespective of the option used in creating the initial fracture. In this case however, the fracture stress is a different input quantity, SIGSEP, a single value for the entire problem. SIGSEP is generally set to a small negative value to prevent separation on small spurious oscillations about zero stress which occasionally occur in the solution. Note that if there are voids or gaps (see Section 4.1) in the initial problem configuration, they are treated as if a fracture had occurred at some time prior to the start of the calculation.

The logic for this spall and join routine is accomplished through the use of two arrays of logical indicators, QFRACT and PFRACT. At the beginning of a calculation both of these arrays are initialized to FALSE. When either of the above fracture criteria are met, the values of QFRACT and PFRACT for that zone boundary are set to TRUE. This signals to the code that two free surfaces occur at that zone boundary. It is now necessary to



store extra values for the position  $x$  and velocity  $u$ . The values of  $x$  and  $u$  for the left side of the fracture are stored in the locations normally reserved for that zone, and the values of  $x$  and  $u$  for the right side of the fracture are stored in the separate array **TABLE** according to the scheme given in the accompanying sketch. In addition, the storage locations  $JN_j$  (see Chapter 7) of the zones currently separated are stored in the array **ITABLE**.



During subsequent cycles at a zone boundary where a fracture has occurred, a test is made to see if the value of  $x$  at the left side exceeds the value of  $x$  at the right side of the fracture. If it does, the fractured surfaces have come together during that cycle. The values of  $x$  and  $u$  for the rejoined zone boundary are then calculated from the previous left and right hand values via

$$u_j^{n+1/2} = \frac{z_L u_L + z_R u_R}{z_L + z_R} \quad (3.126)$$

where

$$z_L = \rho_{j-1/2}^n c_{j-1/2}^n, \quad z_R = \rho_{j+1/2}^n c_{j+1/2}^n$$

and

$$x_j^{n+1} = \frac{\rho_{j-1/2}^n x_L^n + \rho_{j+1/2}^n x_R^n}{\rho_{j-1/2}^n + \rho_{j+1/2}^n} \quad (3.127)$$

At the same time QFRACT is set back to FALSE and PFRACT remains TRUE. A value of FALSE for QFRACT signals that the zone is henceforth to be treated as an ordinary interior zone, while PFRACT = TRUE indicates that subsequent tests for fracture are to be made with the second criterion with the fracture stress replaced by SIGSEP.

If it is desired to suppress these fracture calculations, the records specifying the data for the fracture criteria can be left out of the input instructions. The code then defaults the appropriate input parameters to FCRT = 2. and  $\sigma_{\text{FRACT}} = -1.0 \times 10^{100}$ .

Whenever a fracture occurs or fractured surfaces collide, appropriate messages are printed on the standard output medium giving the cycle, time, and zone number. A maximum of 50 current fractures are allowed. If this number is exceeded, an appropriate error message is printed on the standard output medium and execution is terminated.

#### 4. INITIAL AND BOUNDARY CONDITIONS

The analysis of wave propagation is an initial value problem, and requires that the variables in the problem be defined at time zero, and that appropriate boundary conditions be imposed.

##### 4.1 Initial Variable Definition

Initial conditions are specified by assigning values to all principal quantities for all zones. This is accomplished in subroutine GENERAT. The variables of interest here are listed in the following table.

Report Symbol	Fortran Symbol	Definition
c	C	Sound Speed
$\mathcal{E}$	E	Energy per unit mass
m	M	Mass
p	P	Pressure
q	Q	Artificial Viscosity
$\rho$	R	Density
$\sigma$	S	Normal Stress
u	U	Velocity
x	X	Position
$\phi$	Z	Difference Between Principal Stresses ( $\sigma_x - \sigma_y$ )

The artificial viscosity q is always initialized to zero for all zones. The sound speed c, is initialized to the value input as the second equation

of state constant for each material layer and corresponds to the ambient sound speed of the uncompressed material.

The quantities  $m$  and  $x$  are derived from other input data. The initial positions of the zone boundaries  $x_j$  are calculated from the initial zoning described in Section 4.2. Then, based on the geometry,  $m$  is computed via (2.43). This latter quantity is used in the mass equation (2.42) and is changed only by the REZONE routine.

The remaining variables,  $\mathcal{E}$ ,  $p$ ,  $\rho$ ,  $\sigma$ ,  $u$ , and  $\phi$ , are input as initial conditions and are usually assumed to be constant for each material layer, although the user may introduce more complex initial states if he desires (see Section 5.1). The values of all of these quantities, with the exception of the velocity, must be fully compatible with the equation of state model being employed. Thus to start a calculation with a material initially compressed to some pressure  $p$ , values of  $\mathcal{E}$ ,  $\rho$ ,  $\sigma$ , and  $\phi$  appropriate for this compression from the ambient uncompressed state must be used. Note that in this case the initial density  $\rho$  will be different from the reference density  $\rho_0$ , specified as the first equation of state constant. The latter quantity is generally the reference density of the material in the uncompressed ambient condition. The ability to specify initial conditions in this manner allows the user to program equations of state which utilize non-zero reference conditions (e.g., The Ideal Gas Law, Section 3.3).

The initialization of velocity is a little more complicated since the velocity refers to the zone boundaries. The velocities of all boundaries within a given material layer are initialized to the input quantity UZERO, while the boundary between this layer and the next is set to UZEROI. The latter quantity is used to minimize starting transients when a calculation is initiated with a velocity discontinuity (e.g., flying plate or plate

impact problem). As an example consider plate A to impact plate B, initially at rest, with a velocity  $u_A$ . The initial velocities for plates A and B should be set to  $u_A$  and zero respectively. The initial velocity of the interface between the two materials  $u_i$  (the problem is assumed to start at the instant of impact) should be calculated from the shock impedances  $Z$  of the two materials via

$$u_i = \frac{Z_A u_A + Z_B u_B}{Z_A + Z_B} \quad , \quad (4.1)$$

where for this example  $u_B$  is zero and  $Z$  can be approximated by the acoustic impedance,

$$Z = \rho_0 c_0 \quad . \quad (4.2)$$

For symmetric impact (materials A and B identical),  $u_i$  is equal to the average of the two velocities, in this case  $u_A/2$ .

Some difficulties may arise in problems where the difference in initial velocities of adjacent layers is very large. Since the time step is determined by the stability criterion (Section 2.6), on the first cycle the time increment essentially reduces to

$$\Delta t = \frac{\Delta x}{c_0} \quad .$$

Thus when the impact velocity approaches the sound speed  $c_0$ , it may happen that the interface moves beyond one of its neighboring zone boundaries, leading to a negative density in that zone. To correct this situation it is necessary to choose an initial time increment small enough so that the zones on either side of the interface do no change volume by more than a few percent during this time step. This value for  $\Delta t$  is entered in the

input as DELT (4) (see Section 2.6). A similar problem is likely to occur when large pressure discontinuities are introduced as initial conditions. Again reducing the initial time step usually corrects the difficulty.

#### 4.2 Initial Zoning

The initial spatial mesh is defined independently for each material layer or plate by four input parameters each. They are THKNS, the plate thickness; NOMESH, the number of zones in the plate; DELTAX, the size of the first zone in the plate; and DELTAX1, the size of the last zone in the plate. These input parameters are used to determine the coefficients of a cubic equation which specifies the mesh for the entire plate. Thus the position  $x_i$  of the right-hand boundary of the  $i^{\text{th}}$  zone in the plate is given by

$$x_i = x_L + P_1 i + P_2 i^2 + P_3 i^3, \quad (4.3)$$

$$(i = 1, 2, \dots, \text{NOMESH})$$

where the coefficients are given by

$$P_3 = \frac{\text{NOMESH}(\text{DELTA}X + \text{DELTA}X1) - 2 \cdot \text{THKNS}}{\text{NOMESH}(\text{NOMESH} - 1)(\text{NOMESH} - 2)},$$

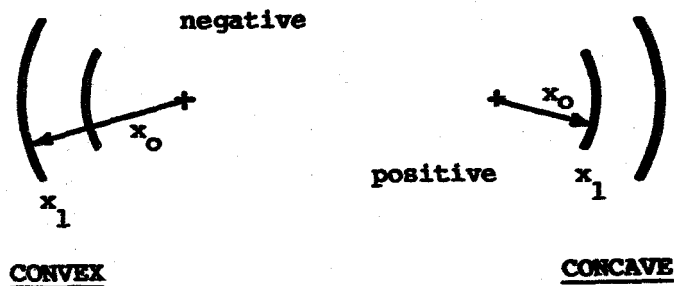
$$P_2 = \frac{\text{DELTA}X1 - \text{DELTA}X}{2(\text{NOMESH} - 1)} - 1.5 \cdot \text{NOMESH} \cdot P_3,$$

$$P_1 = \text{DELTA}X - P_2 - P_3,$$

and  $x_L$  is the position of the left-hand boundary of the plate. The sizes of the individual zones, say the  $k^{\text{th}}$  zone in the plate, are therefore

$$\Delta x_k = x_k - x_{k-1}. \quad (4.4)$$

The left-hand boundary  $x_L$  for the first plate is set to the input parameter XZERO. In slab geometry,  $\alpha = 1$ , the choice of this position does not affect the results and should normally be set to zero to minimize round-off error. However, in the cylindrical and spherical geometries,  $\alpha = 2$  and 3, XZERO determines the radius of the left-hand boundary. If this boundary is to be convex, then XZERO must be negative, while if the boundary is to be concave, XZERO must be positive. To take full advantage of the activity test, the former situation should be employed for problems in which the motion originates at the outside of the configuration, while the concave geometry should be used for problems in which the motion starts interior to the cylinder or sphere. For subsequent plates after the first,  $x_L$  is set equal to the position of the right-hand boundary of the previous plate, that is  $x_{NOMESH}$  for the previous plate.



It is also possible to introduce a gap or void between successive material layers. This situation is treated as if a fracture had occurred at some time prior to the start of the calculation. The size of the gap, XGAP, is input for each material layer and refers to the interface to the right of that layer. Thus the value of XGAP for the last material has no significance. If XGAP is read as a non-zero quantity, the indicators and variables necessary to signal a fracture condition (QFRACT, ITABLE, and TABLE) are appropriately initialized in subroutine GENERAT, and  $x_L$  for

the next plate is increased by the size of the gap, i.e.,

$$x_L = x_L + XGAP \quad . \quad (4.5)$$

Otherwise initialization proceeds in a normal fashion.

Note that in this scheme the left-hand boundary of the problem is assigned an index  $j = 1$ . Thus the index of the right-hand boundary of any plate is one greater than the sum of the numbers of zones through that plate. For example, a problem consisting of two plates with 100 and 200 zones respectively will have the right-hand boundary of the first plate at  $j = 101$  and the right-hand boundary of the second plate at  $j = 301$ .

For given values of plate thickness, initial zone size and last zone size, the relative zone size distribution within the plate is dependent upon the specified value of NOMESH. The parameters  $\delta x_1$  and  $\delta x_2$  can be defined as

$$\begin{aligned} \delta x_1 &= \text{Min}(\text{DELTA}X, \text{DELTA}X1) \quad , \\ \delta x_2 &= \text{Max}(\text{DELTA}X, \text{DELTA}X1) \quad . \end{aligned}$$

Then choosing NOMESH such that

$$\text{NOMESH} < \frac{3 \cdot \text{THKNS}}{\delta x_1 + 2 \cdot \delta x_2} \quad (4.6)$$

produces a zone size distribution with larger mid-plate zones, i.e., resolution is coarser in the interior of the plate than at the boundaries.

Setting NOMESH such that

$$\text{NOMESH} > \frac{3 \cdot \text{THKNS}}{2 \cdot \delta x_1 + \delta x_2} \quad (4.7)$$



yields interior zones which are smaller than those near the boundaries and consequently the interior resolution is finer. For values of NOMESH between these limits, i.e.,

$$\frac{3 \cdot \text{THKNS}}{\delta x_1 + 2 \cdot \delta x_2} < \text{NOMESH} < \frac{3 \cdot \text{THKNS}}{2 \cdot \delta x_1 + \delta x_2} ,$$

the zone sizes in the plate are monotone increasing or decreasing as  $\text{DELTA}X < \text{DELTA}X_1$  or  $\text{DELTA}X > \text{DELTA}X_1$  respectively. It should be noted that whenever

$$\text{NOMESH} > \frac{3 \cdot \text{THKNS}}{\delta x_2} , \quad (4.8)$$

zones of negative size will occur. The code tests for this condition, and if (4.8) is satisfied, NOMESH is adjusted to

$$\text{NOMESH} = \frac{3 \cdot \text{THKNS}}{\delta x_2} .$$

Several general "rules of thumb" regarding zoning should be mentioned at this point. First, no material layer should have fewer than about 10 zones, i.e.,

$$\text{NOMESH} > 10 . \quad (4.9)$$

Second, the zone sizes  $\Delta x_j$  should be small enough so that any non-constant initial condition (e.g., deposited energy density) varies only by small amounts from zone to zone. To put this in a quantitative form, if  $\Psi$  is any non-constant initial condition variable for a given plate, then

$$.9 < \left| \frac{\Psi_j}{\Psi_{j+1}} \right| < 1.1 . \quad (4.10)$$

Applying this last rule along with (4.9) to the zone sizes  $\Delta x_j$  yields approximately

$$\text{DELTA}X < \frac{\text{THKNS}}{10} ,$$

(4.11)

$$\text{DELTA}X1 < \frac{\text{THKNS}}{10} .$$

A final rule regarding the sizes of the zones on either side of a material interface can be stated

$$\frac{\Delta x}{c} \Big|_A \approx \frac{\Delta x}{c} \Big|_B ,$$

(4.12)

where A and B refer to the two materials and the values of c are the sound speeds for the two materials. This is equivalent to saying that the transit times for the two zones on either side of the interface should be approximately equal. Many other criteria for matching of zone sizes across interfaces have been suggested, but it has been shown<sup>26</sup> that violating (4.12) introduces permanent density errors into the solution.

Many problems will arise for which one or more of the rules (4.9) through (4.12) cannot be satisfied. This will not necessarily invalidate the calculational results. However to maintain accuracy, it is suggested that these rules be followed as closely as possible.

### 4.3 Boundary Conditions

Boundary conditions of four different types are provided at both the left and right boundaries of the problem. They are 1) a fixed boundary or reflection plane, 2) a fixed minimum position at the left-hand boundary or a fixed maximum position at the right-hand boundary, 3) a free surface,

and 4) an arbitrary boundary condition programmed in subroutine BOUNDRY, which is generally provided by the user. The indicators LHBT and RHBT are used to choose the desired boundary conditions from the above options for the left and right hand boundaries respectively. Note that the boundary condition affects only the calculation of the acceleration, velocity, and position at the boundary in equations (2.39), (2.40), and (2.41).

For a type 1 or fixed boundary, computation of the acceleration and velocity are omitted and the position is left unchanged.

The type 2 boundary condition is a special case of a fixed boundary. It can be used to describe such problems as the collapse on the center of a hollow sphere or a cylindrical tube, or the impact of material such as explosive detonation products on a rigid containing wall which is initially spaced some distance from the material. The calculation proceeds as follows. Each boundary is calculated as if it were a free surface, and then tested to see if it has gone beyond its "fixed" position. Thus, on the left, if  $x_j^{n+1} < X_{MIN}$ , it is set to  $X_{MIN}$ , and on the right, if  $x_{LMAX}^{n+1} > X_{MAX}$ , it is set to  $X_{MAX}$ . At the same time that the positions are fixed the appropriate velocities are set to zero. For a type 2 boundary  $X_{MIN}$  and  $X_{MAX}$  are input to the program as LBCON and RBCON respectively.

To treat a type 3 boundary or a free surface, use is made of the virtual zones outside the boundary. The values of  $\sigma$ ,  $\phi$ , and  $\rho$  are initialized to zero in these zones and their values are not changed during the problem. The application of equations (2.39), (2.40), and (2.41) at the boundaries then leads to the correct acceleration, velocity and position for the free surface.

A type 4 boundary condition results in a call to subroutine BOUNDRY which is used to insert appropriate parameters in the virtual zones

outside the boundaries. By suitably programming this routine arbitrary boundary conditions can be employed with WONDY. As an example of this type of boundary condition, a BOUNDRY subroutine is included which applies a time varying load on either boundary of the form

$$\sigma = \sigma_0 + \sigma_1 \exp(-kt) \quad , \quad (4.13)$$

where  $\sigma_0$ ,  $\sigma_1$ , and  $k$  are constants. If  $\sigma_0 = 0$ , an exponentially decaying load is applied, and if  $\sigma_1 = 0$ , a step function load is applied. Note that the time constant  $k$  should be input to the code as a positive quantity. The constants necessary to use this option are input via the array ADDATA (see Section 5.1). The values of  $\sigma_0$ ,  $\sigma_1$ , and  $k$  for the left hand boundary are the 8th, 9th, and 10th quantities in ADDATA respectively, while the 11th, 12th and 13th quantities are the same parameters for the right hand boundary. The input constants LBCON and RBCON are also available for use with user programmed versions of BOUNDRY.

When an applied stress boundary condition is employed, the work done at the boundary should be added to the total energy when performing energy checks (see Section 5.3). This work is simply the applied force times the distance moved by the boundary, i.e., in finite difference form,

$$w^{n+1/2} = \frac{A}{2} (\sigma^{n+1} + \sigma^n)(x^{n+1} - x^n) \quad , \quad (4.14)$$

where the appropriate spatial index for the virtual zones is used for both the stress  $\sigma$  and the position  $x$ . Here  $A$  is the area over which the stress is applied,

$$A = 2 \pi (\alpha - 1) \left( \frac{x^{n+1} + x^n}{2} \right)^{\alpha-1} \quad ,$$

for cylindrical and spherical geometries, and  $A = 1$  for rectangular geometry.

#### 4.4 MORSTOR (Energy Deposition)

The subroutine GENERAT initializes the ten principal storage arrays associated with the problem as described in the Section 4.1. However it is often desired to run a problem with more than these ten basic variables. This is accomplished (see Section 7.1) by inputting the number of variables, NVAR, as a number greater than ten. If these extra variables require a zero initial value no problem arises since the entire STORE array is set to zero at the beginning of WONDY, before any other initialization takes place. On the other hand, extra variables requiring non-zero initial values need special treatment. The subroutine MORSTOR is provided for this purpose. There are many possible uses to which these extra quantities can be put, so in general, this subroutine should be supplied by the individual user.

One specific use for this subroutine involves initiating a calculation with the addition of energy at a constant rate for a given time duration. More complex subroutines which determine the energy deposition due to various specific kinds of external energy sources may be programmed as required.

The version of MORSTOR included here accepts as input a series of up to 100 pairs of numbers where the first of a given pair is an Eulerian coordinate position on the initial zone structure, and the second is the total energy per unit mass associated with this point. The absolute magnitude of the energies are immaterial since the routine normalizes them to a given total absorbed energy fluence. Thus this routine can be used to input to WONDY any arbitrary energy density profile desired.

The normalization is accomplished by taking an integral, across the Lagrangian grid, of the curve defined by the arbitrary input profile. This results in a quantity with dimensions of energy per unit area. The

input values for energy density are then scaled so that this integral equals an input quantity, ENOR.\* (If ENOR is set to zero, the normalization is bypassed and the energy densities are used exactly as input.) The total deposited energy densities for each zone are then linearly interpolated or extrapolated from these normalized values and stored as the array DATB (1) which is in turn overlaid with the main STORE array (see Section 6.2). Since difficulties might arise if energy appears in gaps that might exist between individual plates, care must be taken to insure that the position of the points representing the energy density profile are actually located within the material in the problem. In addition, at least two points should be provided for each material layer, even though the layer may have no energy deposited in it.

The energy is deposited at a constant rate for a total time TDEP; after this time no further energy is added. The actual energy addition is accomplished in the equation of state subroutine (note that only STAT1, STAT4, and STAT6 contain this option). A series of statements at the beginning of the subroutine calculate, at the beginning of a cycle, the fraction of the total energy, DEP, deposited in that cycle via

$$DEP = \frac{\Delta t^{n+1/2}}{TDEP} \quad . \quad (4.15)$$

Then the energy added to a particular zone in that cycle,  $\Delta Q_{j-1/2}$ , is the product of DEP and DATB(1) for that zone. The value is then added to the

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\*As with the rest of the code, this subroutine can be used with any consistent set of units. To conform with conventional usage, it is often desirable to employ units which do not meet this criterion. The constant ECON, defined on record number MOR 22, is provided for this purpose. Adjusting ECON appropriately will allow ENOR to appear in any units desired.

appropriate total energies so that the energy checks can be correctly performed (see Section 5.3). If the way in which the energy is to be added is changed (e.g. non-constant rate) these statements in the equation of state subroutine must be appropriately altered.

To best approximate the generally continuous energy deposition phenomena, it is important to keep the amount of energy added on any given time cycle small. Since the time step for stability may be too large for this purpose,  $\Delta t$  is constrained to be no larger than one hundredth of the deposition time TDEP, while deposition is occurring. Also, to preserve accuracy, it is suggested that the rezone time (see Section 6.1) always be set greater than TDEP.

#### 4.5 Activity Test

In many problems the motion initiates at or near the left-hand boundary. For a significant portion of the calculation, a large number of zones may be inactive. In order to save computer time, an activity test is incorporated. A quantity LACT is provided in the input. The computation is performed normally from  $j = 1$  to  $j = \text{LACT}$ . If the value of  $\sigma$  in the last zone to be computed, i.e.,  $j = \text{LACT}$ , is less than a quantity SIGACT which is also an input variable, the computation is interrupted and advanced to the next time cycle. However, if  $\sigma$  is greater than SIGACT, then LACT is advanced by one and the computation is advanced normally.

Thus, zones are activated as needed as a pulse propagates from left to right. The value of LACT should be specified in the input to be greater than any zone number at which motion is expected in the first few cycles. To give an example of its use, consider a plate impact problem in which the first layer has a positive velocity, while the second layer has a zero

velocity. Then LACT is given an integer value greater than the interface zone number by, say 5. As the shock initiated at the interface moves to the right into the second layer, zones are progressively activated just ahead of the shock.

Note that zones are activated from left to right. Under no circumstances must LACT be less than jMAX unless it is absolutely certain that no disturbances originate in the non-active region. The value of SIGACT should be a little greater than possible roundoff or spurious oscillations.

Since the rezoning scheme employed in WONDY IV can very effectively reduce the number of zones being used in inactive regions of the problem, it is important to use the activity test mainly when the REZONE routine is bypassed. Accordingly, if LACT is left blank in the input, the code defaults it to jMAX.



In the actual process of operating the WONDY code, various means are available for both the input of information to define the problem and materials, and to obtain computed results. In addition, a series of checks are made on the computed state of the problem to give the user the status of some important integrated quantities, such as total energy, momentum, etc.

### 5.1 Input (Restarts)

WONDY V contains a semi-free form for input in the sense that since the individual input records are numbered, their order in the input file is immaterial. In addition, a great number of input quantities are defaulted to "standard" values if they are left blank. Specific input instructions are given in detail in Appendix E. Most input quantities for which the meaning and use are not obvious have been discussed in earlier sections of this report. (For example, all the equation of state parameters are described in Chapter 3.) However there are a few points which do require additional comment.

One method for starting a calculation involves using the RESTART feature. The second output option described in Section 5.2 writes a binary tape on logical unit 20 from which a calculation can be reinitiated. To perform a normal restart the previously prepared dump tape 20 should be equipped to unit 25. With the exception of the variables JTAPE and NSTART the input remains the same as with the original run. A non-zero value for NSTART indicates that the current calculation is a restart run. The value of NSTART is the cycle number at which the restart is to take place and JTAPE is the number of material layers to be read from the binary tape.

Possible values for NSTART and JTAPE are obtained from the dump message printed by the original run for any cycle on which a binary dump is performed.

Under certain conditions it is possible to change the problem slightly when restarting. To give an example of how this may be done, consider a plate impact problem in which the first material layer has a positive initial velocity while the second plate has a zero velocity. The problem is run to a stage where the shock wave originating at the impact interface has not yet reached the right-hand boundary of the second material. At this stage a number of zones adjacent to this right-hand boundary have not yet undergone any motion and are uncompressed. If a dump is performed at this stage and the problem is restarted, it is possible to add more or change material layers beyond the original right-hand boundary of the second material by suitably altering the input data. This feature is particularly useful when a parametric study is involved in which, say, a third material layer is to be added and the effects of the thickness or composition of this layer are to be investigated. It is unnecessary to rerun the unchanged first part of the problem. Very great care is necessary to insure that changes are made only in or beyond undisturbed material layers. Under no circumstances may changes be made in plates that have already undergone motion or compression.

It is also possible to use the restart feature to introduce complex initial conditions not provided for in this version of the code. A binary tape may be prepared by a suitable program, written for the purpose, to initialize the variables contained in the STORE array and in the other arrays on the dump tape. The necessary sequence and format for the variables

can be determined by examining the output statements contained in subroutine DUMP. Great care must be exercised to insure that the data contained on the tape are fully compatible with the input data as well as with the equations of state that are being used. If these values are not completely compatible, totally false results will likely result.

One additional input quantity, the array ADDATA, has been provided to communicate with the user-programmed portions of the code such as the output routines or the BOUNDARY subroutine. Up to 100 words can be placed in this normally vacant array. The position of quantities to be read into ADDATA via the input is also an input quantity, thus variables can easily be placed in any location within this array.

## 5.2 Output

WONDY V contains four basic output routines, each of which has its own general purpose. Their use is based on real problem time and controlled independently by a series of times ( $S_i$  in the input) and time increments ( $\Delta_i$  in the input). A given output option is first called at time  $S_1$ , with subsequent calls at intervals of  $\Delta_1$  until time  $S_2$ . Time intervals of  $\Delta_2$  are used between times  $S_2$  and  $S_3$  and so forth. Setting  $\Delta_1 = 0$  calls the appropriate output every cycle between times  $S_i$  and  $S_{i+1}$ . Leaving out all the output time parameters results in a given output routine being bypassed for the entire run. Setting  $s_1$  to some value greater than the problem run time has the same effect.

The first output option, the standard edit, lists ten variables for all active zones at the appropriate times and a summary of the current momentum and energy sums and errors. The edit contains the zone number  $L$ ,

$x_L^n, u_L^n, \rho_{L-1/2}^n, \sigma_{L-1/2}^n, p_{L-1/2}^n, \phi_{L-1/2}^n, \epsilon_{L-1/2}^n, Q_{L-1/2}^n, C_{L-1/2}^n$ , and

$M_{L-1/2}^n$ . The latter six may be replaced by other variables if the user so specifies in the input stream, Records 21 and 22. The coding for this edit is located in the main program. For this output option the data is written on file TAPF21 rather than being printed directly. In normal use this file is equivalenced to the standard output medium, but if large amounts of output are required this equivalencing can be omitted and the data can thus be stored directly on magnetic tape.

Normally the first of these standard edits will be given at the appropriate  $S_1$ ; however, if it is desired to check the initial conditions or zoning, the input parameter NOL can be set to a non-zero value. This will initiate a standard edit before any calculations have taken place. Note that setting  $S_1$  to zero yields a cycle one edit rather than a cycle zero edit. This "zero cycle" or initial edit is given automatically on a restart calculation. The edit of the last cycle before termination of the problem will be given if the parameter NUL is set to a non-zero value. This value of NUL causes WONDY to terminate NUL seconds before allotted computer time is reached to allow for subsequent operations, such as plotting, etc. Just prior to the final edit, a message containing the average rate of computation is printed (zone cycles/hour).

The second output option is the binary dump and is located in subroutine DUMP. This option is employed whenever a restart calculation is desired. This could result from long calculations requiring examination at intermediate stages or parameter variation studies such as described above. For cycles meeting the appropriate time criteria the dump is written on file TAPE20. In addition, a message is printed on the standard output giving the time, cycle number and number of material layers written on the file. The latter two quantities will be needed to perform the restart.

The remaining two output options are contained in subroutines OUTN and OUTPL: Since it is difficult to anticipate the various types of special output which may be required, these routines are generally supplied by the individual users. Note that the array ADDATA can be used to communicate with these routines.

For cycles meeting the appropriate time criteria each routine is called at the end of the cycle and thus has access to all zonal variables stored in the main STORE array. These routines can be used to prepare tapes for subsequent calculation or plotting, or to calculate derived quantities such as momenta or non-normal stresses not carried in the STORE array. For example, the three principal stresses

$$\sigma_x = \sigma_{j-1/2}^{n+1} ,$$

$$\sigma_y = \sigma_{j-1/2}^{n+1} + \phi_{j-1/2}^{n+1} , \quad (5.1)$$

and 
$$\sigma_z = 3p_{j-1/2}^{n+1} - 2\sigma_{j-1/2}^{n+1} - \phi_{j-1/2}^{n+1} ,$$

can easily be calculated and printed with either of the two routines (here all stresses are taken positive in compression).

The version of OUTN included here will output the principal zonal variables for the zones on either side of the material interfaces specified by ADDATA(1) through ADDATA(7). Note that the left hand boundary is considered to be interface number 1. The subroutine OUTPL listed here prepares a file named TAPE23 suitable for subsequent plotting. The default variables written include all positions  $x_j$ , velocities  $u_j$ , densities  $\rho_{j-1/2}$ , stresses  $\sigma_{j-1/2}$ , and internal energies  $\mathcal{E}_{j-1/2}$ , as well as the current fracture

status. Thus this file contains adequate data for virtually any desired plot. Should the user want to plot other variables, provision has been made in the input instructions to select other variables.

### 5.3 Error Checks

A number of features are included which permit checking for errors or to speed up the computation, and in some cases, to halt the calculation if errors become serious. These are described below.

Energy and Momentum Checks -- The mass  $M$  in a zone can be related to  $m$

(2.43) by

$$M_{j-1/2} = k' m_{j-1/2} \quad , \quad (5.2)$$

where

$$k' = 1 \quad , \quad \text{for } \alpha = 1 \quad ,$$

$$k' = \pi \quad , \quad \text{for } \alpha = 2 \quad ,$$

$$k' = \frac{4}{3} \pi \quad , \quad \text{for } \alpha = 3 \quad .$$

Note that  $m$  is not the mass in a zone except in the rectangular case  $\alpha = 1$ .

The momentum\* in a zone may be written in finite difference form, within the factor  $k'$ , as

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\*On occasion, the user may want to calculate the specific momentum, or impulse (momentum/unit area),  $\mathcal{I}$ , in one or more zones. The calculation is as follows:

$$\mathcal{I} = M\bar{u}/A = \begin{cases} m\bar{u} & , \quad \alpha = 1 \\ mk' \bar{u} / 2\pi \bar{x} = \frac{m}{2\bar{x}} \bar{u} & , \quad \alpha = 2 \\ mk' \bar{u} / 4\pi \bar{x}^2 = \frac{m}{3\bar{x}^2} \bar{u} & , \quad \alpha = 3 \end{cases}$$

where  $\bar{x} = (x_l + x_r)/2$ ,  $\bar{u} = (u_l + u_r)/2$ , and the subscripts  $l$  and  $r$  refer to the left and right boundaries of the zone being considered.

$$H_{j-1/2}^{n+1/2} = \frac{1}{2} m_{j-1/2} \left( u_j^{n+1/2} + u_{j-1}^{n+1/2} \right) \quad (5.3)$$

The kinetic energy in a zone is given within the factor  $k'$ , as

$$K_{j-1/2}^{n+1/2} = \frac{1}{6} m_{j-1/2} \left\{ \left( u_j^{n+1/2} \right)^2 + u_j^{n+1/2} u_{j-1}^{n+1/2} + \left( u_{j-1}^{n+1/2} \right)^2 \right\} \quad (5.4)$$

while the internal energy in a zone is given, within the factor  $k'$ , as

$$E_{j-1/2}^{n+1/2} = \frac{1}{2} m_{j-1/2} \left( \mathcal{E}_{j-1/2}^{n+1} + \mathcal{E}_{j-1/2}^n \right) \quad (5.5)$$

These quantities (5.3), (5.4), and (5.5) are computed at each zone and cycle and could be called out via one of the special output routines, if desired. Various sums of these quantities over specified numbers of zones are also occasionally of interest.

It is possible to check whether momentum and energy are conserved during the calculation. In particular, for the momentum,

$$k' \sum_{j=2}^{j_{\max}} H_{j-1/2}^{n+1/2} = \text{constant} \quad (5.6)$$

This sum is computed initially from the input data (HTOT). It is subsequently computed on each cycle (HT). A test is made to see if momentum is conserved by testing if

$$\left| \frac{HT - HTOT}{HTOT} \right| > HMAX \quad (5.7)$$

where HMAX is the allowable relative momentum error and is specified as an input variable. If this relative error is exceeded, the computation is terminated, an error message is printed, and standard printed output is

initiated. If no value is inserted for HMAX, read as zero, a value of 10<sup>100</sup> is used to defeat this test.

If the initial conditions are such that the initial momentum HTOT is zero, the calculation of the relative momentum error is bypassed and it is set to zero. Consequently this test is effectively bypassed for this type of problem. Note also that if a problem involves boundary conditions other than free surfaces, the appropriate momentum transfer and reflection is not taken into account, thus the default option for HMAX should be used for this class of problem.

The energy balance is more difficult since energy may be added by energy sources [ $\Delta Q_{j-1/2}$  in (2.46)] as well as by work done on boundaries by an applied load (in subroutine BOUNDRY). The sum of kinetic and internal energy over all zones is

$$E_{\text{sum}}^{n+1/2} = k' \sum_{j=2}^{j_{\text{max}}} \left( K_{j-1/2}^{n+1/2} + E_{j-1/2}^{n+1/2} \right) . \quad (5.8)$$

This sum is computed initially from the input data (ETOT). It is subsequently computed on each cycle (ET). In addition, if energy sources exist, these must be computed and stored in one of the additional storage arrays available in STORE. The energy added by these energy sources in each cycle must be summed over all the zones and the result stored in SUMQE. This calculation can be done in the equation of state subroutine. If a load is applied to either the left-hand or right-hand boundary, the work done in each cycle must be computed and stored in WL or WR for the left-hand and right-hand boundaries respectively. This calculation can be done in the BOUNDRY subroutine (see Section 4.3). Thus the reference



energy ETOT must be updated each cycle by

$$ETOT = ETOT + k' \cdot SUMQE + WL + WR \quad . \quad (5.9)$$

A check is then made to determine if the energy is conserved by testing if

$$\left| \frac{ET - ETOT}{ETOT} \right| > EMAX \quad , \quad (5.10)$$

where EMAX is the allowable relative energy error and is specified as an input variable. If this relative error is exceeded, the computation is terminated, an error message is printed and a standard printed output is initiated. In addition, if the relative energy error exceeds 10 per cent of EMAX, a cautionary message is printed. If no value is inserted for EMAX, the code defaults it to 1.0 or 100 percent. Thus to defeat this check a large value, say  $10^{100}$ , must be actually input to the program.

The total energy, kinetic energy, internal energy, and momentum summed over all zones are printed in the standard printed output, as are the relative energy error and relative momentum error.

Occasionally when complex energy sources or boundary loads are used for experimental runs, it is convenient to omit calculation of SUMQE, WL, and WR. However, for normal calculations the energy and momentum checks are very valuable in halting the computation if an error occurs and should generally be used.

Overflow Test -- When instabilities occur, oscillations usually grow exponentially with time until overflow occurs in the computer. If the problem is terminated due to overflow, an abnormal exit occurs and no diagnostics are possible. For this reason an overflow test is incorporated.

If the stress  $\sigma$  in any zone exceeds a maximum value  $\sigma_{MAX}$ , an input variable, the computation is terminated and a standard printed output is initiated together with an error message

## 6. REZONING

The rezoning features of WONDY V both improve the resolution and generally shorten the computer time used for a given problem by inserting finer zoning in regions where compression or release waves exist and removing unnecessarily fine zoning in regions where there is no discernible shock or release activity. Chapter 7 contains a description of the storage arrangement necessitated by the use of rezoning.

### 6.1 Rezone Criteria

The rezone criteria locate those active regions of a problem where finer zoning is needed for better resolution and conversely the inactive regions where a coarser zoning can be used to minimize the number of calculations while preserving adequate resolution. These rezone criteria are applied in subroutine MOTION for every zone-cycle calculation.

Active regions are in general characterized by relatively large zone to zone stress variations. This suggests a rezone criterion that locates active regions by searching for high stress variations and then subdivides zones to reduce the variation to within some specified limit. Thus an active, high stress variation region would be defined with many small zones to keep the zone to zone stress variations within these limits. Conversely, inactive, low stress variation regions will require fewer zones to hold the zone to zone stress variations within bounds. The "ideal" mesh for a given problem then would be that which resolves the problem with the smallest number of zones whose stress variations are within these limits.<sup>27</sup>

This concept is implemented in WONDY V for an arbitrary zone number  $j$  as follows (note that actual tests on zone  $j$  take place after zone  $j+1$  has been processed). Based upon  $\sigma_j$ , the stress in zone  $j$ , the maximum

allowable stress variation,  $\sigma_{lim}$ , is calculated as

$$\sigma_{lim} = \text{Max} (RSCRIT, RZCO|\sigma_j - RZC|) \quad , \quad (6.1)$$

where RSCRIT, RZCO, and RZC are input variables. One set of these variables is given for each plate in the problem. Equation (6.1) defines the stress resolution  $\sigma_{lim}$  as a fraction, RZCO, of the local stress  $\sigma_j$ , with an absolute fixed lower limit, RSCRIT. This criterion is designed to yield a limiting stress which will result in nearly the same relative resolution as the local stress increases, and yet will prevent the resolution from becoming infinitesimally fine as  $\sigma_j$  approaches zero.

Consistently good results have been obtained with values of RSCRIT in the neighborhood of  $10^{-4} \times \rho_0 c_0^2$ , although greater computer time savings, at the expense of resolution, can be realized by using larger values for RSCRIT. In complicated problems, a short trial run will be found valuable in determining a suitable value for RSCRIT. The variable RZC in equation (6.1) provides a means of calculating  $\sigma_{lim}$  as a fraction of the local stress relative to some stress level other than zero. This offset is particularly useful in treating materials whose ambient stress is non-zero, such as perfect gases.

Once the maximum stress variation,  $\sigma_{lim}$ , has been determined by (6.1), the actual stress variations  $|\sigma_j - \sigma_{j-1}|$  and  $|\sigma_j - \sigma_{j+1}|$  are calculated and compared with  $\sigma_{lim}$ . If either of these actual variations is greater than  $\sigma_{lim}$ , zone j is to be divided. If, on the other hand, both of these variations are less than  $0.66 \sigma_{lim}$ , then zone j is flagged as being "combinable." The inclusion of the factor 0.66 prevents combining to create a new zone whose stress variations are large enough to cause redivision on the next

time cycle. When two adjacent zones are "combinable," they are combined (subject to further restrictions detailed in Section 6.3).

In some applications where stress gradients are changing rapidly due to energy addition, e.g. problems involving high explosives or energy deposition, equation (6.1) may not divide zones quickly enough to achieve the desired resolution. In such instances the problem can be avoided by setting up a fine initial zoning and delaying rezoning in the particular plates involved until the energy addition is completed. An additional input parameter, RZTIME, specifies the earliest time at which rezoning can take place in a given plate. Since in general the time during which energy addition takes place is small relative to the total problem time, this delayed rezoning has no appreciable effect on the computational efficiency. Setting RZTIME to a value greater than the total problem time can be used to completely avoid rezoning in a given plate.

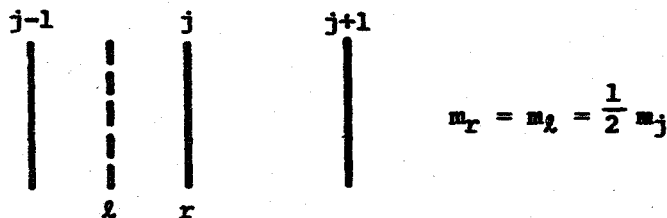
The basic stress variation criterion is complemented in WONDY V by an implicit rule that no zone can be more than twice as large as either of its adjacent zones. Thus whenever a zone is more than twice as large as either neighbor, it is divided, even though its stress variations may be well within the limit,  $\sigma_{lim}$ , of equation (6.1). The effect of this is that whenever a zone becomes divided because of its stress variations, as when a disturbance moves into that zone, this "2:1 rule" causes several zones in front of the disturbance to be divided, insuring that the disturbance will always be moving into regions which have already been predivided. Conversely two zones cannot be combined if the resultant zone would be more than twice as large as either adjacent neighbor. This prevents zones from combining too quickly behind a disturbance.

## 6.2 Dividing Zones

The  $\sigma_{lim}$  and 2:1 rules of the previous section determine those zones that are to be divided. If there were no further restrictions imposed upon the divide process, then in the region of a shock the rezone scheme would keep dividing and redividing in an attempt to resolve the discontinuity. A simple means of limiting the divide process involves specifying a lower limit on the size of zones which can be divided. The plate input variable, DXMIN, provides this necessary limit.\*

When it has been determined that a zone is to be divided, program control is passed to subroutine REZONE, where the actual zone division is performed, and the physical properties of the newly created zones are determined.

A zone  $j$  is divided by inserting a mesh point which divides  $j$  into



two equally massed zones,  $r$  and  $l$ . The densities  $\rho_l$  and  $\rho_r$  of the new zones are set equal to  $\rho_j$ . The position  $x_r$  of zone  $r$  is set equal to  $x_j$ . Clearly the position  $x_l$  of the new mesh point must be such that it divides zone  $j$  into two equal volumes, i.e.,

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\*The DXMIN limitation does not affect the initial zoning where zones much finer than DXMIN may be desired in certain applications, e.g. energy deposition.

$$x_l = \left[ \frac{1}{2} (x_j^\alpha + x_{j-1}^\alpha) \right]^{1/\alpha}, \quad (6.2)$$

where  $\alpha$  is the geometry coefficient.

The expressions for the momentum and kinetic energy of zone  $j$  are given as (see Section 5.3)

$$H_j = \frac{m_j}{2} (u_j + u_{j-1}), \quad (6.3)$$

and

$$K_j = \frac{m_j}{6} (u_j^2 + u_j u_{j-1} + u_{j-1}^2), \quad (6.4)$$

respectively. It is easily shown that by setting

$$\left. \begin{aligned} u_r &= u_j, \\ u_l &= \frac{1}{2} (u_j + u_{j-1}), \end{aligned} \right\} \quad (6.5)$$

one obtains

$$H_r + H_l = H_j,$$

and

$$K_r + K_l = K_j,$$

that is, for these measures, both momentum and kinetic energy are conserved in the divide process.

The remaining zonal variables, sound speed, viscosity, internal energy density, stress-like quantities, and any additional variables are set to their values in zone  $j$  for both zones  $l$  and  $r$ .

### 6.3 Combining Zones

When two adjacent zones,  $j$  and  $j-1$ , are found to satisfy the  $\sigma_{lim}$  criterion for combinability (Section 6.1), they are considered for combining, subject to several additional restrictions imposed by "physical acceptability" and the need to prevent the introduction of any appreciable distortion into the solution. "Physical acceptability" prohibits the combining of zones lying in different materials, i.e. a material interface lies between them as well as zones with a fracture or void between them. Because a fracture which has rejoined remains, in general, a weak point in the material, zones with a rejoined fracture between them are not combined in order to preserve this property.

In problems in which a material melts or vaporizes the absence of significant stress gradients can allow the combining of zones in different phases and thus obscure many real physical effects. A convenient means of preventing this involves limiting the combining of zones to zones of very nearly the same densities. In WONDY V the combining of zones  $j$  and  $j-1$  can take place only if

$$|\rho_j - \rho_{j-1}| < RCCOMB \cdot \rho_0 \quad , \quad (6.6)$$

where  $RCCOMB$  is a plate input parameter preferably 0.1 or less. This density limitation on combine also preserves the resolution of other phenomena which manifest themselves in density gradients rather than stress gradients, e.g. the contact discontinuities produced by shock collisions.

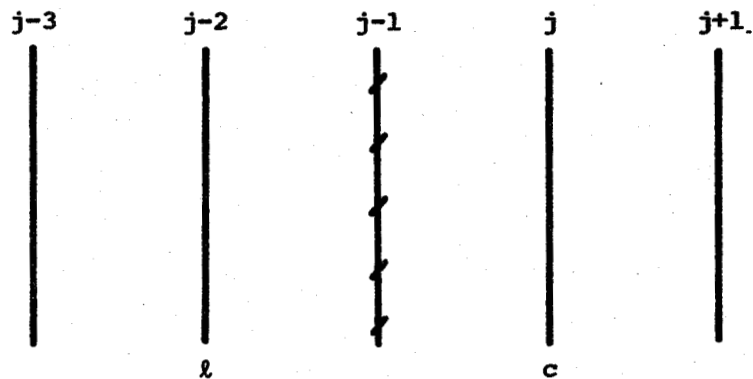
Finally the zones  $j$  and  $j-1$  will not be combined if the size of the resultant zone would exceed  $DXMAX$ , another plate input variable. This limits the maximal size zone that can occur in a given plate, and conversely



insures that the plate will always have at least some minimal number of zones in it.

When two adjacent zones have satisfied the tests above, program control is transferred to subroutine REZONE, where the actual combination is performed, and the physical properties of the new zone are determined.

Zones  $j$  and  $j-1$  are combined to create a new zone  $c$  by removing the mesh point,  $j-1$ , lying between them, where  $x_c = x_j$  and  $x_l = x_{j-2}$ . The



mass,  $m_c$ , of the newly created zone is set equal to  $m_j + m_{j-1}$ , and the density,  $\rho_c$ , becomes

$$\rho_c = \begin{cases} \frac{m_c}{(x_j - x_{j-2})} & , \quad \text{for } \alpha = 1, \\ \frac{m_c}{(x_j - x_{j-2})(x_j + x_{j-2})} & , \quad \text{for } \alpha = 2, \\ \frac{m_c}{(x_j - x_{j-2})(x_j^2 + x_j x_{j-2} + x_{j-2}^2)} & , \quad \text{for } \alpha = 3, \end{cases} \quad (6.7)$$

where  $\alpha$  is the geometry coefficient. If  $v_j$ ,  $v_{j-1}$ , and  $v_c$  are the volumes of zones  $j$ ,  $j-1$ , and  $c$  respectively, given by  $m/\rho$  with appropriate subscripts, then  $\rho_c$  can be alternately expressed as

$$\rho_c = \frac{m_c}{v_c} = \frac{m_j + m_{j-1}}{v_j + v_{j-1}} = \frac{\rho_j v_j + \rho_{j-1} v_{j-1}}{v_j + v_{j-1}} \quad (6.8)$$

Internal energy is conserved by setting

$$e_o = \frac{m_j e_j + m_{j-1} e_{j-1}}{m_c} \quad (6.9)$$

In order to exactly conserve momentum and kinetic energy small adjustments must be made to  $u_\ell = u_{j-2}$  and  $u_c = u_j$ . The adjusted velocities  $u_\ell$  and  $u_c$  are determined from solutions of the simultaneous equations

$$MO = MR \cdot u_c + ML \cdot u_\ell \quad (6.10)$$

and

$$KEO = MR \cdot u_c^2 + ML \cdot u_\ell^2 + MB \cdot u_c \cdot u_\ell + KER1 \cdot u_c + KEL1 \cdot u_\ell \quad (6.11)$$

where

$$MO = m_{j+1} u_j + m_j u_j + m_j u_{j-1} + m_{j-1} u_{j-1} + m_{j-1} u_{j-2} + m_{j-2} u_{j-2}$$

$$MR = m_{j+1} + m_j + m_{j-1}$$

$$ML = m_j + m_{j-1} + m_{j-2}$$

$$MB = m_C = m_j + m_{j-1} \quad ,$$

$$KER1 = m_{j+1}u_{j+1} \quad ,$$

$$KEL1 = m_{j-2}u_{j-3} \quad ,$$

and

$$\begin{aligned} KEO = & M_{j+1} (u_j^2 + u_{j+1}u_j) + m_j (u_j^2 + u_ju_{j-1} + u_{j-1}^2) \\ & + m_{j-1}(u_{j-1}^2 + u_{j-1}u_{j-2} + u_{j-2}^2) + M_{j-2}(u_{j-2}^2 + u_{j-2}u_{j-3}) \quad . \end{aligned}$$

Equations (6.10) and (6.11) express conservation of momentum and kinetic energy respectively. The equations (6.10) and (6.11) will give two sets of solutions,  $(u_C, u_\ell)$  and  $(u_C^i, u_\ell^i)$ , both real or both complex. If the solutions are complex then only the real part is used, which conserves momentum exactly and minimizes the kinetic energy error. If both solutions are real, then the solution "closest" to the original  $u_j$  and  $u_{j-2}$  is chosen, i.e., if

$$(u_C - u_j)^2 + (u_\ell - u_{j-2})^2 < (u_C^i - u_j)^2 + (u_\ell^i - u_{j-2})^2 \quad ,$$

then  $(u_C, u_\ell)$  is chosen, otherwise  $(u_C^i, u_\ell^i)$  is chosen.

By combining equation (3.9) with (2.7), the equation of state can be made to take the form

$$p = f_1(\rho) + \Gamma\rho\mathcal{E} \quad . \quad (6.12)$$

Because the combine criteria restrict  $\rho_j$  and  $\rho_{j-1}$  to being nearly equal, it can be assumed that, between  $\rho_j$  and  $\rho_{j-1}$ ,  $f_1(\rho)$  is locally linear and  $\Gamma$  is constant. Letting  $\psi$  represent any linear function of  $\rho$ , from the properties of linearity and equation (6.8)

$$\begin{aligned} \Psi_c \equiv \Psi(\rho_c) &= \Psi \frac{\rho_j v_j + \rho_{j-1} v_{j-1}}{v_j + v_{j-1}} \\ &= \frac{v_j \Psi(\rho_j) + v_{j-1} \Psi(\rho_{j-1})}{v_j + v_{j-1}} \end{aligned}$$

or

$$\Psi_c = \frac{v_j \Psi_j + v_{j-1} \Psi_{j-1}}{v_j + v_{j-1}} \quad (6.13)$$

It is easily shown, using equations (6.8) and (6.9), that  $\rho_c^e$  satisfies the relation (6.13), i.e.,

$$\rho_c^e = \frac{v_j \rho_j^e + v_{j-1} \rho_{j-1}^e}{v_j + v_{j-1}} \quad (6.14)$$

Applying equations (6.13) and (6.14) to equation (6.12), with the assumptions of  $f_1(\rho)$  "locally" linear and  $\Gamma$  "locally" constant, yields for the pressure of the combined zone,

$$P_c = \frac{v_j P_j + v_{j-1} P_{j-1}}{v_j + v_{j-1}} \quad (6.15)$$

that is  $p$  as given by (6.12) can be interpolated using (6.13).

Because the assumption of "local" linearity with respect to  $\rho$  is reasonable in the inactive regions where combining occurs, equation (6.13) is used to interpolate all remaining zonal variables including all stress components, sound speed, artificial viscosity and any additional variables being used when  $NVAR > 10$  (see Section 7.2).

## 7. STORAGE ARRANGEMENT

### 7.1 The Main STORE Array

WONDY V carries all zonal variables in one large singly subscripted array named STORE. This method has several advantages over more conventional techniques. First, all the needed information can, before any rezoning takes place, be "close-packed" within the array. This allows a fairly simple method for changing the computer core arrangement of the code, i.e., changing the dimensioned size of the STORE array (along with the parallel arrays PFRACT, QFRACT, NEXT and DUM) permits the adjustment of the code to fit virtually any available core size. Of course, the maximum number of zones that can be used in the code depends on the size of this array. In fact the total number of zones  $L_{TOT}$  must be such that

$$L_{TOT} < \frac{DIM}{NVAR} - 2 \quad (7.1)$$

rounded to the next lower integer. Here DIM is the dimensioned size of the STORE array and NVAR is the number of variables to be used. In addition, with the scheme outlined below, most of the calculations can be carried out using unsubscripted variables, resulting in a considerable saving in computer time.

Before any rezoning takes place the STORE array consists of blocks of variables NVAR quantities long, the first block containing all the variables associated with zone 1, the second block containing all the variables for zone 2, and so forth. The order of the variables within these blocks is given by the appropriate "parameter index,"  $i$ , as indicated in the accompanying table. Thus  $i$  ranges from 1 to NVAR where for most problems

Parameter Index	Report Symbol	Fortran Symbol	Definition
1	$c_{j-1/2}^n$	C	sound speed
2	$\mathcal{E}_{j-1/2}^n$	E	internal energy
3	$m_{j-1/2}$	M	mass
4	$p_{j-1/2}^n$	P	pressure
5	$q_{j-1/2}^n$	Q	artificial viscosity
6	$\rho_{j-1/2}^n$	R	density
7	$\sigma_{j-1/2}^n$	S	normal stress
8	$u_j^{n-1/2}$	U	particle velocity
9	$x_j^n$	X	position
10	$\phi_{j-1/2}^n$	Z	difference in principal stresses
11	$\mathcal{E}_{\text{dep } j-1/2}$	DATB(1)	total energy deposited in zone j (Section 4.4)
12	$K_j^n$	DATB(2)	integral damage (Section 3.6)
13	$d_{p\mathcal{E}_{j-1/2}^n}$	DATB(3)	plastic work in STAT1 (Section 3.1.2)
13-20	---	DATB(3) - DATB(10)	various parameters associated with STAT4 (Section 3.4)

NVAR will be 10; cases for which NVAR is greater than 10 are explained in Section 7.3. Therefore using the difference scheme given in Section 2.4, and before any rezoning takes place, the first variable in the STORE array is the sound speed for the virtual zone outside the left-hand boundary. Similarly an applied stress on the left-hand boundary would be placed in STORE(7); the position of the left boundary is STORE(9); and assuming NVAR = 10, the sound speed of the first real mass element or second zone is stored as STORE(11).

In order to maintain this sequential storage, it would be necessary to completely rearrange the STORE array every time a zone boundary is added or removed with a rezoning operation. To avoid this time consuming process, an array named NEXT is employed, which is parallel to the STORE array. The values in NEXT indicate block numbers in the STORE array where the NVAR quantities for a given zone can be found. Specifically, the value of NEXT for a given zone  $j$  is the block number within STORE for the variables associated with zone  $j + 1$ .<sup>\*</sup> Thus when a zone boundary is eliminated by the REZONE routine the values in NEXT are altered so that on the next pass through the problem mesh the appropriate block in STORE is bypassed. Similarly, when a zone boundary is added, the variables for the newly created zone are placed in the first empty block in the STORE array and NEXT is adjusted accordingly. More specific details on how this is accomplished as well as the interrelationship between the STORE and NEXT arrays can be found in Reference 27.

With this preliminary discussion some specific examples can now be given. A general variable  $\Psi[i]$ , with parameter index  $i$ , for  $j^{\text{th}}$  zone

---

\*NEXT is a link list for blocks in the STORE array linking the storage block for a given zone to that of the next zone in sequence.

and  $n^{\text{th}}$  time step, can be found in the STORE array at

$$\Psi[1]_j^n = \text{STORE} (J_j + 1) \quad , \quad (7.2)$$

where

$$J_j = (\text{JN}_j - 1) \cdot \text{NVAR} \quad , \quad (7.3)$$

and JN is obtained from the recursion relation

$$\text{JN}_j = \text{NEXT} (\text{JN}_{j-1}) \quad , \quad (7.4)$$

where

$$\text{JN}_1 \equiv 1 \quad . \quad (7.5)$$

Note that here and in the following, J always refers to a specific location within the STORE array, where JN refers to a block number within STORE and is obtained from the NEXT array. As an explicit example, if the stress for the  $j^{\text{th}}$  zone is desired, (7.2) is written as

$$\sigma_{j-1/2}^n = \text{STORE} (J_j + 7) \quad ,$$

with  $J_j$  obtained from (7.3) through (7.5).

Since rezoning adds and removes zone boundaries, a given zone number  $j$  does not refer to the same mass element throughout the course of a calculation. Thus to retrieve a specific variable for a specific mass element the appropriate numerical value for  $j$  must be recomputed for each cycle the variable is needed. The most straightforward way to accomplish this is to use the array NOMESH which contains the updated number of zones in each plate in order. Note that if rezoning is completely bypassed, this process is unnecessary.



To clarify the use of equations (7.2) through (7.5) a few specific FORTRAN examples will now be given. Consider again the problem of retrieving the general variable  $\Psi[i]_j^n$  for a given zone  $j$  on the time cycle  $n$ . A generic segment of a FORTRAN routine which will accomplish this is:

```

[Calculate j, say from NOMESH (PLATE)]

      JN = 1

      DO 10 L = 1, LACT

      IF (L - j) 10, 20, 10

10     JN = NEXT (JN)

      GO TO 30

20     J = (JN - 1) * NVAR

       $\Psi[i]_j^n = \text{STORE}(J + i)$ 

      [Output or further process  $\Psi[i]_j^n$ ]

30     CONTINUE

```

Here LACT is the current number of the last active zone. The statement after 10, GO TO 30, is reached only if the desired zone,  $j$ , has not yet become active. As before,  $i$  is the parameter index of the desired variable. A more common situation involves the output of one or more variables for all active zones in the problem mesh. The following program segment will accomplish this:

```

      JN = 1

      DO 10 L = 1, LACT

      J = (JN - 1) * NVAR

```

$\Psi[i]_L^n = \text{STORE } (J + i)$

[Retrieve other  $\Psi[i]$ , place  $\Psi[i]_L^n$

in new array with subscript L, or

further process  $\Psi[i]_L^n$ ]

10 JN = NEXT (JN)

As was mentioned earlier, most calculations using these stored variables employ unsubscripted counterparts according to the following scheme. For the zone-cycle calculation at time  $n + 1$  and spatial position  $j$ , the general variable  $\Psi$  has the following forms:

$$\left. \begin{aligned} \Psi_B &= \Psi_{j-1}^n \\ \Psi &= \Psi_j^n \\ \Psi_A &= \Psi_{j+1}^n \\ \Psi_{BN} &= \Psi_{j-1}^{n+1} \\ \Psi_N &= \Psi_j^{n+1} \end{aligned} \right\}$$

(7.6)

and

Note that A and B refer to "ahead" and "behind" in space, where N refers to "new" in time. The logic for performing these transfers is similar to the examples given above and is located in subroutine JLOOP. At the beginning of the zone-cycle calculation for time  $n + 1$  and zone  $j$  the following transfers are made:

JNA = NEXT (JN)

JA = (JNA - 1) \* NVAR

$\Psi_B = \Psi$	}	$i = 1, NVAR$
$\Psi = \Psi_A$		
$\Psi_A = \text{STORE}(JA + i)$		
$\Psi_{EN} = \Psi_N$		

When these transfers are made,  $J$ ,  $JN$ ,  $\Psi$ ,  $\Psi_A$ , and  $\Psi_N$  are available from the previous zone-cycle at  $n + 1$  and  $j - 1$ . At the end of the zone-cycle calculation, the new variables for  $n + 1$  and  $j$  have been computed and placed in  $\Psi_N$ . They are then transferred to the STORE array and the parameters  $J$  and  $JN$  are updated with

```

STORE (J + i) =  $\Psi_N$  , i = 1, NVAR

JN = JNA

J = JA

```

Here again,  $i$  is the parameter index of  $\Psi$ . Thus for each of the  $NVAR$  variables only two references to the STORE array are made for each zone-cycle; all other computations and transfers are made with their unsubscripted counterparts.

## 7.2 The Preprocessor

The current version of WONDY V contains storage allocations for 20 materials/plates and  $NVAR * \text{No. Zones} = 19851$ , which nearly fills the small core memory of a CDC 7600. If smaller storage limits are desired for economy purposes, or required for computers with smaller memory, a preprocessor may be accessed to operate on the main storage arrays to the user's specifications. The preprocessor reads input data that specify the total number

of zones in the problem, the number of variables desired, and the number of plates/materials to be used. The preprocessor writes update instructions that are read during the subsequent attachment of the WONDY V code. The altered WONDY V code is then compiled and run with the normal input information (that must, of course, be compatible with the original preprocessor size request).

It should be noted that the use of the preprocessor does not influence the overall operational efficiency of the code, but simply removes excess storage from the code. Use of the rezone capability may require that allowance should be made for an increased number of zones due to zone division.

### 7.3 Additional Variables

As was suggested in the previous section, WONDY V carries an adjustable number of variables, where the input quantity NVAR is that number. NVAR must be at least 10 for any calculation, but if the requirement exists it can be made as large as 110. The extra variables are stored sequentially with the 10 basic variables in the STORE array and are transferred within a given zone-cycle calculation via the array DATB(k) where  $k = i - 10$ , and  $i$  is the appropriate parameter index. If, for example, one extra variable is to be used, NVAR is set to 11. This extra variable for zone 1 (left-hand virtual zone) is then stored in STORE (11), and before rezoning takes place, its value for zone 2 (the first real mass element) is placed in STORE (22), and so forth. Within a zone-cycle it is carried as DATB(1). Retrieval of the extra variables is achieved exactly as described in the previous section by using the appropriate parameter index.

Since the entire STORE array is initialized to zero, no problem arises with regard to the initial values of these extra variables unless non-zero

values are desired. If the latter is the case, a special subroutine MORSTOR may have to be written for the purpose. However, if the extra variables are used only by the equations of state, non-zero values can be set in the initialization sections of the appropriate state routines.

In the present version of the code a number of these extra variables have been assigned uses. The first (NVAR > 11) is used for the total deposited energy density when initiating a problem with an external energy source as described in Section 4.4. The second (NVAR > 12) is the time integral damage, equation (3.124), needed when employing the cumulative damage fracture criterion of Section 3.6. The third extra variable (NVAR > 13) is the plastic work, equation (3.62), used in the work hardening elastic plastic material model of STAT1 as described in Section 3.1.2. The remaining extra variables currently programmed are employed by STAT4 (see Section 3.4.1). Here parameter index  $i = 13$  refers to the equilibrium stress deviator  $\sigma_e^d$ . If the isotropic hardening model is used  $i = 14$  refers to the plastic work (3.84). When the anisotropic hardening model is employed  $14 < i < 20$  refer to the elemental stress deviators  $\sigma_{xi}^d$  described by equations (3.91) and (3.92). These uses for extra variables have been included in the table of Section 7.1.

Note that NVAR must be equal to or greater than the indicated value only if the appropriate option is being employed. In addition, other uses for a given extra variable can be programmed as long as the corresponding uses presented here are not duplicated.

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## APPENDIX A

### CODE STRUCTURE AND GLOSSARY OF VARIABLES APPEARING IN COMMON

The WONDY V code contains the following subroutines and entry points  
(indented):

WONDY	Main program
GENERAT	Generates and initializes arrays
ZONER	Zone generator
MORSTOR	Initializes extra arrays -- may be dummy
JLOOP	Handles logic to advance through problem mesh; fracture logic
BOUNDRY	Handles special boundary conditions -- may be dummy
LBOUND	Entry point for left boundry
RBOUND	Entry point for right boundry
MOTION	Computes conservation of mass and momentum, artificial viscosity, and rezone tests.
INDSET	
MESHES	
REZONE	Combines and divides zones
COMBINE	Entry point for combining
DIVIDE	Entry point for dividing



STAT1	
STIN1	
STAT2	
STIN2	
STAT3	Computes equations of state -- up to 5 may be dummies Entry point for initialization
STIN3	
STAT4	
STIN4	
STAT5	
STIN5	
STAT6	
STIN6	
OUTN	Handles special output -- may be dummy
OUTPL	Handles special output (plotting) -- may be dummy
DUMP	Prepares restart tape
DUMPF	Writes restart tape
RESTART	Reads restart tape

The following logical tape units are referenced in the code and should be defined if their functions are desired.

TAPE10	Available for user programmed routines
TAPE11	Available for user programmed routines
TAPE20	Binary output for restart

TAPE21	BCD output when this unit not equivalenced to standard output medium
TAPE22	Available for user programmed routines
TAPE23	Binary tape written by OUTPL
TAPE25	Binary input tape for restart -- format identical with TAPE20
TAPE60	Equivalenced to input

---

The common blocks contained in WONDY are

/CON/

/VAR/

/IND/

/ / (blank)

These common blocks may be accessed for use in a new subroutine by the statement

\*CALL, SHORTEN

in the subroutine.

The following table provides a list of variables contained in the common blocks in WONDY.

**GLOSSARY OF VARIABLES  
APPEARING IN COMMON**

FORTRAN NAME	COMMON BLOCK		DESCRIPTION
ADDATA(100)	CON		dummy array for additional input data
ALPHA2	IND	$2(\alpha - 1)$	geometry constant
ANGLE	CON		not used
B1	CON	$B_1$	quadratic viscosity coefficient
B2	CON	$B_2$	linear viscosity coefficient
C	VAR	$c_{j-1/2}^n$	sound speed
CAPE	VAR	$E_{j-1/2}^{n+1/2}$	internal energy
CAPH	VAR	$H_{j-1/2}^{n+1/2}$	momentum
CAPK	VAR	$K_{j-1/2}^{n+1/2}$	kinetic energy
CES(42,20)	CON		equation of state constants for plates
CHG(6,20)	CON		not used
CN	VAR	$C_{j-1/2}^{n+1}$	sound speed
DATB(100)	VAR		storage for additional variables
DE	CON		not used
DELRHO	VAR	$\Delta\rho/2\rho^2$	

FORTRAN NAME	COMMON BLOCK	DESCRIPTION
DELT(5)	VAR	$\Delta t^{n+1/2}, \Delta t^{n-1/2}, \Delta t_{j-1/2}^{n+1/2},$ $\Delta t^{n+3/2}, \frac{1}{2}(\Delta t^{n+1/2} + \Delta t^{n-1/2})$
DELTAX(20)	CON	initial size of first zone in plate
DELTAX1(20)	CON	initial size of last zone in plate
DELXJ	VAR	$x_j^{n+1} - x_{j-1}^{n+1}$
DELXJ1	VAR	$x_{j-1}^{n+1} - x_{j-2}^{n+1}$
DELXJ2	VAR	$x_{j-2}^{n+1} - x_{j-3}^{n+1}$
DEP	VAR	$\Delta t^{n+1/2} / \text{TDEP fraction of energy added}$
DUM(9025)		temporary storage area for variables to be dumped to restart tape plot tape
DXMAX	CON	largest allowable zone size
DXMIN	CON	smallest zone size
DXMN(20)	CON	smallest zone size by plate
DXMX(20)	CON	largest zone size by plate

FORTRAN NAME	COMMON BLOCK	DESCRIPTION
E	VAR	$\epsilon_{j-1/2}^n$ internal energy density
EN	VAR	$\epsilon_{j-1/2}^{n+1}$ internal energy density
ETOT	VAR	total initial energy plus added energy
EXIT	IND	exit indicator
EZERO(20)	CON	initial energy density in each plate
FCONST(20)	CON	constant for fracture criteria for plate
FCONSTI(20)	CON	constant for fracture criteria for interface
FCRIT(20)	CON	fracture criterion for plate
FCRITI(20)	CON	fracture criterion for interface
HTOT	VAR	total initial momentum
IALPHA	IND	$\alpha - 2$ geometry constant
IDUMP	IND	buffer out indicator

FORTRAN NAME	COMMON BLOCK	DESCRIPTION
IPL1	IND	
IPL2	IND	
IPL3	IND	output plot variables
IPL4	IND	
IPL5	IND	
ITABLE(50)	VAR	storage for zone numbers at fractures
J	IND	STORE Index for zone j
JA	IND	STORE Index for zone j+1
JAN	IND	position in NEXT corresponding to zone j+1
JBBN	IND	position in NEXT corresponding to zone j-2
JBN	IND	position in NEXT corresponding to zone j-1
JN	IND	position in NEXT corresponding to zone j
J3BN	IND	position in NEXT corresponding to zone j-3
J4BN	IND	position in NEXT corresponding to zone j-4
JONE	IND	indicates first zone
JTAPE	IND	number of plates to be read from restart tape

FORTRAN NAME	COMMON BLOCK	DESCRIPTION
KM(3)	CON	symmetry constants: 1; $\pi$ ; $4\pi/3$
KFI	CON	time constant in stability criterion
KIZ	IND	not used
L	IND	zone number
LACT	IND	number of active zones
LBCN	CON	minimum position of left boundary
LDT	IND	zone controlling time- step
LDUMP	IND	not currently used
LHBT	IND	left hand boundary type
LMAX	IND	maximum number of zones
LOL	IND	zone at left of plate
LOR	IND	zone at right of plate
LPHA	IND	$\alpha$ symmetry coefficient
M	VAR	$m_j^{-1/2}$ zone mass constant
MCTR	IND	not used
MORE	IND	flag for MORSTOR

FORTRAN NAME	COMMON BLOCK		DESCRIPTION
N	IND		cycle number
NAME1(20)	CON		not used
NAME2(20)	CON		not used
NE	IND		not used
NEWPLAT	IND		indicator for first zone in plate
NEXT(1805)			array of zone sequence linkages
NEXTEM	IND		first unused position in NEXT
NOMESH(20)	CON		number of zones in plate
NONE	IND		indicator for first cycle
NOP	IND		number of plates
NSTART	IND		cycle to restart from dump tape
NTWO	IND		indicator for restart cycle
NVAR	IND		number of variables
P	VAR	$n$ $p_{j-1/2}$	pressure
PA	VAR	$n$ $p_{j+1/2}$	pressure
PFRACT(1805)			indicator for rejoined fracture zone



FORTRAN NAME	COMMON BLOCK	DESCRIPTION
PLATE	IND	plate index
PN	VAR	$n+1$ $P_{j-1/2}$ pressure
PNUM(20)	CON	not used
PRINTE	IND	indicator for normal editing
PRINTL	IND	indicator for calling OUTL
PRINTN	IND	indicator for calling OUTN
PZERO(20)	CON	initial pressure in plate
Q	VAR	$n$ $Q_{j-1/2}$ viscosity
QA	VAR	$n$ $Q_{j+1/2}$ viscosity
QFRACT(1805)		indicator for fractured zone
QN	VAR	$n+1$ $Q_{j-1/2}$ viscosity
R	VAR	$n$ $\rho_{j-1/2}$ density
RA	VAR	$n$ $\rho_{j+1/2}$ density
RBCN	CON	maximum position of right boundary

FORTRAN NAME	COMMON BLOCK	DESCRIPTION
RCCOMB	CON	density gradient combine criteria
RCMB(20)	CON	RCCOMB by plate
RHBT	IND	right hand boundary type
RHODOT	VAR	$\dot{\rho}/\rho$
RN	VAR	$\rho_{j-1/2}^{n+1}$ density
RSC(20)	CON	RSCRIT by plate
RSCRIT	CON	constant stress resolution
RSCO(20)	CON	RZCO by plate
RSC1(20)	CON	RZC1 by plate
RZCO	CON	fractional stress resolution
RZC1	CON	reference stress level
RZERO(20)	CON	initial density of plate
RZTIME	CON	beginning rezone time
RZTM(20)	CON	RZTIME by plate
R1	VAR	indicator of combina- bility of zones j-1 and j-2

FORTRAN NAME	COMMON BLOCK	DESCRIPTION
R2	VAR	indicator of combinability/divisibility of zone j-1
S	VAR	$\sigma_{j-1/2}^n$ stress
SA	VAR	$\sigma_{j+1/2}^n$ stress
SBN	VAR	$\sigma_{j-3/2}^{n+1}$ stress
SIGACT	CON	active stress
SIGMAF(20)	CON	constant for fracture criteria for plate
SIGMAIF(20)	CON	constant for fracture criteria for interface
SIGMAO(20)	CON	constant for fracture criteria for plate
SIGMAOI(20)	CON	constant for fracture criteria for interface
SIGMAX	CON	maximum stress
SIGSEP	CON	separation stress
SN	VAR	$\sigma_{j-1/2}^{n+1}$ stress
STATE(20)	CON	equation of state indicator
STORE(19870)		main storage array
SUMH	VAR	sum of momentum

FORTRAN NAME	COMMON BLOCK		DESCRIPTION
SUMIE	VAR		sum of internal energy
SUMKE	VAR		sum of kinetic energy
SUMQE	VAR		sum of added energy
SZERO(20)	CON		initial stress in plate
T	VAR		time
TABLE(2,50)	VAR		storage for u, x at fracture
TDEP	CON		deposition time
THKNS(20)	CON		initial plate thickness
TITLE(10)	CON		problem title
U	VAR	$n-1/2$ $u_j$	velocity
UA	VAR	$n-1/2$ $u_{j+1}$	velocity
UB	VAR	$n-1/2$ $u_{j-1}$	velocity
UBBN	VAR	$n+1/2$ $u_{j-2}$	velocity
UBN	VAR	$n+1/2$ $u_{j-1}$	velocity
UN	VAR	$n+1/2$ $u_j$	velocity
UZERO(20)	CON		initial velocity of plate

FORTRAN NAME	COMMON BLOCK		DESCRIPTION
UZEROI(20)	CON		initial velocity of interface
WL	VAR		work at the left boundary
WR	VAR		work at the right boundary
W4020	IND		indicator for printed output
X	VAR	$x_j^n$	position
XA	VAR	$x_{j+1}^n$	position
XB	VAR	$x_{j-1}^n$	position
XBN	VAR	$x_{j-1}^{n+1}$	position
XGAP(20)	CON		size of gap between plates
XN	VAR	$x_j^{n+1}$	position
XRATIO(20)	CON		ratio of initial zone sizes
XZERO	CON		initial position of left boundary
Z	VAR	$z_{j-1/2}^n$	difference in principal stresses, $\phi$
ZA	VAR	$z_{j+1/2}^n$	difference in principal stresses, $\phi$
ZN	VAR	$z_{j-1/2}^{n+1}$	difference in principal stresses, $\phi$
ZZERO(20)	CON		initial $\phi$ in plate

## APPENDIX B

### RESEARCH MATERIAL STUDIES

Although the numerical techniques discussed in this report are important in terms of accuracy and stability, an area that remains open to extensive research is the material characterization. Materials under shock loading behave in complex ways, and the code contains some common characterizations that are believed to be satisfactory for general classes of materials as described in Section 3. There are many materials that are not covered by these models, and formulations exist to cover limited classes of materials. Many of these models are formulated with internal state variables, involving micromechanical concepts, and are conveniently incorporated into the equation-of-state routines of the wavecode. They are not a part of the permanent wavecode structure principally because of their specialized nature.

The purpose of this appendix is to briefly describe several models which have been developed and successfully used in WONDY. WONDY often serves as a proving code for the validity of a model, and is frequently an important step to subsequent incorporation into multi-dimensional codes. The list here is meant only to be a sampling, and by no means represents all of the models that have been developed.

Recalling that the equation of state routine is entered with new (current cycle) values of the density ( $\rho^{n+1}$ ), current strain rates, and the old (previous cycle) values of the density ( $\rho^n$ ), stresses, energy, and state variables unique to the model, the routine must return new values of the stresses, energy, and state variables. The equations in many of these models may be written as

$$\dot{\sigma}_{ij} = f(\rho, \xi_i, \dots) \quad (\text{B-1})$$

$$\dot{\xi}_k = g_k(\rho, \xi_k, \sigma_{ij} \dots) \quad (\text{B-2})$$

where  $\dot{\sigma}_{ij}$  is the stress rate, and  $\dot{\xi}_k$  the rate equations for the internal state variables. The complex coupling in these equations usually renders direct differencing ineffective to advance them in time. A method found to be quite successful is to retain the rate form, define the density variation with time over the timestep to be

$$\rho = \frac{\rho^{n+1} - \rho^n}{t^{n+1} - t^n} (t - t^n) + \rho^n \quad (\text{B-3})$$

then use an ordinary differential equation system solver such as ODE<sup>B1</sup> to integrate equations (B-1,2) from  $t^n$  to  $t^{n+1}$ . Extra internal state variables are conveniently stored in the DATB array.

### Fracture Models

The dynamic fracture of ductile metals and many rocks is often not characterized by a unique "fracture stress." Instead, the concept of continuous "damage" accrual seems to be a more precise way to describe the onset of failure. To model such a process, rate equations for the damage are driven by some stress or strain measure, and the material moduli are written as variables that decrease with increasing damage.

Ductile aluminum has been modeled with the damage related to void nucleation and growth in the metal, as well as complementary rate equations for dislocation multiplication and visoplastic slip.<sup>B2,B3</sup> Oil shale is representative of many rocks that exhibit strain-rate dependent fracture

strengths, and its fracture and fragmentation properties have been described by damage rate equations.<sup>B4</sup>

Loss of strength occurs in a continuous sense until the material cannot support further tension, but may disperse impinging compressive waves much as would a porous material.

### Piezoelectric/Ferroelectric Materials

Piezoelectric and ferroelectric materials are classes of solids that exhibit electromechanical coupling.<sup>B5,B6,B7</sup> Under local mechanical loading, an electric signal is generated which could cause a global response (depending on the circuit), hence distinguishing these materials from simpler materials in which information travels only at the solid wave speed. This electrical coupling requires that a circuit equation be added to the code, and that subsequent to the normal equation of state calculation, an overall implicit iteration is required. The ferroelectric materials are characterized by complex coupling of several rate equations, notably for dipole dynamics and domain switching<sup>B7</sup>, while the simpler piezoelectric materials respond in a time-independent mode. Under low amplitude loading conditions, the ferroelectrics behave like piezoelectric solids.

### Viscoelasticity

The dynamic (mechanical) behavior of a non-linear viscoelastic material may be characterized by a generalized Maxwell model,

$$\dot{\sigma} - F(\sigma, \eta)\dot{\eta} + G(\sigma, \eta) = 0$$

where  $\sigma$  is the stress and  $\eta$  is the bulk strain.  $F(\sigma, \eta)$  is the instantaneous modulus, and  $G(\sigma, \eta)$  is the relaxation function, providing viscous damping to the model<sup>B8,B9</sup>. Depending upon the characteristic relaxation time of the



material and the code timestep, a subcycling technique provides the means to obtain the stress state.

A specific application of this model is to regular composites<sup>B9</sup>, where the functions F and G are determined from specified combinations of the constituent properties.

A modification of this model currently resides in STAT5 of WONDY.

### Explosives

The current models for explosives in the code (Section 3.2) treat the chemical energy release as a predetermined process -- "programmed-burn". Although this approximation is sufficient for many applications, it does not address cases where stress waves may initiate the explosive, or the detonation is boundary dependent. These latter issues require that not only the product gases be defined, but that an equation of state for the unreacted explosive also be available. Laws governing the mixture of these two states may then be written, where a state variable that accounts for the extent of reaction,  $x$ , is defined<sup>B10,B11,B12</sup>. A rate equation governing the change in  $x$ , and driven by the temperature (T) (for which a rate equation is also required), and/or the pressure (p), and current extent of the reaction may take the form

$$\dot{x} = f(T, p, x, \dots)$$

In such a model, the propagation of shocks into the explosive is allowed, and eventually leads to reaction, or detonation failure. Although two-dimensional boundary effects are particularly interesting explosive problems, the ability to more accurately simulate the transient wave phenomena in explosives is an important capability.

### Mixture Theories (Granular and Porous Materials; Bubbly Liquids)

The porous material model described in Section 3.5 is a limiting case of a more general class of materials that can be modelled with mixture theories<sup>B13,B14</sup>. These theories have been applied to, for example, porous and granular materials, mixtures such as alumina/epoxy, and bubbly liquids. The granular aspect has been incorporated into a model for explosives with hot spot formation<sup>B12</sup>, where rate equations for changes in phase are required, and marked influence of grain size is seen on the reaction properties.

The wave motion through a bubbly liquid has been studied using a two-phase model. The micromechanics of bubble growth and collapse, both mechanically and thermally induced, involves vaporization and condensation of the fluid, local heat transfer, local pressure imbalances, and inertia of the bubble growth.

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## APPENDIX C

### NON-DIMENSIONALIZATION

Since there are no dimensional constants in WONDY (except for the BKW explosives model), any consistent set of units can be used for running problems. However it is also possible to run problems in non-dimensional form. This occasionally has advantages for parametric studies, although since most materials are non-linear and possess at least three non-dimensional properties ( $\Gamma_0$ ,  $s$ , and  $v$ ), the only scaling that is practical is geometric scaling.

To perform non-dimensional calculations no changes are required in the coding, and any version of WONDY can be rendered non-dimensional in exactly the same way, providing that no dimensional constants have been inserted.\* The only requirement is that the input parameters be specified properly.

To begin, one of the two independent variables, position or time, must be chosen as a reference quantity. This will be either a characteristic length,  $x_r$ , or a characteristic time,  $t_r$ . The former might be the thickness of one or more material layers, where the latter could be chosen as a characteristic pulse width or relaxation time.

To complete the non-dimensionalization two of the dependent variables must be chosen as reference quantities. These may be any two of velocity, density, stress, and energy. Note that since WONDY carries the energy as energy per unit mass, the pair, velocity and energy, is not a possible choice. The reference velocity,  $u_r$ , could be chosen as a material sound speed or input velocity; the density,  $\rho_r$ , would normally be a material

---

\* Certain energy deposition routines for subroutine MORSTOR have been coded with dimensional constants. These must be removed if these routines are to be used in non-dimensional calculations.

density; the reference stress,  $\sigma_r$ , may be one of the material moduli or an input stress; and the reference energy,  $\mathcal{E}_r$ , could be chosen as a material sublimation or melt energy or possibly a deposited energy density. There are, of course, many other choices which can be made for these reference quantities, in fact, the unit value may well be chosen for any reference quantity.

Once an appropriate set of three reference quantities has been chosen, all dimensional code input parameters can be converted to their non-dimensional forms by utilizing the relations given in the accompanying table. To obtain the proper non-dimensional output, which is non-dimensionalized with the same relations used for the input, it is essential that all code input variables be non-dimensionalized with respect to the same set of reference quantities.

To illustrate the procedure, the input parameters necessary to non-dimensionalize two sample problems will be given here. Both problems involve the same single material. It can be characterized with the following set of material properties and will utilize the normal equation of state for solids, STAT1.

$$\text{Density, } \rho_0 = 2.7 \text{ gm/cm}^3$$

$$\text{Sound speed, } c_0 = 5.0 \times 10^5 \text{ cm/sec}$$

$$U_s/u_p \text{ slope, } s = 1.5$$

$$\text{Gruneisen parameter, } \Gamma_0 = 2.0$$

$$\text{Poisson's ratio, } \nu = .333$$

$$\text{Yield strength, } Y_0 = 3.0 \text{ kb}$$

$$\text{Sublimation energy, } \mathcal{E}_s = 10^{11} \text{ ergs/gm}$$

Vapor  $\gamma - 1$  ,  $H = .25$

Spall strength,  $\sigma_{SPALL} = 20 \text{ kb}$

Note that the quantities  $s$ ,  $\Gamma_0$ ,  $v$ , and  $H$  are already non-dimensional.

This material represents a generic form of aluminum.

### Sample Problem No. 1

A 0.2 cm flyer plate impacts a target plate of the same material, 0.8 cm thick, at a velocity of  $1.0 \times 10^5 \text{ cm/sec}$ . The problem is to be run for  $2.0 \times 10^{-6} \text{ sec}$ .

The problem is non-dimensionalized by choosing:

$x_r =$  Flyer plate thickness (0.2 cm)

$\rho_r =$  Material density ( $2.7 \text{ gm/cm}^3$ )

$u_r =$  Material bulk sound speed ( $5.0 \times 10^5 \text{ cm/sec}$ )

The main problem input parameters therefore become:

Flyer plate thickness = 1.0       $\rho_0^* = 1.0$

Target plate thickness = 4.0       $c_0^* = 1.0$

Flyer plate velocity = 0.2       $\Gamma_0^* = 4.44 \times 10^{-3}$

Maximum problem time = 5.0       $\epsilon_s^* = 0.4$

Spall strength,  $\sigma_{SPALL}^* = 2.96 \times 10^{-2}$

Sample Problem No. 2

A single material layer 1.0 cm thick is subjected to a uniform energy deposition of  $1.5 \times 10^{10}$  ergs/gm applied over  $1.0 \times 10^{-7}$  sec. Maximum problem time is again  $2.0 \times 10^{-6}$  sec.

Non-dimensionalize with respect to:

$x_r$  = Plate thickness (1.0 cm)

$\mathcal{E}_r$  = Material sublimation energy ( $1.0 \times 10^{11}$  ergs/gm)

$\sigma_r$  = Material bulk modulus ( $K_0 = \rho_0 c_0^2 = 675$  kb)

The main problem input parameters are then

Plate thickness = 1.0		$\rho_0^* = 0.4$
Deposited energy density = .15		$c_0^* = 1.58$
Deposition time, TDEP* = $3.162 \times 10^{-2}$		$\gamma_0^* = 4.44 \times 10^{-3}$
Maximum problem time = 0.6324		$\mathcal{E}_s^* = 1.0$
Spall strength, $\sigma_{SPALL}^* = 2.96 \times 10^{-2}$		

It should be emphasized that success in running non-dimensional calculations is directly dependent on the proper choice of non-dimensional input parameters. In other words all incidental code parameters such as zone sizes, edit times, and activity stress, etc., must be put in the appropriate non-dimensional form.

Nondimensionalize with respect to	Dependent Variables				Independent Variables			
	$\sigma^*$	$\rho^*$	$u^*$	$\mathcal{E}^*$	And, in addition, non-dimensionalize with respect to			
					Length $x_r$		Time $t_r$	
					$x^*$	$t^*$	$x^*$	$t^*$
Velocity $u_r$ and Density $\rho_r$	$\frac{\sigma}{\rho_r u_r^2}$	$\frac{\rho}{\rho_r}$	$\frac{u}{u_r}$	$\frac{\mathcal{E}}{u_r^2}$	$\frac{x}{x_r}$	$\frac{t u_r}{x_r}$	$\frac{x}{u_r t_r}$	$\frac{t}{t_r}$
Stress $\sigma_r$ and Density $\rho_r$	$\frac{\sigma}{\sigma_r}$	$\frac{\rho}{\rho_r}$	$u \sqrt{\frac{\rho_r}{\sigma_r}}$	$\frac{\mathcal{E} \rho_r}{\sigma_r}$	$\frac{x}{x_r}$	$\frac{t}{x_r} \sqrt{\frac{\sigma_r}{\rho_r}}$	$\frac{x}{t_r} \sqrt{\frac{\rho_r}{\sigma_r}}$	$\frac{t}{t_r}$
Stress $\sigma_r$ and Velocity $u_r$	$\frac{\sigma}{\sigma_r}$	$\frac{\rho u_r^2}{\sigma_r}$	$\frac{u}{u_r}$	$\frac{\mathcal{E}}{u_r^2}$	$\frac{x}{x_r}$	$\frac{t u_r}{x_r}$	$\frac{x}{u_r t_r}$	$\frac{t}{t_r}$
Energy $\mathcal{E}_r$ and Density $\rho_r$	$\frac{\sigma}{\rho_r \mathcal{E}_r}$	$\frac{\rho}{\rho_r}$	$\frac{u}{\sqrt{\mathcal{E}_r}}$	$\frac{\mathcal{E} \rho_r}{\mathcal{E}_r}$	$\frac{x}{x_r}$	$\frac{t \sqrt{\mathcal{E}_r}}{x_r}$	$\frac{x}{t_r \sqrt{\mathcal{E}_r}}$	$\frac{t}{t_r}$
Energy $\mathcal{E}_r$ and Stress $\sigma_r$	$\frac{\sigma}{\sigma_r}$	$\frac{\rho \mathcal{E}_r}{\sigma_r}$	$\frac{u}{\sqrt{\mathcal{E}_r}}$	$\frac{\mathcal{E}}{\mathcal{E}_r}$	$\frac{x}{x_r}$	$\frac{t \sqrt{\mathcal{E}_r}}{x_r}$	$\frac{x}{t_r \sqrt{\mathcal{E}_r}}$	$\frac{t}{t_r}$

\*Quantities which have been non-dimensionalized.



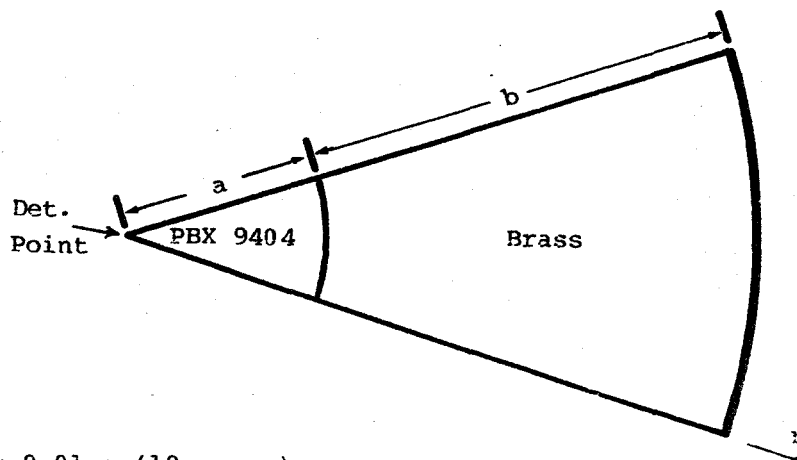
## APPENDIX D

### SAMPLE PROBLEMS

This appendix contains three problems, providing a sampling of the options available in WONDY: spherical divergence of a wave from an explosive source into a thick shell, energy deposition into a plate, and plate impact with subsequent spall. The record set required for each problem is reproduced, followed by selected output for that problem. The number of zones in each problem has purposely been kept small to allow printed output to be included. All problems are done with mks units (length - m, time - s, density -  $\text{kg/m}^3$ , stress -  $\text{N/m}^2(\text{Pa})$ ).

The printed output from each sample problem has been edited down to selected times to be representative of what the user can expect. The plots are excised from a larger figure, and may not contain all of the information the user will see upon executing this problem. As with the printed output, the plots are meant only to provide a representation of the more extensive output available to the user.

1. Thick spherical brass shell containing a charge of PBX-9404 explosive that is detonated at its center.



$$a = 0.01 \text{ m (10 zones)}$$

$$b = 0.04 \text{ m (40 zones)}$$

PBX-9404

$\rho_o = 1840 \text{ kg/m}^3$   
 $c_o = 8800 \text{ m/s}$   
 $D = 8800 \text{ m/s}$   
 $\gamma = 2.658$

Brass

$\rho_o = 8450 \text{ kg/m}^3$   
 $c_o = 3726 \text{ m/s}$   
 $s = 1.434$   
 $\Gamma_o = 2.04$   
 $\nu = 0.32$   
 $\gamma_o = 0.2 \text{ GPa}$

Input file for Sample Problem 1, with plotting instructions:

```

1      SAMPLE PROBLEM 1 - PBX9404 IN BRASS SHELL
2          3      2      10      1      3      5      1      1      15
3          1.0E-05          -1.0E+06      1.0E+06      0.0
5          0.0          1.0E-06      1.0
6          0.0          1.0E-08      1.0
10     1      2.0          20.0          0.01          0.0005      0.0005
15     1      1840.0      8800.0          2.658          8800.0          0.0          2.5
10     2      1.0          40.0          0.01          0.00025      0.00025
15     2      8450.0      3726.0
16     2      0.0          1.434
17     2      1.0          2.04
19     2      1.0          2.0E+08
{EOR}
{EOR}
XPLOT          0.0          1.0E-06      1.0
XDATA          1      0      1      -0.005      0.055      1
XLABEL        POSITION (( ) M)
YDATA          4      0      1      -1.0E10      7.0E+10
YLABEL        STRESS (( ) PA)
END

```

SAMPLE PROBLEM 1 - PBX9404 IN BRASS SHELL

CARD NO.	LPHA	NOF	NVAR	LIHT	RHBT	LACT	NJOB	NIL	NOL	NUL	MORE	JTAP	NSTART
2	3	2	10	1	3	5	1	0	0	15	0	0	0
CARD NO.	TMAX		DEL T (4)		SIGSEP		SIGACT		XZERO		LBCN		RBCN
3	7.0000E-06		0.0000E+00		-1.0000E+06		1.0000E+06		0.0000E+00		0.0000E+00		0.0000E+00
CARD NO.	KT1		KT2		B1		B2		SIGMAX		EMAX		HMAX
4	9.5000E-01		1.0500E+00		2.0000E+00		1.0000E-01		1.0000+100		1.0000E+00		1.0000+100
STANDARD EDIT													
CARD NO.	S1		DEL T1		E1-S2		DEL T2		E2-S3		DEL T3		E3
5	1.0000E-06		1.0000E-06		2.0000E-06		4.0000E-06		6.0000E-06		0.0000E+00		0.0000E+00
OUTPL													
CARD NO.	S1		DEL T1		E1-S2		DEL T2		E2-S3		DEL T3		E3
6	0.0000E+00		1.0000E-06		1.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00
OUTN													
CARD NO.	S1		DEL T1		E1-S2		DEL T2		E2-S3		DEL T3		E3
7	0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00
DUMP													
CARD NO.	S1		DEL T1		E1-S2		DEL T2		E2-S3		DEL T3		E3
8	0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00
PLOT VARIABLES													
CARD NO.	IPL1	IPL2	IPL3	IPL4	IPL5								
23	9	8	6	7	2								



PLATE	2	RT INT AT X	2. 0000E-02	RT INT AT ZONE	61			
CARD NO		STAIF	NOMESHES	THICKNESS	DELTA X	DELTA X1	XRATIO	XGAP
10		1. 0000E+00	4. 0000E+01	1. 0000E-02	2. 5000E-04	2. 5000E-04	0. 0000E+00	0. 0000E+00
CARD NO		RZTIME	DXMIN	DXMAX	RCCOMB	RSCRIT	RZCO	RZCI
11		1. 0000E+30	1. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00
CARD NO.		EZERO	PZERO	RZERO	SZERO	UZERO	UZERO1	ZZERO
12		0. 0000E+00	0. 0000E+00	8. 4500E+03	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00
CARD NO.		FCRIT	SIGMAF	FCNST	SIGMA0			
13		2. 0000E+00	-1. 0000+100	0. 0000E+00	0. 0000E+00			
CARD NO.		FCRIT1	SIGMAIF	CONST1	SIGMA01			
14		2. 0000E+00	-1. 0000+100	0. 0000E+00	0. 0000E+00			

HYDRO-VAPOR-ELASTIC-PLASTIC

CARD NO.		CES( 1)	CES( 2)	CES( 3)	CES( 4)	CES( 5)	CES( 6)	CES( 7)
15		8. 4500E+03	3. 7260E+03	0. 0000E+00	0. 0000E+00	0. 0000E+00	3. 2000E-01	0. 0000E+00
CARD NO.		CES( 8)	CES( 9)	CES(10)	CES(11)	CES(12)	CES(13)	CES(14)
16		0. 0000E+00	0. 0000E+00	1. 4340E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00
CARD NO		CES(15)	CES(16)	CES(17)	CES(18)	CES(19)	CES(20)	CES(21)
17		1. 0000E+00	2. 0400E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00
CARD NO.		CES(22)	CES(23)	CES(24)	CES(25)	CES(26)	CES(27)	CES(28)
18		0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00
CARD NO		CES(29)	CES(30)	CES(31)	CES(32)	CES(33)	CES(34)	CES(35)
19		1. 0000E+00	2. 0000E+08	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00

SAMPLE PROFILE # 1 - PBX9404 IN BRASS SHELL

CYCLE =	09	TIME =	1 00571E-06	TIME STEP =	2 51877E-08	LDT =	18							
	1	POSITION	VELOCITY	DENSITY	STRESS	PRESSURE	PHI	ENERGY	ART VISC	SND SPR	MASS			
1	0	0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00			
2	5	6253E-04	-5 7697E+00	1 2971E+03	8 7763E+09	2 8409E-08	0 0000E+00	4 1994E+06	1 1159E+07	4 3014E+03	2 3000E-07			
3	1	1082E-03	8 6516E+00	1 3600E+03	9 0482E+09	8 5227E-08	0 0000E+00	4 0103E+06	0 0000E+00	4 2040E+03	1 6100E-06			
4	1	6912E-03	-4 1286E+00	1 3912E+03	9 0228E+09	1 4205E-07	0 0000E+00	3 9117E+06	7 2669E+06	4 1519E+03	4 3700E-06			
5	2	1934E-03	4 0412E+00	1 4044E+03	9 0437E+09	1 7806E-07	0 0000E+00	3 8783E+06	0 0000E+00	4 1342E+03	8 5100E-06			
6	2	7357E-03	-1 8407E+01	1 4142E+03	9 0280E+09	2 5560E-07	0 0000E+00	3 8505E+06	1 9285E+07	4 1193E+03	1 4030E-05			
7	3	2829E-03	-4 4022E+01	1 4040E+03	8 7355E+09	3 1290E-07	0 0000E+00	3 7527E+06	2 8433E+07	4 0667E+03	2 0730E-05			
8	3	8271E-03	2 6977E+01	1 4128E+03	8 8234E+09	3 6932E-07	0 0000E+00	3 7667E+06	0 0000E+00	4 0743E+03	2 9210E-05			
9	4	3591E-03	1 5243E+02	1 4515E+03	9 3965E+09	4 2614E-07	0 0000E+00	3 9046E+06	0 0000E+00	4 1482E+03	3 8870E-05			
10	4	8754E-03	3 1005E+02	1 5101E+03	1 0374E+10	4 8295E-07	0 0000E+00	4 1434E+06	0 0000E+00	4 2731E+03	4 9910E-05			
11	5	3763E-03	4 7145E+02	1 5772E+03	1 1601E+10	5 3977E-07	0 0000E+00	4 4361E+06	0 0000E+00	4 4215E+03	6 2330E-05			
12	5	8632E-03	6 4517E+02	1 6494E+03	1 3013E+10	5 9659E-07	0 0000E+00	4 7585E+06	0 0000E+00	4 5794E+03	7 6130E-05			
13	6	3333E-03	8 3086E+02	1 7320E+03	1 4788E+10	6 5341E-07	0 0000E+00	5 1427E+06	0 0000E+00	4 7606E+03	9 1310E-05			
14	6	7937E-03	1 0291E+03	1 8196E+03	1 6789E+10	7 1023E-07	0 0000E+00	5 5650E+06	0 0000E+00	4 9522E+03	1 0787E-04			
15	7	2369E-03	1 2812E+03	1 9210E+03	1 9365E+10	7 6705E-07	0 0000E+00	6 0769E+06	0 0000E+00	5 1750E+03	1 2581E-04			
16	7	6625E-03	1 5394E+03	2 0475E+03	2 2850E+10	8 2386E-07	0 0000E+00	6 7308E+06	0 0000E+00	5 4463E+03	1 4513E-04			
17	8	0776E-03	1 2614E+03	2 1496E+03	2 5963E+10	8 8068E-07	0 0000E+00	7 2849E+06	3 8500E+08	5 6661E+03	1 6583E-04			
18	8	5177E-03	5 0382E+02	2 0661E+03	1 2863E+10	9 3750E-07	0 0000E+00	6 8840E+06	4 7388E+09	8 8000E+03	1 8791E-04			
19	9	0004E-03	1 3852E+01	1 4021E+03	1 6799E+09	9 9432E-07	0 0000E+00	6 4139E+06	2 3632E+09	8 8000E+03	2 1137E-04			
20	9	5000E-03	0 0000E+00	1 8412E+03	0 0000E+00	1 0511E-06	0 0000E+00	6 3842E+06	2 2501E+07	8 8000E+03	2 3621E-04			
WORK AT LBOUND		0 0000E+00		WORK AT RBOUND		0 0000E+00		INTERNAL ENERGY		0 39787E+05		KINETIC ENERGY		0 23844E+04
ADDED ENERGY		0 0000E+00		TOTAL MOMENTUM		0 44109E+03		REL ENERGY ERROR		0 0000E+00		REL. MDMENTUM ERROR		0 0000E+00

158

SAMPLE PROBLEM 1 - PHX9404 IN MASS SHELL

CYCLE = 80      TIME = 2.02712E-06      TIME STEP = 2.85415E-08      LDT = 37

L	POSITION	VELOCITY	DENSITY	STRESS	PRESSURE	PHI	ENERGY	ART VISC	SND SPD	MASS
1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2	5.6259E-04	8.2240E+00	1.2917E+03	8.9889E+09	2.8409E-08	0.0000E+00	4.1973E+06	0.0000E+00	0.0000E+00	2.3000E-07
3	1.1097E-03	2.8341E+00	1.3549E+03	8.9434E+09	8.5227E-08	0.0000E+00	3.9813E+06	0.0000E+00	0.0000E+00	4.3700E-06
4	1.6539E-03	5.6467E+00	1.3838E+03	8.8955E+09	1.4205E-07	0.0000E+00	3.8771E+06	0.0000E+00	0.0000E+00	1.6100E-06
5	2.1975E-03	4.9347E+00	1.3979E+03	8.8980E+09	1.9886E-07	0.0000E+00	3.8392E+06	0.0000E+00	0.0000E+00	4.1336E+03
6	2.7405E-03	5.4047E+00	1.4071E+03	8.9088E+09	2.5568E-07	0.0000E+00	3.8187E+06	0.0000E+00	0.0000E+00	8.5100E-06
7	3.2819E-03	-1.2912E+00	1.4172E+03	8.9575E+09	3.1250E-07	0.0000E+00	3.8121E+06	0.0000E+00	0.0000E+00	4.1023E+03
8	3.8236E-03	-2.4897E+00	1.4212E+03	8.9637E+09	3.6932E-07	0.0000E+00	3.8040E+06	0.0000E+00	0.0000E+00	4.0987E+03
9	4.3639E-03	-3.4528E+00	1.4287E+03	9.0108E+09	4.2614E-07	0.0000E+00	3.8039E+06	0.0000E+00	0.0000E+00	4.0944E+03
10	4.9024E-03	-5.0708E+01	1.4376E+03	9.1038E+09	4.8295E-07	0.0000E+00	3.8193E+06	0.0000E+00	0.0000E+00	4.0943E+03
11	5.4281E-03	-2.4847E+02	1.4800E+03	9.8070E+09	5.3977E-07	0.0000E+00	3.8193E+06	0.0000E+00	0.0000E+00	4.9910E-05
12	5.9271E-03	-5.4443E+02	1.5765E+03	1.1653E+10	5.9659E-07	0.0000E+00	4.9581E+06	0.0000E+00	0.0000E+00	6.2330E-05
13	6.4032E-03	-7.4938E+02	1.6812E+03	1.3929E+10	6.5341E-07	0.0000E+00	4.9973E+06	0.0000E+00	0.0000E+00	7.6130E-05
14	6.8718E-03	-7.3768E+02	1.7411E+03	1.5219E+10	7.1023E-07	0.0000E+00	5.2721E+06	0.0000E+00	0.0000E+00	8.584E+08
15	7.3601E-03	-4.6441E+02	1.6951E+03	1.4031E+10	7.6705E-07	0.0000E+00	5.2721E+06	0.0000E+00	0.0000E+00	4.8202E+03
16	7.8769E-03	-2.5998E+02	1.6123E+03	1.2197E+10	8.2386E-07	0.0000E+00	4.5626E+06	0.0000E+00	0.0000E+00	1.6906E+03
17	8.3990E-03	-1.3562E+02	1.5980E+03	1.1854E+10	8.8069E-07	0.0000E+00	4.4740E+06	0.0000E+00	0.0000E+00	4.6906E+03
18	8.9311E-03	6.0707E+01	1.5673E+03	1.1233E+10	9.3750E-07	0.0000E+00	4.3228E+06	0.0000E+00	0.0000E+00	1.4841E+03
19	9.4666E-03	1.9677E+02	1.5541E+03	1.0948E+10	9.9432E-07	0.0000E+00	4.2488E+06	0.0000E+00	0.0000E+00	4.4404E+03
20	1.0004E-02	3.5464E+02	1.5468E+03	1.0777E+10	1.0511E-06	0.0000E+00	4.2023E+06	0.0000E+00	0.0000E+00	4.3647E+03
21	1.0541E-02	4.8688E+02	1.5421E+03	1.0808E+10	1.1080E-06	0.0000E+00	4.2270E+06	0.0000E+00	0.0000E+00	4.4404E+03

MATERIAL INTERFACE

22	1.0752E-02	4.8084E+02	9.0375E+03	1.0956E+10	1.0822E+10	-2.0000E+08	1.1961E+05	0.0000E+00	5.2890E+03	6.4973E-04
23	1.0965E-02	4.7630E+02	9.0706E+03	1.1326E+10	1.1193E+10	-2.0000E+08	1.1156E+05	0.0000E+00	5.3090E+03	6.8221E-04
24	1.1179E-02	4.7149E+02	9.0947E+03	1.1572E+10	1.1438E+10	-2.0000E+08	1.0432E+05	0.0000E+00	5.3222E+03	7.1548E-04
25	1.1394E-02	4.7382E+02	9.1252E+03	1.1980E+10	1.1847E+10	-2.0000E+08	1.0039E+05	0.0000E+00	5.3434E+03	7.4954E-04
26	1.1610E-02	4.7097E+02	9.1592E+03	1.2499E+10	1.2366E+10	-2.0000E+08	9.9288E+04	0.0000E+00	5.3697E+03	7.8440E-04
27	1.1826E-02	4.7147E+02	9.1809E+03	1.2815E+10	1.2681E+10	-2.0000E+08	9.7692E+04	0.0000E+00	5.3856E+03	8.2005E-04
28	1.2044E-02	4.8483E+02	9.2212E+03	1.3417E+10	1.3368E+10	-7.3046E+07	9.9886E+04	0.0000E+00	5.4193E+03	8.5649E-04
29	1.2261E-02	4.8635E+02	9.2704E+03	1.4286E+10	1.4243E+10	-6.3760E+07	1.0435E+05	0.0000E+00	5.4613E+03	8.9372E-04
30	1.2479E-02	4.9779E+02	9.3068E+03	1.4819E+10	1.4897E+10	-1.0984E+08	1.0743E+05	0.0000E+00	5.4918E+03	9.3174E-04
31	1.2698E-02	5.0129E+02	9.3529E+03	1.5681E+10	1.5745E+10	9.6599E+07	1.1277E+05	0.0000E+00	5.5312E+03	9.7056E-04
32	1.2916E-02	5.1883E+02	9.3886E+03	1.6275E+10	1.6406E+10	2.0000E+08	1.1681E+05	0.0000E+00	5.5612E+03	1.0102E-03
33	1.3135E-02	5.2553E+02	9.4508E+03	1.7602E+10	1.7621E+10	2.8203E+07	1.2643E+05	0.0000E+00	5.6147E+03	1.0506E-03
34	1.3354E-02	5.4352E+02	9.4795E+03	1.8035E+10	1.8169E+10	2.0000E+08	1.3007E+05	0.0000E+00	5.6384E+03	1.0918E-03
35	1.3572E-02	5.5007E+02	9.5504E+03	1.9738E+10	1.9605E+10	-2.0000E+08	1.4291E+05	0.0000E+00	5.6991E+03	1.1338E-03
36	1.3792E-02	4.6173E+02	9.5294E+03	1.9261E+10	1.9127E+10	-2.0000E+08	1.3651E+05	5.7898E+08	5.6791E+03	1.1765E-03
37	1.4018E-02	2.8793E+02	9.2866E+03	1.4292E+10	1.4158E+10	-2.0000E+08	8.5977E+04	1.7431E+09	5.4577E+03	1.2201E-03
38	1.4255E-02	9.9071E+01	8.9015E+03	7.1609E+09	7.0205E+09	-2.0000E+08	2.6666E+04	1.9567E+09	5.0871E+03	1.2645E-03
39	1.4501E-02	2.0931E+01	8.5874E+03	2.0371E+09	1.9038E+09	-2.0000E+08	2.3520E+03	5.0279E+08	4.7671E+03	1.3096E-03
40	1.480E-02	6.8469E+00	8.4781E+03	5.2721E+08	3.9388E+08	-2.0000E+08	1.2597E+02	5.9843E+07	4.6606E+03	1.3556E-03
41	1.5000E-02	2.0405E+00	8.4592E+03	2.0010E+08	1.2756E+08	-1.0881E+08	1.4302E+01	1.8977E+07	4.6414E+03	1.4023E-03
42	1.530E-02	4.8124E-01	8.4376E+03	5.6628E+07	3.6170E+07	-3.0688E+07	1.1595E+00	6.0209E+06	4.6347E+03	1.4498E-03
43	1.5500E-02	9.1067E-02	8.4506E+03	1.2800E+07	8.1869E+06	-6.9195E+06	9.9866E-02	1.4958E+06	4.6326E+03	1.4988E-03
44	1.5750E-02	1.2742E-02	8.4501E+03	2.2991E+06	1.4841E+06	-1.2224E-03	1.7612E-03	3.0028E+05	4.6321E+03	1.5473E-03
45	1.6000E-02	0.0000E+00	8.4500E+03	2.2869E+05	1.6798E+05	-9.1059E+04	2.2845E-05	3.9494E+04	4.6320E+03	1.5972E-03

WORK AT LBOUND      0.0000E+00  
INTERNAL ENERGY    0.41129E+05  
ADDED ENERGY        0.00000E+00  
REL. ENERGY ERROR   0.00000E+00

WORK AT RBOUND      0.00000E+00  
KINETIC ENERGY      0.80765E+04  
TOTAL MOMENTUM        0.31010E+02  
REL. MOMENTUM ERROR   0.00000E+00

SAMPLE PROBLEM 1 - PBX9404 IN BRASS SHELL

CYCLE = 214 TIME = 6.00197E-06 TIME STEP = 2.89097E-08 LDT = 22

	POSITION	VELOCITY	DENSITY	STRESS	PRESSURE	PHI	ENERGY	ART VISC	SND SPD	MASS
1	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00
2	7 1615E-04	8 2260E+01	5 2073E+02	1 1622E+09	2 8409E-08	0 0000E+00	1 3461E+06	0 0000E+00	2 4356E+03	2 3000E-07
3	1 4841E-03	9 1102E-01	5 6947E+02	1 1192E+09	8 5227E-08	0 0000E+00	1 1853E+06	7 3238E+06	2 2859E+03	1 6100E-06
4	2 1734E-03	-1 4601E+02	6 2448E+02	1 2641E+09	1 4205E-07	0 0000E+00	1 2209E+06	1 3335E+08	2 3196E+03	4 3700E-06
5	2 8356E-03	-2 3224E+02	6 7904E+02	1 4512E+09	1 9886E-07	0 0000E+00	1 2890E+06	1 2420E+08	2 3834E+03	8 5100E-06
6	3 4733E-03	-3 0025E+02	7 3450E+02	1 6970E+09	2 558E-07	0 0000E+00	1 3935E+06	1 2207E+08	2 4781E+03	1 4030E-05
7	4 0872E-03	-2 9044E+02	7 9345E+02	2 0141E+09	3 1250E-07	0 0000E+00	1 5310E+06	4 0852E+07	2 5975E+03	2 0930E-05
8	4 6952E-03	-2 4958E+02	8 2907E+02	2 2236E+09	3 6932E-07	0 0000E+00	1 6176E+06	1 1133E+07	2 6700E+03	2 9210E-05
9	5 3029E-03	-2 0735E+02	8 5268E+02	2 3954E+09	4 2614E-07	0 0000E+00	1 6661E+06	3 5840E+06	2 7097E+03	3 8870E-05
10	5 9133E-03	-1 8604E+02	8 6535E+02	2 4231E+09	4 8295E-07	0 0000E+00	1 6888E+06	6 6319E+06	2 7281E+03	4 9910E-05
11	6 5209E-03	-1 6845E+02	8 8393E+02	2 5471E+09	5 3977E-07	0 0000E+00	1 7380E+06	5 1639E+06	2 7675E+03	6 2330E-05
12	7 1315E-03	-1 3037E+02	8 9136E+02	2 5907E+09	5 9659E-07	0 0000E+00	1 7530E+06	0 0000E+00	2 7794E+03	7 6130E-05
13	7 7363E-03	-1 0284E+02	9 1007E+02	2 7328E+09	6 5341E-07	0 0000E+00	1 8111E+06	0 0000E+00	2 8252E+03	9 1310E-05
14	8 3372E-03	-6 0939E+01	9 2599E+02	2 8414E+09	7 1023E-07	0 0000E+00	1 8507E+06	0 0000E+00	2 8559E+03	1 0787E-04
15	8 9278E-03	-2 8095E+01	9 5254E+02	3 0322E+09	7 6705E-07	0 0000E+00	1 9199E+06	0 0000E+00	2 9088E+03	1 2581E-04
16	9 5127E-03	2 7481E+01	9 7258E+02	3 1823E+09	8 2386E-07	0 0000E+00	1 9735E+06	0 0000E+00	2 9491E+03	1 4513E-04
17	1 0096E-02	1 2488E+02	9 8515E+02	3 2769E+09	8 8068E-07	0 0000E+00	2 0062E+06	0 0000E+00	2 9734E+03	1 6583E-04
18	1 0692E-02	3 0225E+02	9 7283E+02	3 1623E+09	9 3750E-07	0 0000E+00	1 9606E+06	0 0000E+00	2 9394E+03	1 8791E-04
19	1 1312E-02	5 3802E+02	9 3917E+02	2 8703E+09	9 9432E-07	0 0000E+00	1 8433E+06	0 0000E+00	2 8501E+03	2 1137E-04
20	1 1967E-02	8 2918E+02	8 8671E+02	2 4557E+09	1 0511E-06	0 0000E+00	1 6703E+06	0 0000E+00	2 7131E+03	2 3621E-04
21	1 2681E-02	1 1906E+03	8 0643E+02	1 9290E+09	1 1080E-06	0 0000E+00	1 4427E+06	0 0000E+00	2 5215E+03	2 6243E-04
----- MATERIAL INTERFACE -----										
22	1 2839E-02	1 1647E+03	8 4288E+03	1 2441E+09	1 1108E+09	-2 0000E+08	8 1665E+04	0 0000E+00	4 6944E+03	6 4973E-04
23	1 3001E-02	1 1328E+03	8 3745E+03	2 9053E+08	1 5720E+08	-2 0000E+08	7 0053E+04	1 1593E+07	4 6269E+03	6 8221E-04
24	1 3167E-02	1 0943E+03	8 3078E+03	-7 7651E+08	-9 0984E+08	-2 0000E+08	6 1162E+04	4 0204E+07	4 5478E+03	7 1548E-04
25	1 3342E-02	1 0528E+03	8 2307E+03	-1 9084E+09	-2 0418E+09	-2 0000E+08	5 5704E+04	5 4768E+07	4 4593E+03	7 4954E-04
26	1 3519E-02	1 0100E+03	8 1605E+03	-2 8817E+09	-3 0151E+09	-2 0000E+08	5 3178E+04	6 2526E+07	4 3793E+03	7 8440E-04
27	1 3701E-02	9 5995E+02	8 1008E+03	-3 6779E+09	-3 8113E+09	-2 0000E+08	5 2199E+04	9 9182E+07	4 3110E+03	8 2005E-04
28	1 3887E-02	9 0018E+02	8 0720E+03	-4 0761E+09	-4 2095E+09	-2 0000E+08	5 0699E+04	9 2763E+07	4 2636E+03	8 5649E-04
29	1 4076E-02	8 3280E+02	8 0800E+03	-4 0228E+09	-4 1561E+09	-2 0000E+08	4 7653E+04	2 1114E+08	4 2821E+03	8 9372E-04
30	1 4267E-02	7 6697E+02	8 1018E+03	-3 7984E+09	-3 9317E+09	-2 0000E+08	4 4191E+04	2 1435E+08	4 3031E+03	9 3174E-04
31	1 4460E-02	7 0668E+02	8 1370E+03	-3 4040E+09	-3 5373E+09	-2 0000E+08	4 0303E+04	1 9354E+08	4 3388E+03	9 7056E-04
32	1 4654E-02	6 5027E+02	8 1665E+03	-3 0739E+09	-3 2072E+09	-2 0000E+08	3 6886E+04	1 8180E+08	4 3681E+03	1 0102E-03
33	1 4851E-02	6 0094E+02	8 1866E+03	-2 8552E+09	-2 9886E+09	-2 0000E+08	3 4099E+04	1 5039E+08	4 3875E+03	1 0506E-03
34	1 5049E-02	5 5885E+02	8 2042E+03	-2 6645E+09	-2 7978E+09	-2 0000E+08	3 1610E+04	1 1852E+08	4 4042E+03	1 0918E-03
35	1 5249E-02	5 2324E+02	8 2226E+03	-2 4597E+09	-2 5930E+09	-2 0000E+08	2 9292E+04	9 1616E+07	4 4219E+03	1 1338E-03
36	1 5451E-02	4 9275E+02	8 2420E+03	-2 2330E+09	-2 3664E+09	-2 0000E+08	2 7123E+04	7 1797E+07	4 4412E+03	1 1765E-03
37	1 5655E-02	4 6629E+02	8 2624E+03	-1 9971E+09	-2 1304E+09	-2 0000E+08	2 5150E+04	5 7277E+07	4 4609E+03	1 2201E-03
38	1 5860E-02	4 4307E+02	8 2827E+03	-1 7632E+09	-1 8966E+09	-2 0000E+08	2 3371E+04	4 6314E+07	4 4803E+03	1 2645E-03
39	1 6066E-02	4 2242E+02	8 3004E+03	-1 5434E+09	-1 6767E+09	-2 0000E+08	2 1781E+04	3 8083E+07	4 4982E+03	1 3096E-03
40	1 6274E-02	4 0392E+02	8 3167E+03	-1 3465E+09	-1 4799E+09	-2 0000E+08	2 0371E+04	3 1517E+07	4 5141E+03	1 3556E-03
41	1 6483E-02	3 8729E+02	8 3312E+03	-1 1715E+09	-1 3048E+09	-2 0000E+08	1 9125E+04	2 5888E+07	4 5282E+03	1 4023E-03
42	1 6694E-02	3 7230E+02	8 3442E+03	-1 0126E+09	-1 1459E+09	-2 0000E+08	1 8008E+04	2 1179E+07	4 5409E+03	1 4498E-03
43	1 6905E-02	3 5870E+02	8 3560E+03	-8 6890E+08	-1 0029E+09	-2 0000E+08	1 7006E+04	1 7262E+07	4 5522E+03	1 4982E-03
44	1 7118E-02	3 4635E+02	8 3665E+03	-7 4112E+08	-8 7445E+08	-2 0000E+08	1 6094E+04	1 3793E+07	4 5623E+03	1 5473E-03
45	1 7333E-02	3 3507E+02	8 3758E+03	-6 2798E+08	-7 6131E+08	-2 0000E+08	1 5276E+04	1 0953E+07	4 5712E+03	1 5972E-03
46	1 7548E-02	3 2477E+02	8 3837E+03	-5 2895E+08	-6 6228E+08	-2 0000E+08	1 4556E+04	8 3894E+06	4 5787E+03	1 6479E-03
47	1 7765E-02	3 1535E+02	8 3907E+03	-4 4364E+08	-5 7697E+08	-2 0000E+08	1 3932E+04	6 0778E+06	4 5855E+03	1 6994E-03
48	1 7982E-02	3 0667E+02	8 3967E+03	-3 7337E+08	-5 0671E+08	-2 0000E+08	1 3356E+04	4 2704E+06	4 5910E+03	1 7517E-03
49	1 8201E-02	2 9861E+02	8 4013E+03	-3 1896E+08	-4 5229E+08	-2 0000E+08	1 2856E+04	2 8821E+06	4 5952E+03	1 8047E-03
50	1 8421E-02	2 9109E+02	8 4047E+03	-2 7928E+08	-4 1261E+08	-2 0000E+08	1 2415E+04	1 6647E+06	4 5983E+03	1 8536E-03
51	1 8642E-02	2 8410E+02	8 4075E+03	-2 4994E+08	-3 8330E+08	-2 0000E+08	1 1933E+04	5 0902E+05	4 6006E+03	1 9133E-03
52	1 8864E-02	2 7763E+02	8 4097E+03	-2 2715E+08	-3 6048E+08	-2 0000E+08	1 1501E+04	0 0000E+00	4 6034E+03	1 9687E-03
53	1 9086E-02	2 7156E+02	8 4114E+03	-2 0849E+08	-3 4187E+08	-2 0000E+08	1 1197E+04	0 0000E+00	4 6034E+03	2 0250E-03
54	1 9310E-02	2 6565E+02	8 4134E+03	-1 8742E+08	-3 2076E+08	-2 0000E+08	1 0799E+04	0 0000E+00	4 6036E+03	2 0820E-03



SAMPLE PROBLEM 1 - PBX9404 IN BRASS SHELL

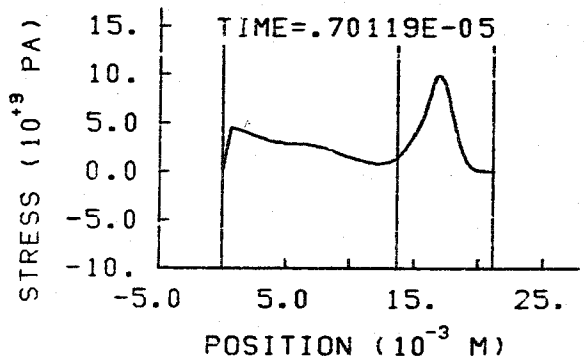
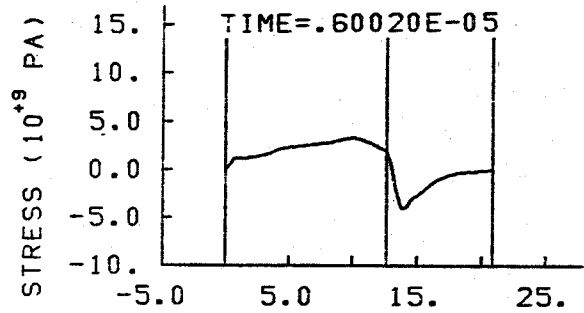
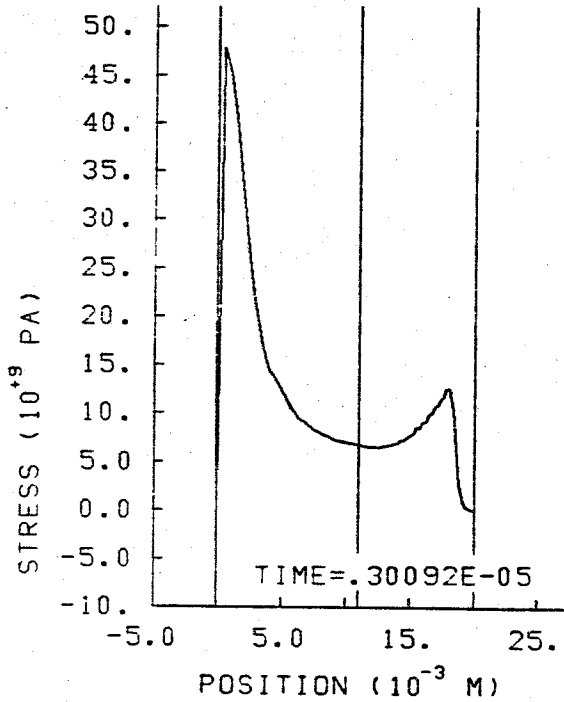
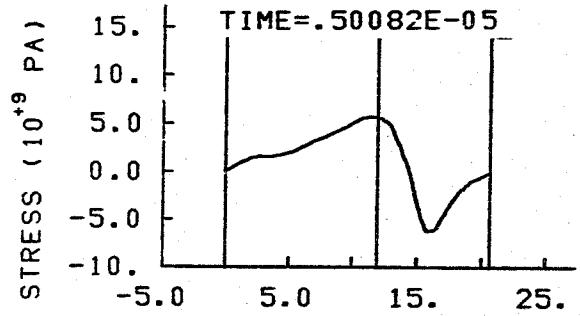
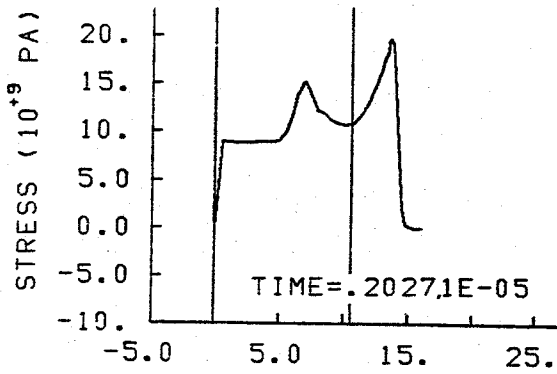
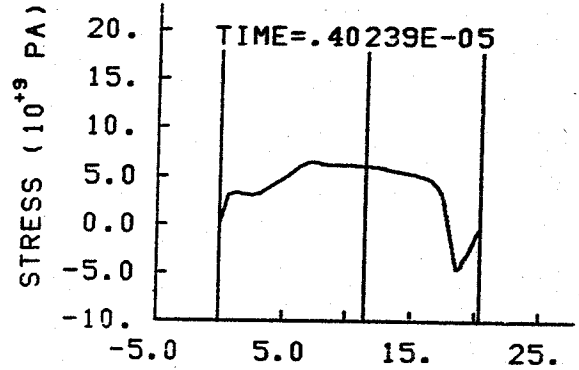
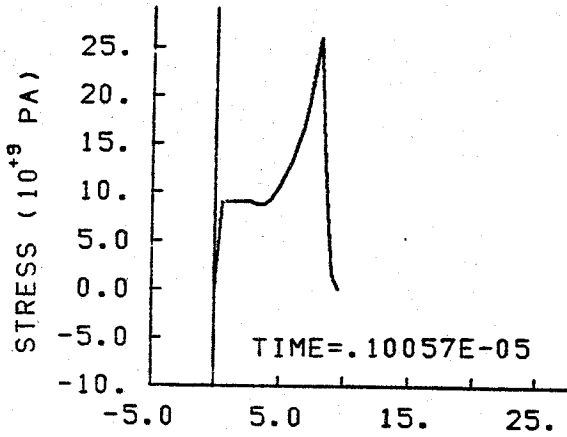
CYCLE = 214      TIME = 6 00197E-06      TIME STEP = 2.89097E-08      LDT = 22

L	POSITION	VELOCITY	DENSITY	STRESS	PRESSURE	PHI	ENERGY	ART VISC	SND SPD	MASS
55	1 9535E-02	2 5980E+02	8 4158E+03	-1.6306E+08	-2.9639E+08	-2.0000E+08	1.0276E+04	0.0000E+00	4.6075E+03	2.1398E-03
56	1 9760E-02	2 5400E+02	8 4182E+03	-1.3859E+08	-2.7192E+08	-2.0000E+08	9.8290E+03	0.0000E+00	4.6095E+03	2.1985E-03
57	1 9987E-02	2 4825E+02	8 4207E+03	-1.1298E+08	-2.4632E+08	-2.0000E+08	9.3012E+03	1.2802E+05	4.6115E+03	2.2579E-03
58	2 0214E-02	2 4261E+02	8 4231E+03	-8.8211E+07	-2.2154E+08	-2.0000E+08	8.7585E+03	3.7365E+05	4.6135E+03	2.3181E-03
59	2 0442E-02	2 3713E+02	8 4258E+03	-6.3325E+07	-1.9666E+08	-2.0000E+08	8.0329E+03	3.7225E+05	4.6155E+03	2.3791E-03
60	2 0670E-02	2 3187E+02	8 4296E+03	-3.8300E+07	-1.7163E+08	-2.0000E+08	6.4169E+03	1.8313E+05	4.6178E+03	2.4409E-03
61	2 0899E-02	2 2681E+02	8 4358E+03	-1.2376E+07	-1.4571E+08	-2.0000E+08	2.9707E+03	2.7322E+04	4.6205E+03	2.5034E-03

-----MATERIAL INTERFACE-----

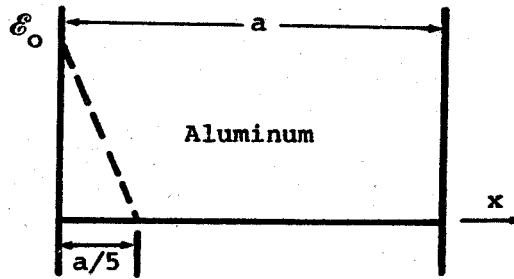
WORK AT LBOUND	0.00000E+00	WORK AT RBOUND	0.00000E+00
INTERNAL ENERGY	0.18790E+05	KINETIC ENERGY	0.30398E+05
ADDED ENERGY	0.00000E+00	TOTAL MOMENTUM	0.10649E+03
REL. ENERGY ERROR	0.00000E+00	REL. MOMENTUM ERROR	0.00000E+00

SAMPLE PROBLEM 1 - PBX9404 IN BRASS SHELL



2. Energy deposition in a thick aluminum plate.

$a = 0.01$  m  
(50 zones)



Triangular deposition of energy,  $\epsilon_0 = 1.0 (10^6)$  Joules/kg.  
Deposition time = 1 ns

Aluminum

$\rho_0 = 2785$  kg/m<sup>3</sup> (Fracture suppressed)  
 $c_0 = 5355$  m/g  
 $s = 1.345$   
 $\Gamma_0 = 2.1$   
 $\nu = 0.333$   
 $Y_0 = 75$  MPa

Input file for Sample Problem 2, with plotting instructions:

```

1      SAMPLE PROBLEM 2 - ENERGY DEPOSITION IN ALUMINUM PLATE
2      1      1      11      3      3      25      1      1      15      1
3      5.0E-06      1.0E-09      -1.0E+06      1.0E+06      0.0
5      0.0      0.5E-06      1.0
6      0.0      1.0E-08      1.0
10     1      1.0      50.0      0.01      0.0002      0.0002
15     1      2785.0      5355.0
16     1      0.0
17     1      2.0      2.1      -1.0
19     1      1.0      7.5E+07
{EOR}
31     1.0E-09      0.0
38     2      0.0      1.0E+06      0.002      0.0
{EOR}
{EOR}
XPLOT      0.0      0.1E-06      1.0
XDATA      1      0      1      -0.002      0.04      1
XLABEL      POSITION (( ) M)
YDATA      4      0      1      -0.3E+10      1.7E+10
YLABEL      STRESS (( ) PA)
END
    
```

## SAMPLE PROBLEM 2 -- ENERGY DEPOSITION IN ALUMINUM PLATE

CARD NO.	LPHA	NOP	NVAR	LHMT	RHBT	LACT	NJOB	NIL	NOL	NUL	MORE	JTAFE	NSTART
2	1	1	11	3	3	25	1	0	0	15	1	0	0
CARD NO		TMAX		DEL T(4)		SIGSEP		SIGACT		XZERO		LBCN	RBCN
3		3.0000E-06		1.0000E-09		1.0000E+06		1.0000E+06		0.0000E+00		0.0000E+00	0.0000E+00
CARD NO		KT1		KT2		B1		B2		SIGMAX		EMAX	HMAX
4		9.5000E-01		1.0500E+00		2.0000E+00		1.0000E-01		1.0000+100		1.0000E+00	1.0000+100
STANDARD EDIT													
CARD NO.		S1		DEL T1		E1=S2		DEL T2		E2=S3		DEL T3	E3
5		1.0000E-06		1.0000E-06		2.0000E-06		0.0000E+00		0.0000E+00		0.0000E+00	0.0000E+00
OUTPL													
CARD NO.		S1		DEL T1		E1=S2		DEL T2		E2=S3		DEL T3	E3
6		0.0000E+00		2.0000E-07		1.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00	0.0000E+00
OUTN													
CARD NO.		S1		DEL T1		E1=S2		DEL T2		E2=S3		DEL T3	E3
7		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00	0.0000E+00
DUMP													
CARD NO		S1		DEL T1		E1=S2		DEL T2		E2=S3		DEL T3	E3
8		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00		0.0000E+00	0.0000E+00
PLOT VARIABLES													
CARD NO.		IPL1		IPL2		IPL3		IPL4		IPL5			
23		9		8		6		7		2			

PLATE	1	RT INT AT X	1	0000E-02	RT INT AT ZONE	51											
CARD NO	10	STATE	1	0000E+00	NOMENBES	5	0000E+01	THICKNESS	1.0000E-02	DELTA X	2.0000E-04	DELTA X1	2.0000E-04	XNATIO	0.0000E+00	XCAP	0.0000E+00
CARD NO	11	R7TIME	1	0000E+30	DXMIN	1.0000E+00		DXMAX	0.0000E+00	RCCOMB	0.0000E+00	HSCRIT	0.0000E+00	RZCD	0.0000E+00	RZCI	0.0000E+00
CARD NO	12	EZERO	0	0000E+00	PZERO	0.0000E+00		RZERO	2.7850E+03	SZERO	0.0000E+00	UZERO	0.0000E+00	UZERO1	0.0000E+00	ZZERO	0.0000E+00
CARD NO.	13	FCRIT	2	0000E+00	SIGMAF	-1.0000E+100		FCONST	0.0000E+00	SIGMA0	0.0000E+00						
CARD NO	14	FCRIT1	2	0000E+00	SIGMA1F	-1.0000E+100		CONST1	0.0000E+00	SIGMA01	0.0000E+00						

HYDRO-VAPOR-ELASTIC-PLASTIC

CARD NO.	15	CES(1)	2	7850E+03	CES(2)	5	3550E+03	CES(3)	0.0000E+00	CES(4)	0.0000E+00	CES(5)	0.0000E+00	CES(6)	3.3300E-01	CES(7)	0.0000E+00
CARD NO.	16	CES(8)	0	0000E+00	CES(9)	0	0000E+00	CES(10)	1.3450E+00	CES(11)	0.0000E+00	CES(12)	0.0000E+00	CES(13)	0.0000E+00	CES(14)	0.0000E+00
CARD NO.	17	CES(15)	2	0000E+00	CES(16)	2	1000E+00	CES(17)	-1.0000E+00	CES(18)	0.0000E+00	CES(19)	0.0000E+00	CES(20)	0.0000E+00	CES(21)	0.0000E+00
CARD NO.	18	CES(22)	0	0000E+00	CES(23)	0	0000E+00	CES(24)	0.0000E+00	CES(25)	0.0000E+00	CES(26)	0.0000E+00	CES(27)	0.0000E+00	CES(28)	0.0000E+00
CARD NO.	19	CES(29)	1	0000E+00	CES(30)	7	5000E+07	CES(31)	0.0000E+00	CES(32)	0.0000E+00	CES(33)	0.0000E+00	CES(34)	0.0000E+00	CES(35)	0.0000E+00

## ARBITRARY ENERGY DENSITY PROFILE DATA

TIME DURATION OF ENERGY SOURCES 0.100E-08

INPUT ABSORBED FLUX 0.000000E+00

ACTUAL ABSORBED FLUX 0.278500E+07

NO OF POINTS IN ARBITRARY DEPOSITION PROFILE 2

POSITION	ARBITRARY ENERGY DENSITY	NORMALIZED ENERGY DENSITY
-0.0000E+00	0.100E+07	0.100E+07
0.2000E-02	0.000E+00	0.000E+00

## ENERGY DENSITIES FOR PLATE 1

2 0.950E+06	3 0.850E+06	4 0.750E+06	5 0.650E+06	6 0.550E+06	7 0.450E+06
8 0.350E+06	9 0.250E+06	10 0.150E+06	11 0.500E+05	12 0.000E+00	13 0.000E+00
14 0.000E+00	15 0.000E+00	16 0.000E+00	17 0.000E+00	18 0.000E+00	19 0.000E+00
20 0.000E+00	21 0.000E+00	22 0.000E+00	23 0.000E+00	24 0.000E+00	25 0.000E+00
26 0.000E+00	27 0.000E+00	28 0.000E+00	29 0.000E+00	30 0.000E+00	31 0.000E+00
32 0.000E+00	33 0.000E+00	34 0.000E+00	35 0.000E+00	36 0.000E+00	37 0.000E+00
38 0.000E+00	39 0.000E+00	40 0.000E+00	41 0.000E+00	42 0.000E+00	43 0.000E+00
44 0.000E+00	45 0.000E+00	46 0.000E+00	47 0.000E+00	48 0.000E+00	49 0.000E+00
50 0.000E+00	51 0.000E+00				

SAMPLE PROBLEM 2 - ENERGY DEPOSITION IN ALUMINUM PLATE

CYCLE =	TIME =	TIME STEP =	LDT =											
282	1 02303E-06	2 43427E-08	29	L	POSITION	VELOCITY	DENSITY	STRESS	PRESSURE	PHI	ENERGY	ART VISC	SND SPII	MASS
	1	8 0703E-05	-6 9910E-02	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00	0 0000E+00
	2	1 3406E-04	5 6381E-01	2 5936E+03	-5 0201E+05	-4 3536E+07	-6 4551E+07	8 7110E+05	0 0000E+00	6 2763E+03	5 5700E-01	6 2763E+03	5 5700E-01	6 2763E+03
	3	3 4713E-04	8 4962E-01	2 6141E+03	1 5107E+07	-3 2582E+07	-7 1534E+07	8 0063E+05	0 0000E+00	6 3067E+03	5 5700E-01	6 3067E+03	5 5700E-01	6 3067E+03
	4	5 5861E-04	1 0383E+00	2 6337E+03	2 1792E+07	-2 5667E+07	-7 1188E+07	7 1166E+05	0 0000E+00	6 3358E+03	5 5700E-01	6 3358E+03	5 5700E-01	6 3358E+03
	5	7 6852E-04	1 1980E+00	2 6536E+03	3 1064E+07	-1 8272E+07	-7 4003E+07	6 2216E+05	0 0000E+00	6 3651E+03	5 5700E-01	6 3651E+03	5 5700E-01	6 3651E+03
	6	9 7686E-04	1 0933E+00	2 6734E+03	3 6135E+07	-1 3865E+07	-7 5000E+07	5 3103E+05	1 7903E+05	6 3943E+03	5 5700E-01	6 3943E+03	5 5700E-01	6 3943E+03
	7	1 1837E-03	8 2218E-01	2 6935E+03	4 1025E+07	-8 9753E+06	-7 5000E+07	4 3793E+05	4 6992E+05	6 4241E+03	5 5700E-01	6 4241E+03	5 5700E-01	6 4241E+03
	8	1 3889E-03	7 7684E-01	2 7133E+03	3 6994E+07	-1 3006E+07	-7 5000E+07	3 4364E+05	7 9402E+04	6 4529E+03	5 5700E-01	6 4529E+03	5 5700E-01	6 4529E+03
	9	1 5927E-03	3 5397E-01	2 7332E+03	3 3587E+07	-1 6413E+07	-7 5000E+07	2 4822E+05	7 5105E+05	6 4820E+03	5 5700E-01	6 4820E+03	5 5700E-01	6 4820E+03
	10	1 7950E-03	1 7951E-01	2 7532E+03	2 8913E+07	-2 1087E+07	-7 5000E+07	1 5131E+05	3 1308E+05	6 5113E+03	5 5700E-01	6 5113E+03	5 5700E-01	6 5113E+03
	11	1 9959E-03	7 5474E-02	2 7731E+03	1 9530E+07	-3 0462E+07	-7 5000E+07	5 2863E+04	1 8881E+05	6 5404E+03	5 5700E-01	6 5404E+03	5 5700E-01	6 5404E+03
	12	2 1960E-03	-6 0210E-01	2 7830E+03	1 7480E+07	-3 2320E+07	-7 5000E+07	3 0066E+03	1 2412E+06	6 5556E+03	5 5700E-01	6 5556E+03	5 5700E-01	6 5556E+03
	13	2 3961E-03	-1 2417E+00	2 7833E+03	1 9810E+07	-3 0170E+07	-7 5000E+07	2 7696E+03	1 1714E+06	6 5559E+03	5 5700E-01	6 5559E+03	5 5700E-01	6 5559E+03
	14	2 5963E-03	-2 2199E+00	2 7819E+03	5 7527E+06	-4 4247E+07	-7 5000E+07	2 9063E+03	1 7943E+06	6 5539E+03	5 5700E-01	6 5539E+03	5 5700E-01	6 5539E+03
	15	2 7965E-03	-5 4834E+00	2 7807E+03	-2 0448E+07	-7 0448E+07	-7 5000E+07	2 7875E+03	6 0603E+06	6 5503E+03	5 5700E-01	6 5503E+03	5 5700E-01	6 5503E+03
	16	2 9967E-03	-1 1458E+01	2 7801E+03	-2 0471E+07	-1 2447E+08	-7 5000E+07	2 7817E+03	1 1248E+07	6 5428E+03	5 5700E-01	6 5428E+03	5 5700E-01	6 5428E+03
	17	3 1975E-03	-2 0563E+01	2 7763E+03	-1 8312E+08	-2 3312E+08	-7 5000E+07	2 8072E+03	1 7379E+07	6 5275E+03	5 5700E-01	6 5275E+03	5 5700E-01	6 5275E+03
	18	3 3985E-03	-3 4365E+01	2 7710E+03	-3 3273E+08	-3 8273E+08	-7 5000E+07	2 8543E+03	2 6902E+07	6 5064E+03	5 5700E-01	6 5064E+03	5 5700E-01	6 5064E+03
	19	3 6000E-03	-5 2333E+01	2 7638E+03	-5 3651E+08	-5 8651E+08	-7 5000E+07	3 2288E+03	3 5577E+07	6 4773E+03	5 5700E-01	6 4773E+03	5 5700E-01	6 4773E+03
	20	3 8023E-03	-7 1866E+01	2 7538E+03	-8 1416E+08	-8 6416E+08	-7 5000E+07	4 0315E+03	3 8640E+07	6 4369E+03	5 5700E-01	6 4369E+03	5 5700E-01	6 4369E+03
	21	4 0054E-03	-9 9319E+01	2 7424E+03	-1 1290E+09	-1 1790E+09	-7 5000E+07	5 3434E+03	5 6009E+07	6 3903E+03	5 5700E-01	6 3903E+03	5 5700E-01	6 3903E+03
	22	4 2096E-03	-1 2410E+02	2 7277E+03	-1 5262E+09	-1 5762E+09	-7 5000E+07	7 8307E+03	4 9181E+07	6 3301E+03	5 5700E-01	6 3301E+03	5 5700E-01	6 3301E+03
	23	4 4146E-03	-1 4445E+02	2 7169E+03	-1 8139E+09	-1 8639E+09	-7 5000E+07	1 0166E+04	3 9074E+07	6 2854E+03	5 5700E-01	6 2854E+03	5 5700E-01	6 2854E+03
	24	4 6205E-03	-1 5243E+02	2 7060E+03	-2 1667E+09	-2 1497E+09	-7 5000E+07	2 3468E+07	1 3015E+04	6 2400E+03	5 5700E-01	6 2400E+03	5 5700E-01	6 2400E+03
	25	4 8261E-03	-1 1362E+02	2 7086E+03	-2 1325E+09	-2 1825E+09	-7 5000E+07	1 2830E+04	0 0000E+00	6 2508E+03	5 5700E-01	6 2508E+03	5 5700E-01	6 2508E+03
	26	5 0301E-03	-6 0078E+01	2 7312E+03	-1 5360E+09	-1 4860E+09	-7 5000E+07	6 6046E+03	0 0000E+00	6 3441E+03	5 5700E-01	6 3441E+03	5 5700E-01	6 3441E+03
	27	5 2319E-03	-3 0115E+00	2 7597E+03	-7 5684E+08	-7 0684E+08	-7 5000E+07	2 2443E+03	0 0000E+00	6 4603E+03	5 5700E-01	6 4603E+03	5 5700E-01	6 4603E+03
	28	5 4314E-03	4 9569E+01	2 7892E+03	7 7034E+07	1 2703E+08	7 5000E+07	9 1820E+02	0 0000E+00	6 5783E+03	5 5700E-01	6 5783E+03	5 5700E-01	6 5783E+03
	29	5 6294E-03	8 5789E+01	2 8155E+03	8 4404E+08	8 9404E+08	7 5000E+07	2 4403E+03	0 0000E+00	6 6816E+03	5 5700E-01	6 6816E+03	5 5700E-01	6 6816E+03
	30	5 8261E-03	9 7314E+01	2 8319E+03	1 3377E+09	1 3877E+09	7 5000E+07	4 6805E+03	0 0000E+00	6 7458E+03	5 5700E-01	6 7458E+03	5 5700E-01	6 7458E+03
	31	6 0225E-03	9 6981E+01	2 8357E+03	1 4626E+09	1 5034E+09	6 1160E+07	5 3261E+03	6 3935E+05	6 7606E+03	5 5700E-01	6 7606E+03	5 5700E-01	6 7606E+03
	32	6 2189E-03	1 0650E+02	2 8358E+03	1 4679E+09	1 5056E+09	5 6489E+07	5 3329E+03	0 0000E+00	6 7609E+03	5 5700E-01	6 7609E+03	5 5700E-01	6 7609E+03
	33	6 4151E-03	1 0864E+02	2 8397E+03	1 6673E+09	1 6288E+09	-5 7627E+07	6 1101E+03	0 0000E+00	6 7765E+03	5 5700E-01	6 7765E+03	5 5700E-01	6 7765E+03
	34	6 6113E-03	9 6028E+01	2 8391E+03	1 6542E+09	1 6042E+09	-7 5000E+07	5 9374E+03	2 5978E+07	6 7734E+03	5 5700E-01	6 7734E+03	5 5700E-01	6 7734E+03
	35	6 8080E-03	7 6516E+01	2 8316E+03	1 4275E+09	1 3775E+09	-7 5000E+07	4 4793E+03	4 1379E+07	6 7445E+03	5 5700E-01	6 7445E+03	5 5700E-01	6 7445E+03
	36	7 0054E-03	5 5273E+01	2 8213E+03	1 1145E+09	1 0645E+09	-7 5000E+07	2 7729E+03	4 5039E+07	6 7041E+03	5 5700E-01	6 7041E+03	5 5700E-01	6 7041E+03
	37	7 2036E-03	3 6286E+01	2 8105E+03	7 9280E+06	7 4280E+06	-7 5000E+07	1 4173E+03	3 9419E+07	6 6618E+03	5 5700E-01	6 6618E+03	5 5700E-01	6 6618E+03
	38	7 4024E-03	2 2326E+01	2 8012E+03	5 1992E+08	4 6992E+08	-7 5000E+07	6 0514E+02	2 7993E+07	6 6253E+03	5 5700E-01	6 6253E+03	5 5700E-01	6 6253E+03
	39	7 6018E-03	1 3995E+01	2 7947E+03	3 2933E+08	2 7923E+08	-7 5000E+07	2 3215E+02	1 6108E+07	6 5994E+03	5 5700E-01	6 5994E+03	5 5700E-01	6 5994E+03
	40	7 8013E-03	9 9907E+00	2 7909E+03	2 2051E+08	1 7051E+08	-7 5000E+07	9 4233E+01	7 5290E+06	6 5845E+03	5 5700E-01	6 5845E+03	5 5700E-01	6 5845E+03
	41	8 0010E-03	8 5158E+00	2 7892E+03	1 7034E+08	1 2034E+08	-7 5000E+07	4 9439E+01	8 1736E+05	6 5776E+03	5 5700E-01	6 5776E+03	5 5700E-01	6 5776E+03
	42	8 2008E-03	7 0711E+00	2 7866E+03	1 5312E+08	1 0312E+08	-7 5000E+07	3 6874E+01	8 1736E+05	6 5752E+03	5 5700E-01	6 5752E+03	5 5700E-01	6 5752E+03
	43	8 4005E-03	7 0820E+00	2 7884E+03	1 4539E+08	9 6913E+07	-7 2722E+07	3 2691E+01	1 8237E+06	6 5744E+03	5 5700E-01	6 5744E+03	5 5700E-01	6 5744E+03
	44	8 6003E-03	5 3204E+00	2 7879E+03	1 2501E+08	8 3316E+07	-6 2534E+07	2 4316E+01	3 2610E+06	6 5725E+03	5 5700E-01	6 5725E+03	5 5700E-01	6 5725E+03
	45	8 8002E-03	3 3088E+00	2 7871E+03	9 1713E+07	6 1121E+07	-4 5887E+07	1 3208E+01	3 7264E+06	6 5694E+03	5 5700E-01	6 5694E+03	5 5700E-01	6 5694E+03
	46	9 0001E-03	1 6966E+00	2 7863E+03	5 5768E+07	3 7212E+07	-2 7833E+07	4 9307E+00	2 9771E+06	6 5661E+03	5 5700E-01	6 5661E+03	5 5700E-01	6 5661E+03
	47	9 2000E-03	7 1934E-01	2 7857E+03	2 8033E+07	1 8711E+07	-1 3983E+07	1 2070E+00	1 7971E+06	6 5636E+03	5 5700E-01	6 5636E+03	5 5700E-01	6 5636E+03
	48	9 4000E-03	2 5624E-01	2 7853E+03	1 1689E+07	7 8006E+06	-5 8202E+06	2 2029E-01	8 4871E+05	6 5621E+03	5 5700E-01	6 5621E+03	5 5700E-01	6 5621E+03
	49	9 6000E-03	6 8836E-02	2 7851E+03	4 0814E+06	2 7326E+06	-2 0233E+06	2 7268E-02	3 4649E+05	6 5611E+03	5 5700E-01	6 5611E+03	5 5700E-01	6 5611E+03
	50	9 8000E-03	5 0225E-03	2 7850E+03	1 0050E+06	6 7811E+05	-4 9033E+05	1 6852E-03	1 1299E+05	6 5610E+03	5 5700E-01	6 5610E+03	5 5700E-01	6 5610E+03
	51	1 0000E-02	0 0000E+00	2 7850E+03	6 5119E+04	4 8821E+04	-2 4447E+04	7 9689E-06	7 4907E+03	6 5610E+03	5 5700E-01	6 5610E+03	5 5700E-01	6 5610E+03

MATERIAL INTERFACE

WORK AT LBOUND	0 00000E+00	WORK AT RBOUND	0 00000E+00
INTERNAL ENERGY	0 27379E+07	KINETIC ENERGY	0 45993E+05
ADDED ENERGY	0 00000E+00	TOTAL MOMENTUM	0 15058E-03

SAMPLE PROBLEM 2 - ENERGY DEPOSITION IN ALUMINUM PLATE

CYCLE = 322

TIME -

2.02243E-06

TIME STEP =

2.62276E-08

LDT = 49

L	POSITION	VELOCITY	DENSITY	STRESS	PRESSURE	PHI	ENERGY	ART VISC	SND SLD	MASS
1	-8 1797E-05	-1 2763E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2	1 3296E-04	-1 2606E+00	2.5936E+03	-1.8311E+06	-4.4421E+07	-6.3885E+07	8.7110E+05	0.0000E+00	6.2761E+03	5.5700E-01
3	3 4608E-04	-1 2696E+00	2.6136E+03	-2.0840E+06	-4.4039E+07	-6.2932E+07	8.0063E+05	0.0000E+00	6.4697E+04	5.5700E-01
4	5 5760E-04	-1 2632E+00	2.6332E+03	-7.9258E+05	-3.9661E+07	-6.0680E+07	7.1165E+05	0.0000E+00	6.3336E+03	5.5700E-01
5	7 6756E-04	-1 2334E+00	2.6597E+03	1.4798E+06	-3.7987E+07	-5.9200E+07	6.2216E+05	0.0000E+00	6.3621E+03	5.5700E-01
6	9 7597E-04	-1 2361E+00	2.6725E+03	1.9466E+06	-3.6360E+07	-5.7459E+07	5.3103E+05	0.0000E+00	6.4113E+03	5.5700E-01
7	1 1828E-03	-1 1918E+00	2.6725E+03	1.3540E+06	-3.5413E+07	-5.5150E+07	4.3793E+05	0.0000E+00	6.3909E+03	5.5700E-01
8	1 3882E-03	-1 2633E+00	2.7125E+03	9.9545E+05	-3.6335E+07	-5.5976E+07	3.4364E+05	0.0000E+00	6.4202E+03	5.5700E-01
9	1 5920E-03	-1 3033E+00	2.7325E+03	1.6075E+06	-3.6741E+07	-5.7522E+07	2.4822E+05	0.0000E+00	6.4495E+03	5.5700E-01
10	1 7944E-03	-1 3215E+00	2.7527E+03	3.7115E+06	-3.3810E+07	-5.6283E+07	1.5131E+05	0.0000E+00	6.4791E+03	5.5700E-01
11	1 9952E-03	-1 3115E+00	2.7730E+03	3.6828E+06	-3.4905E+07	-5.7881E+07	5.7863E+04	0.0000E+00	6.5095E+03	5.5700E-01
12	2 1954E-03	-1 2211E+00	2.7831E+03	7.4065E+05	-3.7387E+07	-5.7191E+07	3.0009E+03	0.0000E+00	6.5398E+03	5.5700E-01
13	2 3955E-03	-1 1965E+00	2.7831E+03	-7.3904E+05	-3.7218E+07	-5.4718E+07	3.0009E+03	0.0000E+00	6.5548E+03	5.5700E-01
14	2 5957E-03	-1 3328E+00	2.7831E+03	2.3848E+06	-3.6058E+07	-5.7664E+07	2.9074E+03	0.0000E+00	6.5549E+03	5.5700E-01
15	2 7958E-03	-1 2796E+00	2.7832E+03	2.9002E+06	-3.5435E+07	-5.7503E+07	2.7833E+03	0.0000E+00	6.5551E+03	5.5700E-01
16	2 9959E-03	-1 2527E+00	2.7832E+03	3.3149E+06	-3.5715E+07	-5.8545E+07	2.7727E+03	0.0000E+00	6.5552E+03	5.5700E-01
17	3 1960E-03	-1 2169E+00	2.7832E+03	2.2336E+06	-3.6558E+07	-5.8187E+07	2.7423E+03	0.0000E+00	6.5552E+03	5.5700E-01
18	3 3962E-03	-1 3269E+00	2.7832E+03	2.1965E+06	-3.5887E+07	-5.7126E+07	2.6281E+03	0.0000E+00	6.5550E+03	5.5700E-01
19	3 5963E-03	-1 4174E+00	2.7832E+03	1.5418E+06	-3.6375E+07	-5.6875E+07	2.6189E+03	0.0000E+00	6.5552E+03	5.5700E-01
20	3 7964E-03	-1 3255E+00	2.7831E+03	1.9261E+05	-3.7958E+07	-5.7225E+07	2.5861E+03	0.0000E+00	6.5551E+03	5.5700E-01
21	3 9966E-03	-1 2871E+00	2.7832E+03	2.1202E+06	-3.6557E+07	-5.8016E+07	2.4982E+03	0.0000E+00	6.5549E+03	5.5700E-01
22	4 1967E-03	-1 2143E+00	2.7832E+03	2.0356E+06	-3.6744E+07	-5.8170E+07	2.4852E+03	0.0000E+00	6.5551E+03	5.5700E-01
23	4 3968E-03	-1 0359E+00	2.7831E+03	-9.6065E+05	-3.8863E+07	-5.6854E+07	2.4371E+03	0.0000E+00	6.5551E+03	5.5700E-01
24	4 5970E-03	-7 8779E-01	2.7831E+03	-6.2344E+06	-4.1676E+07	-5.3193E+07	2.3881E+03	0.0000E+00	6.5548E+03	5.5700E-01
25	4 7971E-03	-5 7905E-01	2.7829E+03	-1.2293E+07	-4.7208E+07	-5.2371E+07	2.3647E+03	0.0000E+00	6.5544E+03	5.5700E-01
26	4 9973E-03	-1 3409E-01	2.7826E+03	2.4764E+07	-5.4548E+07	-4.4676E+07	2.3037E+03	0.0000E+00	6.5537E+03	5.5700E-01
27	5 1975E-03	1.4510E+00	2.7821E+03	-4.7528E+07	-7.0512E+07	-3.4476E+07	2.2906E+03	0.0000E+00	6.5527E+03	5.5700E-01
28	5 3978E-03	2.9477E+00	2.7815E+03	7.4321E+07	-8.8296E+07	-2.0962E+07	2.2544E+03	0.0000E+00	6.5505E+03	5.5700E-01
29	5 5980E-03	4.1456E+00	2.7811E+03	-7.1060E+07	-9.9363E+07	-1.2453E+07	2.2185E+03	0.0000E+00	6.5480E+03	5.5700E-01
30	5 7983E-03	5.1101E+00	2.7808E+03	-1.0353E+08	-1.0887E+08	-8.0007E+06	2.1980E+03	0.0000E+00	6.5465E+03	5.5700E-01
31	5 9987E-03	6.0724E+00	2.7803E+03	-1.2055E+08	-1.2307E+08	-3.7792E+06	2.1494E+03	0.0000E+00	6.5451E+03	5.5700E-01
32	6 1991E-03	7.2143E+00	2.7798E+03	-1.3713E+08	-1.3717E+08	-5.9697E+04	2.1439E+03	0.0000E+00	6.5432E+03	5.5700E-01
33	6 3995E-03	9.0785E+00	2.7790E+03	-1.6439E+08	-1.6015E+08	6.3552E+06	2.1111E+03	0.0000E+00	6.5412E+03	5.5700E-01
34	6 6000E-03	1.2112E+01	2.7776E+03	-2.0678E+08	-1.9947E+08	1.0970E+07	2.1080E+03	0.0000E+00	6.5380E+03	5.5700E-01
35	6 8007E-03	1.6538E+01	2.7756E+03	-2.6802E+08	-2.5653E+08	1.7239E+07	2.1220E+03	0.0000E+00	6.5325E+03	5.5700E-01
36	7 0016E-03	2.1955E+01	2.7732E+03	-3.5334E+08	-3.2495E+08	4.2577E+07	2.1570E+03	0.0000E+00	6.5245E+03	5.5700E-01
37	7 2027E-03	3.0553E+01	2.7672E+03	-4.8822E+08	-4.3822E+08	7.5000E+07	2.3252E+03	0.0000E+00	6.5148E+03	5.5700E-01
38	7 4042E-03	4.0827E+01	2.7636E+03	-6.4383E+08	-6.9383E+08	7.5000E+07	2.6467E+03	0.0000E+00	6.4987E+03	5.5700E-01
39	7 6063E-03	5.2117E+01	2.7566E+03	-8.4048E+08	-7.9048E+08	7.5000E+07	3.2600E+03	0.0000E+00	6.4764E+03	5.5700E-01
40	7 8089E-03	6.3298E+01	2.7495E+03	-1.0354E+09	-7.8542E+08	7.5000E+07	3.9775E+03	0.0000E+00	6.4479E+03	5.5700E-01
41	8 0121E-03	6.9794E+01	2.7413E+03	-1.2660E+09	-1.2100E+09	7.5000E+07	5.0107E+03	0.0000E+00	6.4193E+03	5.5700E-01
42	8 2157E-03	6.7465E+01	2.7348E+03	-1.3472E+09	-1.3887E+09	-6.2289E+07	5.7749E+03	0.0000E+00	6.3859E+03	5.5700E-01
43	8 4197E-03	5.8859E+01	2.7303E+03	-1.4982E+09	-1.5082E+09	-7.5000E+07	6.8478E+03	0.0000E+00	6.3589E+03	5.5700E-01
44	8 6242E-03	4.1767E+01	2.7244E+03	-1.6146E+09	-1.6446E+09	-7.5000E+07	8.3453E+03	0.0000E+00	6.3407E+03	5.5700E-01
45	8 8292E-03	1.9948E+01	2.7168E+03	-1.8158E+09	-1.8658E+09	-7.5000E+07	1.0280E+04	0.0000E+00	6.3165E+03	5.5700E-01
46	9 0347E-03	-1.1392E+01	2.7105E+03	-1.9798E+09	-2.0298E+09	-7.5000E+07	1.1970E+04	0.0000E+00	6.2851E+03	5.5700E-01
47	9 2405E-03	-4.3624E+01	2.7071E+03	-2.0690E+09	-2.1190E+09	-7.5000E+07	1.3826E+04	0.0000E+00	6.3309E+03	5.5700E-01
48	9 4456E-03	-8.4429E+01	2.7153E+03	-1.8549E+09	-1.9049E+09	-7.5000E+07	1.5825E+04	0.0000E+00	6.2449E+03	5.5700E-01
49	9 6495E-03	-1.2888E+02	2.7313E+03	-1.4326E+09	-1.4826E+09	-7.5000E+07	1.7789E+03	0.0000E+00	6.2789E+03	5.5700E-01
50	9 8522E-03	-1.5489E+02	2.7485E+03	-9.6901E+08	-1.0190E+09	-7.5000E+07	3.2619E+03	0.0000E+00	6.3446E+03	5.5700E-01
51	1 0053E-02	-1.5674E+02	2.7717E+03	-3.5350E+08	-3.7682E+08	-3.4970E+07	4.9795E+02	0.0000E+00	6.4146E+03	5.5700E-01

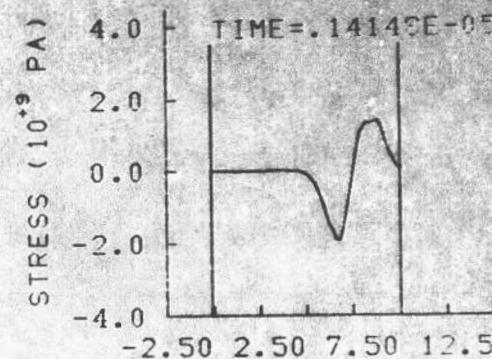
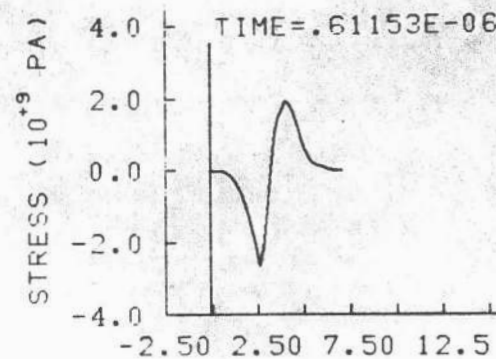
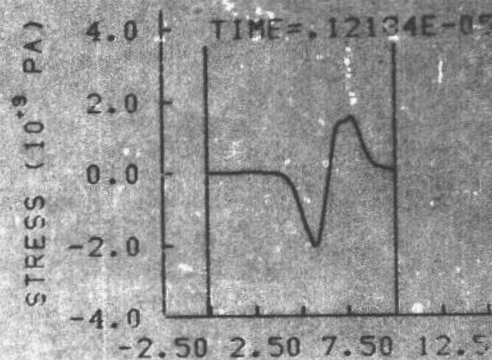
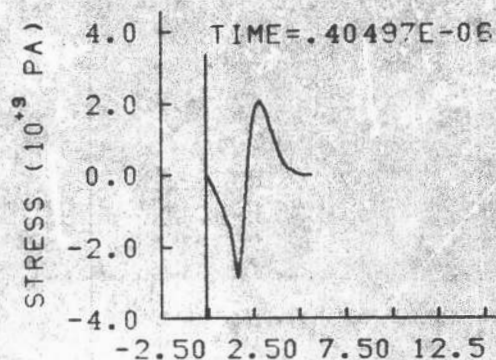
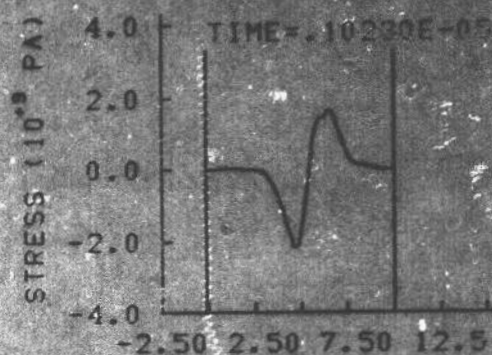
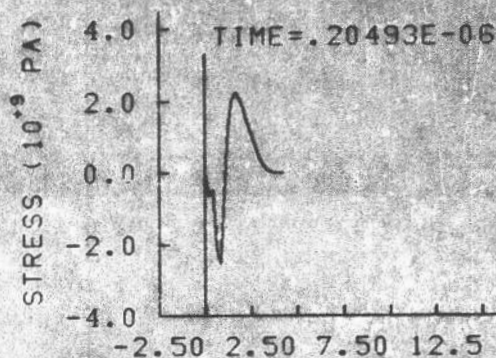
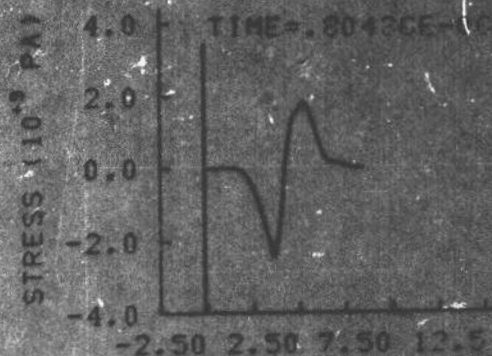
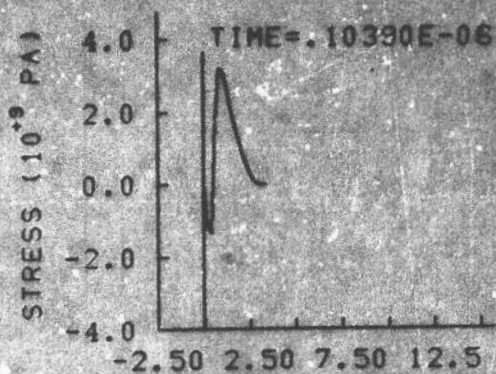
MATERIAL INTERFACE

WORK AT LBOUND 0.0000E+00  
 INTERNAL ENERGY 0.2598E+07  
 ADDED ENERGY 0.0000E+00

WORK AT RBOUND 0.0000E+00  
 KINETIC ENERGY 0.2402E+05  
 TOTAL MOMENTUM 0.2401E-04

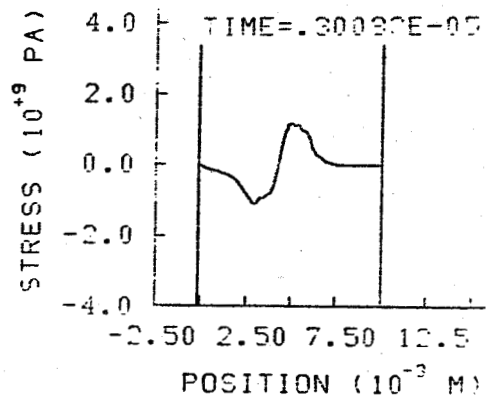
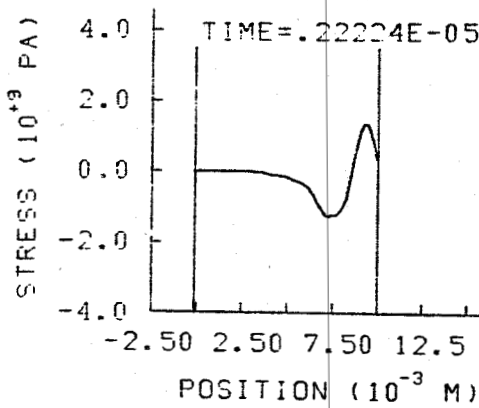
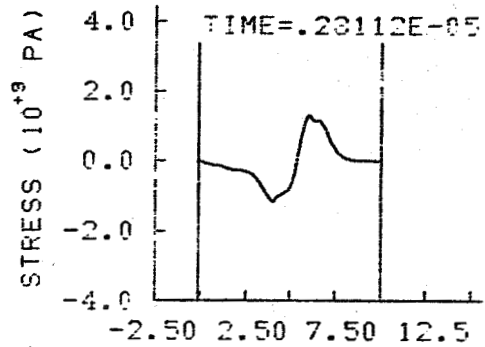
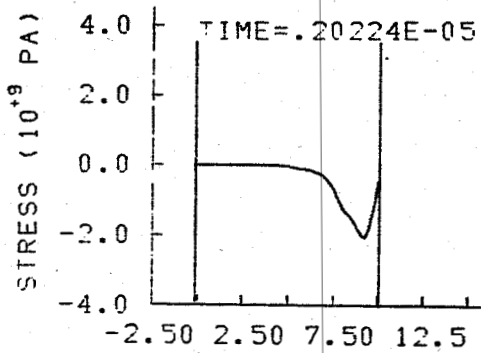
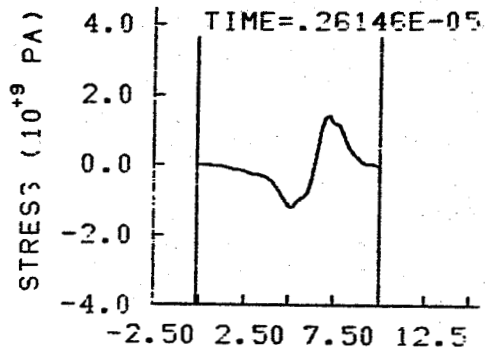
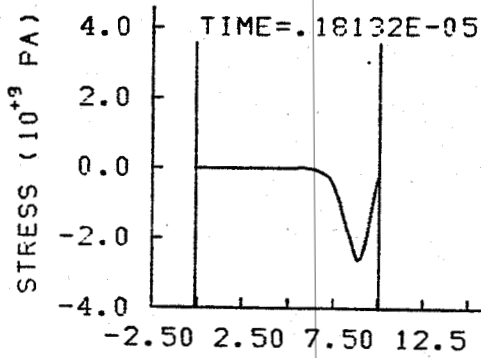
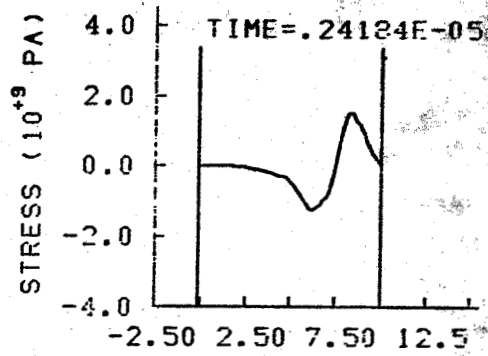
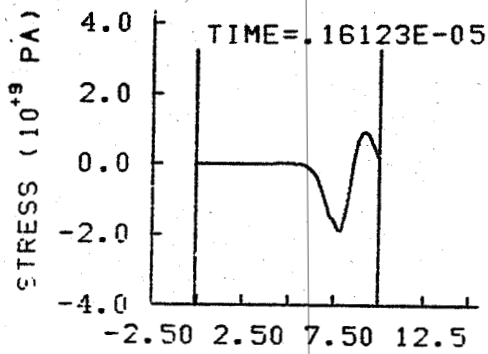


SAMPLE PROBLEM 2 - ENERGY DEPOSITION IN ALUMINUM

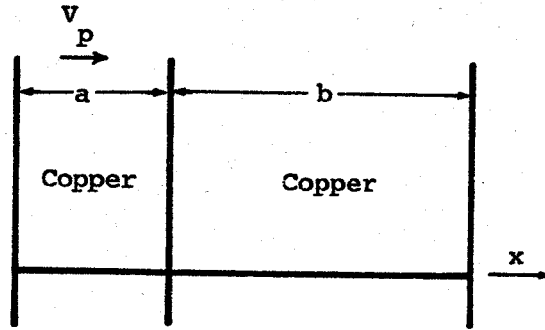


POSITION ( $10^{-3}$  M)

POSITION ( $10^{-3}$  M)



3. Impact of a copper plate onto a copper target.



$a = 0.01 \text{ m (20 zones)}$   
 $b = 0.002 \text{ m (40 zones)}$   
 $v_p = 300 \text{ m/s}$

Copper

$\rho_o = 8930$                       Fracture stress ~ 3.5 GPa.  
 $c_o = 3940$   
 $s = 1.489$   
 $\Gamma_o = 1.99$   
 $\nu = 0.345$   
 $Y_o = 25 \text{ MPa}$

SAMPLE PROBLEM 3 - CU FLYER ON CU TARGET (210M/S)									
	1	2	10	3	3	25	1	1	15
3		1.5E-06			-1.0E+06		1.0E+06		-0.001
5		0.0		1.0E-07	1.0				
6		0.0		1.0E-08	1.0				
10	1	1.0		20.0	0.001		0.00005		0.00005
12	1								210.0
14	1	2.0		-1.0E+06					105.0
15	1	8930.0		3940.0					0.345
16	1	0.0			1.489				
17	1	1.0		1.99					
19	1	1.0		2.5E+07					
10	2	1.0		40.0	0.002		0.00005		0.00005
13	2	2.0		-3.5E+09					
15	2	8930.0		3940.0					0.345
16	2	0.0			1.489				
17	2	1.0		1.99					
19	2	1.0		2.5E+07					
{EOR}									
{EOR}									
XPLOT		0.0		1.0E-07	1.0				
XDATA			1	0	1		-0.0015		0.0045
XLABEL			POSITION (( ) M)						1
YDATA			4	0	1		-4.0E09		16.0E09
YLABEL			STRESS (( ) PA)						
TIMEPLOT		3		0	1				
XDATA				0	1		0.3E-06		2.3E-06
XLABEL				TIME (( ) S)					1
YDATA			2	0	1		0.0		1000.0
YLABEL			VELOCITY (( ) M/S)						
END									

SAMPLE PROBLEM 3 -- CU FLYER ON CU TARGET (210M/S)

CARD NO	LPHA	NDP	NVAR	LHHT	RHBT	LACT	NJOB	NIL	NDI	NUL	MDRE	JTAPE	NSTART
2	1	2	10	3	3	25	1	0	1	15	0	0	0
CARD NO	TMAX		DELT(4)		SIGSEP		SIGACT		XZERO		LIGN		RBCN
3	1	5000E-06	0.0000E+00	-1.0000E+06	1.0000E+06	-1.0000E-03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
CARD NO.	KT1		KT2		B1		B2		SIGMAX		EMAX		HMAX
4	9	5000E-01	1.0500E+00	2.0000E+00	1.0000E-01	1.0000E+100	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+100
STANDARD EDIT													
CARD NO.	S1		DELT1		E1=S2		DELT2		E2=S3		DELT3		E3
5	1	0000E-07	4.0000E-07	5.0000E-07	5.0000E-07	1.0000E-06	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
OUTPL													
CARD NO.	S1		DELT1		E1=S2		DELT2		E2=S3		DELT3		E3
6	0	0000E+00	1.0000E-08	1.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
OUTN													
CARD NO.	S1		DELT1		E1=S2		DELT2		E2=S3		DELT3		E3
7	0	0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
DUMP													
CARD NO.	S1		DELT1		E1=S2		DELT2		E2=S3		DELT3		E3
8	0	0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
PLOT VARIABLES													
CARD NO.	IPL1	IPL2	IPL3	IPL4	IPL5								
23	9	8	6	7	2								

PLATE	1	RT INT AT X = 0.0000E+00		R1 INT AT ZONE 21				
CARD NO	STATE	NDM-SHES	THICKNESS	DELTA X	DELTA X1	XRATIO	XGAP	
10	1.0000E+00	2.0000E+01	1.0000E-03	5.0000E-05	5.0000E-05	0.0000E+00	0.0000E+00	
CARD NO	RZTIME	DXMIN	DXMAX	RCCOMB	RSCRIT	RZCO	RZCI	
11	1.0000E+30	1.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	
CARD NO.	EZERO	PZERO	QZERO	SZERO	UZERO	UZERDI	ZZERO	
12	0.0000E+00	0.0000E+00	8.7300E-03	0.0000E+00	2.1000E+02	1.0500E+02	0.0000E+00	
CARD NO.	FCRIT	SIGMAF	FCRIT	SIGMAD				
13	2.0000E+00	-1.0000E+10	0.0000E+00	0.0000E+00				
CARD NO	FCRITI	SIGMAIF	CONSTI	SIGMADI				
14	2.0000E+00	-1.0000E+06	0.0000E+00	0.0000E+00				

HYDRO-VAPOR-ELASTIC-PLASTIC

CARD NO.	CES( 1)	CES( 2)	CES( 3)	CES( 4)	CES( 5)	CES( 6)	CES( 7)
15	8.7300E+03	3.9400E+03	0.0000E+00	0.0000E+00	0.0000E+00	3.4500E-01	0.0000E+00
CARD NO.	CES( 8)	CES( 9)	CES(10)	CES(11)	CES(12)	CES(13)	CES(14)
16	0.0000E+00	0.0000E+00	1.4870E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
CARD NO.	CES(15)	CES(16)	CES(17)	CES(18)	CES(19)	CES(20)	CES(21)
17	1.0000E+00	1.7900E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
CARD NO.	CES(22)	CES(23)	CES(24)	CES(25)	CES(26)	CES(27)	CES(28)
18	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
CARD NO.	CES(29)	CES(30)	CES(31)	CES(32)	CES(33)	CES(34)	CES(35)
19	1.0000E+00	2.5000E+07	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

PLATE	2	RT INT AT X	2. 0000E-03	RT INT AT ZONE	61				
CARD NO.	10	STATE	NOMESHES	THICKNESS	DELTAX	DELTAX1	XRATIO	XGAP	
		1 0000E+00	4. 0000E+01	2. 0000E-03	5. 0000E-05	5. 0000E-05	0. 0000E+00	0. 0000E+00	
CARD NO.	11	RZTIME	DXMIN	DXMAX	RCCOMB	RSCRIT	RZCO	RZCI	
		1. 0000E+30	1. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	
CARD NO.	12	EZERO	PZERO	RZERO	SZERO	UZERO	UZERO1	ZZERO	
		0. 0000E+00	0. 0000E+00	8. 9300E+03	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	
CARD NO.	13	FCRIT	SIGMAF	FCNST	SIGMAD				
		2. 0000E+00	-3. 5000E+09	0. 0000E+00	0. 0000E+00				
CARD NO.	14	FCRIT1	SIGMA1F	CONST1	SIGMAO1				
		2. 0000E+00	-1. 0000E+100	0. 0000E+00	0. 0000E+00				

HYDRO-VAPOR-ELASTIC-PLASTIC

CARD NO.	15	CES( 1)	CES( 2)	CES( 3)	CES( 4)	CES( 5)	CES( 6)	CES( 7)
		8. 9300E+03	3. 7400E+03	0. 0000E+00	0. 0000E+00	0. 0000E+00	3. 4500E-01	0. 0000E+00
CARD NO.	16	CES( 8)	CES( 9)	CES(10)	CES(11)	CES(12)	CES(13)	CES(14)
		0. 0000E+00	0. 0000E+00	1. 4890E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00
CARD NO.	17	CES(15)	CES(16)	CES(17)	CES(18)	CES(19)	CES(20)	CES(21)
		1. 0000E+00	1. 7700E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00
CARD NO.	18	CES(22)	CES(23)	CES(24)	CES(25)	CES(26)	CES(27)	CES(28)
		0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00
CARD NO.	19	CES(29)	CES(30)	CES(31)	CES(32)	CES(33)	CES(34)	CES(35)
		1. 0000E+00	2. 5000E+07	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00	0. 0000E+00

SAMPLE PROBLEM 3 - CU FLYER ON CU TARGET (210M/S)

CYCLE = 13

TIME = 1.03332E-07

TIME STEP = 8.02705E-09

LDT = 14

L	POSITION	VELOCITY	DENSITY	STRESS	PRESSURE	PHI	ENERGY	ART VISC	SND SPD	MASS
1	-9.7830E-04	2.1000E+02	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2	-9.2830E-04	2.1000E+02	8.9300E+03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	4.4650E-01
3	-8.7830E-04	2.1000E+02	8.9300E+03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	4.4650E-01
4	-8.2830E-04	2.1000E+02	8.9300E+03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	4.4650E-01
5	-7.7830E-04	2.1000E+02	8.9300E+03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	4.4650E-01
6	-7.2830E-04	2.1000E+02	8.9300E+03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	4.4650E-01
7	-6.7830E-04	2.1000E+02	8.9300E+03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	4.4650E-01
8	-6.2830E-04	2.1000E+02	8.9300E+03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	4.4650E-01
9	-5.7830E-04	2.0986E+02	8.9302E+03	4.4857E+06	3.0704E+06	-2.1229E+06	6.2909E-03	5.8737E+05	4.7623E+03	4.4650E-01
10	-5.2831E-04	2.0892E+02	8.9317E+03	3.7494E+07	2.5667E+07	-1.7740E+07	4.3200E-01	4.6446E+06	4.7623E+03	4.4650E-01
11	-4.7835E-04	2.0539E+02	8.9373E+03	1.3024E+08	1.1358E+08	-2.5000E+07	7.1951E+00	1.5450E+07	4.7623E+03	4.4650E-01
12	-4.2856E-04	1.8740E+02	8.9680E+03	6.1182E+08	5.9515E+08	-2.5000E+07	1.6919E+02	8.8545E+07	4.7700E+03	4.4650E-01
13	-3.7920E-04	1.4770E+02	9.0461E+03	1.8694E+09	1.8527E+09	-2.5000E+07	1.5214E+03	2.2951E+08	4.8023E+03	4.4650E-01
14	-3.3028E-04	1.1586E+02	9.1255E+03	3.1925E+09	3.1758E+09	-2.5000E+07	4.1801E+03	1.7943E+08	4.8842E+03	4.4650E-01
15	-2.8151E-04	1.0701E+02	9.1567E+03	3.7245E+09	3.7078E+09	-2.5000E+07	5.5484E+03	4.3245E+07	4.9992E+03	4.4650E-01
16	-2.3278E-04	1.0584E+02	9.1614E+03	3.8060E+09	3.7894E+09	-2.5000E+07	5.8017E+03	5.4201E+06	5.0041E+03	4.4650E-01
17	-1.8405E-04	1.0499E+02	9.1638E+03	3.8475E+09	3.8309E+09	-2.5000E+07	5.9671E+03	3.8862E+06	5.0066E+03	4.4650E-01
18	-1.3532E-04	1.0407E+02	9.1630E+03	3.8274E+09	3.8188E+09	-1.2945E+07	6.0080E+03	0.0000E+00	5.0059E+03	4.4650E-01
19	-8.6605E-05	1.0463E+02	9.1647E+03	3.8666E+09	3.8499E+09	-2.5000E+07	6.2125E+03	2.9605E+06	5.0077E+03	4.4650E-01
20	-3.7875E-05	1.0500E+02	9.1637E+03	3.8564E+09	3.8398E+09	-2.4964E+07	6.4122E+03	0.0000E+00	5.0060E+03	4.4650E-01
21	1.0850E-05	1.0500E+02	9.1637E+03	3.8564E+09	3.8398E+09	-2.4964E+07	6.5776E+03	1.7085E+06	5.0070E+03	4.4650E-01
----- MATERIAL INTERFACE -----										
22	5.9575E-05	1.0463E+02	9.1637E+03	3.8564E+09	3.8398E+09	-2.4964E+07	6.5776E+03	1.7085E+06	5.0070E+03	4.4650E-01
23	1.0830E-04	1.0537E+02	9.1628E+03	3.8239E+09	3.8227E+09	-1.9002E+06	6.4122E+03	0.0000E+00	5.0060E+03	4.4650E-01
24	1.5702E-04	1.0473E+02	9.1647E+03	3.8666E+09	3.8499E+09	-2.5000E+07	6.2125E+03	2.9605E+06	5.0077E+03	4.4650E-01
25	2.0575E-04	1.0501E+02	9.1630E+03	3.8274E+09	3.8188E+09	-1.2945E+07	6.0080E+03	0.0000E+00	5.0059E+03	4.4650E-01
26	2.5448E-04	1.0416E+02	9.1638E+03	3.8475E+09	3.8309E+09	-2.5000E+07	5.9671E+03	3.8862E+06	5.0066E+03	4.4650E-01
27	3.0321E-04	1.0299E+02	9.1614E+03	3.8060E+09	3.7894E+09	-2.5000E+07	5.8017E+03	5.4201E+06	5.0041E+03	4.4650E-01
28	3.5198E-04	9.4141E+01	9.1567E+03	3.7245E+09	3.7078E+09	-2.5000E+07	5.5484E+03	4.3245E+07	4.9992E+03	4.4650E-01
29	4.0090E-04	6.2300E+01	9.1255E+03	3.1925E+09	3.1758E+09	-2.5000E+07	4.1801E+03	1.7943E+08	4.9669E+03	4.4650E-01
30	4.5026E-04	2.2601E+01	9.0461E+03	1.8694E+09	1.8527E+09	-2.5000E+07	1.5214E+03	2.2951E+08	4.8842E+03	4.4650E-01
31	5.0005E-04	4.6062E+00	8.9680E+03	6.1182E+08	5.9515E+08	-2.5000E+07	1.6919E+02	8.8545E+07	4.8023E+03	4.4650E-01
32	5.5001E-04	1.0814E+00	8.9373E+03	1.3024E+08	1.1358E+08	-2.5000E+07	7.1951E+00	1.5450E+07	4.8023E+03	4.4650E-01
33	6.0000E-04	1.3795E-01	8.9317E+03	3.7494E+07	2.5667E+07	-1.7740E+07	4.3200E-01	4.6446E+06	4.7700E+03	4.4650E-01
34	6.5000E-04	0.0000E+00	8.9302E+03	4.4857E+06	3.0704E+06	-2.1229E+06	6.2909E-03	5.8737E+05	4.7623E+03	4.4650E-01
35	7.0000E-04	0.0000E+00	8.9300E+03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	4.7623E+03	4.4650E-01

WORK AT LBOUND 0.0000E+00  
INTERNAL ENERGY 0.41557E+05  
ADDED ENERGY 0.0000E+00  
REL. ENERGY ERROR 0.62711E-02

WORK AT RBOUND 0.0000E+00  
KINETIC ENERGY 0.15328E+06  
TOTAL MOMENTUM 0.18753E+04  
REL. MOMENTUM ERROR -0.19399E-13

SAMPLE PROBLEM 3 · CU FLYER ON CU TARGET (210M/S)

CYCLE = 62 TIME = 5.05187E-07 TIME STEP = 8.34003E-09 LDT = 25

Table with columns: L, POSITION, VELOCITY, DENSITY, STRESS, PRESSURE, PHI, ENERGY, ART VISC, SMD SPD, MASS. Rows 1-21.

MATERIAL INTERFACE

Table with columns: L, POSITION, VELOCITY, DENSITY, STRESS, PRESSURE, PHI, ENERGY, ART VISC, SMD SPD, MASS. Rows 22-54.



SAMPLE PROBLEM 3 - CU FLYER ON CU TARGET (210M/S)

CYCLE = 62      TIME = 5.05187E-07      TIME STEP = 8.34003E-09      LDT = 25

L	POSITION	VELOCITY	DENSITY	STRESS	PRESSURE	PHI	ENERGY	ART VISC	SND SPD	MASS
55	1.7097E-03	1.0704E+02	9.1597E+03	3.7553E+09	3.7548E+09	-7.1025E+05	5.4298E+03	0.0000E+00	5.0021E+03	4.4650E-01
56	1.7584E-03	1.0891E+02	9.1559E+03	3.6733E+09	3.6889E+09	2.3408E+07	5.2543E+03	0.0000E+00	4.9981E+03	4.4650E-01
57	1.8072E-03	1.1301E+02	9.1474E+03	3.5268E+09	3.5435E+09	2.5000E+07	4.8798E+03	0.0000E+00	4.9893E+03	4.4650E-01
58	1.8562E-03	1.2077E+02	9.1289E+03	3.2128E+09	3.2295E+09	2.5000E+07	4.1197E+03	0.0000E+00	4.9702E+03	4.4650E-01
59	1.9053E-03	1.3346E+02	9.0928E+03	2.6059E+09	2.6226E+09	2.5000E+07	2.8135E+03	0.0000E+00	4.9328E+03	4.4650E-01
60	1.9547E-03	1.4690E+02	9.0380E+03	1.7030E+09	1.7197E+09	2.5000E+07	1.2800E+03	0.0000E+00	4.8757E+03	4.4650E-01
61	2.0044E-03	1.5223E+02	8.9691E+03	5.9564E+08	6.1231E+08	2.5000E+07	1.7689E+02	0.0000E+00	4.8035E+03	4.4650E-01

-----MATERIAL INTERFACE-----

WORK AT LBOUND	0.00000E+00	WORK AT RBOUND	0.00000E+00
INTERNAL ENERGY	0.95667E+05	KINETIC ENERGY	0.99117E+05
ADDED ENERGY	0.00000E+00	TOTAL MOMENTUM	0.18753E+04
REL. ENERGY ERROR	0.59985E-02	REL. MOMENTUM ERROR	0.24701E-07

FRACTURE OCCURRED AT X=	0.51127E-04	CYCLE	70	TIME	0.57189E-06	STRESS	-0.20210E+07	ZONE	21
FRACTURED SURFACES COLLIDED AT X=	0.51132E-04	CYCLE	71	TIME	0.58023E-06	STRESS		ZONE	21
FRACTURE OCCURRED AT X=	0.51164E-04	CYCLE	95	TIME	0.78350E-06	STRESS	-0.24088E+07	ZONE	21
FRACTURE OCCURRED AT X=	0.10586E-02	CYCLE	97	TIME	0.80152E-06	STRESS	-0.35723E+10	ZONE	41

## SAMPLE PROBLEM 3 - CU FLYER ON CU TARGET (210M/S)

CYCLE = 121		TIME = 1 00568E-06		TIME STEP = B 74711E-09		LDT = 27				
	POSITION	VELOCITY	DENSITY	STRESS	PRESSURE	PHI	ENERGY	ART VISC	SND SPD	MASS
1	-9 4879E-04	9 3667E-02	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2	-8 9880E-04	9 7180E-02	8.7310E+03	2.3485E+05	1.6527E+07	2.4439E+07	5.7640E+01	0.0000E+00	4.7634E+03	4.4650E-01
3	-8 4880E-04	9 8581E-02	8.9306E+03	6.9378E+05	1.6687E+07	2.3990E+07	2.5335E+02	0.0000E+00	4.7634E+03	4.4650E-01
4	-7 9881E-04	1 0343E-01	8.9307E+03	8.6937E+05	1.6816E+07	2.3920E+07	3.5972E+02	0.0000E+00	4.7633E+03	4.4650E-01
5	-7 4881E-04	1 0809E-01	8.9306E+03	9.0882E+05	1.6684E+07	2.3663E+07	3.9401E+02	0.0000E+00	4.7633E+03	4.4650E-01
6	-6 9882E-04	1 0716E-01	8.9306E+03	8.2348E+05	1.6776E+07	2.3929E+07	4.0888E+02	3.9300E+03	4.7633E+03	4.4650E-01
7	-6 4882E-04	1 0113E-01	8.9306E+03	8.3678E+05	1.6509E+07	2.3509E+07	4.1939E+02	2.5682E+04	4.7633E+03	4.4650E-01
8	-5 9882E-04	9 8315E-02	8.9305E+03	9.7411E+05	1.6032E+07	2.2586E+07	4.3027E+02	1.1962E+04	4.7633E+03	4.4650E-01
9	-5 4882E-04	9 4053E-02	8.9305E+03	1.2511E+06	1.5577E+07	2.1489E+07	4.4193E+02	1.8129E+04	4.7632E+03	4.4650E-01
10	-4 9883E-04	8 6031E-02	8.9304E+03	1.4753E+06	1.4512E+07	1.9555E+07	4.5590E+02	3.4123E+04	4.7631E+03	4.4650E-01
11	-4 4883E-04	8 4116E-02	8.9303E+03	1.3283E+06	1.3576E+07	1.8371E+07	4.7107E+02	8.1476E+03	4.7631E+03	4.4650E-01
12	-3 9883E-04	9 0393E-02	8.9303E+03	1.1321E+06	1.2622E+07	1.7235E+07	4.8880E+02	0.0000E+00	4.7630E+03	4.4650E-01
13	-3 4883E-04	9 9051E-02	8.9302E+03	9.4111E+05	1.1973E+07	1.6548E+07	5.1007E+02	0.0000E+00	4.7630E+03	4.4650E-01
14	-2 9883E-04	9 1421E-02	8.9301E+03	1.0900E+06	1.1217E+07	1.5191E+07	5.3564E+02	3.2455E+04	4.7629E+03	4.4650E-01
15	-2 4883E-04	9 3312E-02	8.9301E+03	7.4055E+05	1.1160E+07	1.5630E+07	5.6791E+02	0.0000E+00	4.7629E+03	4.4650E-01
16	-1 9883E-04	1 0271E-01	8.9299E+03	3.6201E+05	8.6441E+06	1.2423E+07	6.0626E+02	0.0000E+00	4.7627E+03	4.4650E-01
17	-1 4883E-04	1 1232E-01	8.9299E+03	3.7260E+05	1.0494E+07	1.5185E+07	6.6125E+02	0.0000E+00	4.7628E+03	4.4650E-01
18	-9 8829E-05	1 0551E-01	8.9297E+03	6.0188E+05	8.3883E+06	1.1680E+07	7.3433E+02	2.8962E+04	4.7626E+03	4.4650E-01
19	-4 8827E-05	1 0438E-01	8.9297E+03	4.7281E+05	1.0303E+07	1.4745E+07	8.5565E+02	4.8293E+03	4.7627E+03	4.4650E-01
20	1 1778E-06	1 0864E-01	8.9291E+03	1.1386E+05	6.2769E+06	9.2445E+06	1.1278E+03	0.0000E+00	4.7624E+03	4.4650E-01
21	5 1182E-05	1 1701E-01	8.9292E+03	-3.7850E+04	9.7926E+06	1.4747E+07	1.2469E+03	0.0000E+00	4.7626E+03	4.4650E-01
-----MATERIAL INTERFACE-----										
FRACTURE										
22	5 5742E-05	1 1392E+02	8.8992E+03	-4.6857E+08	-4.5212E+08	2.4670E+07	1.3369E+03	8.4588E+04	4.7309E+03	4.4650E-01
23	1 5641E-04	1 1187E+02	8.8430E+03	-1.2999E+09	-1.3045E+09	-6.9043E+06	1.8610E+03	8.4834E+06	4.6712E+03	4.4650E-01
24	2 0710E-04	1 0044E+02	8.8081E+03	-1.8098E+09	-1.8244E+09	-2.5000E+07	2.3142E+03	5.1022E+07	4.6337E+03	4.4650E-01
25	2 5776E-04	7 5326E+01	8.8134E+03	-1.7319E+09	-1.7486E+09	-2.5000E+07	2.1327E+03	1.2378E+08	4.6394E+03	4.4650E-01
26	3 0817E-04	4 2446E+01	8.8574E+03	-1.0773E+09	-1.0940E+09	-2.5000E+07	1.3805E+03	1.7273E+08	4.6862E+03	4.4650E-01
27	3 5831E-04	1 7111E+01	8.9050E+03	-3.5194E+08	-3.6861E+08	-2.5000E+07	1.0002E+03	1.2851E+08	4.7386E+03	4.4650E-01
28	4 0832E-04	5 8967E+00	8.9278E+03	-3.8815E+08	-1.7055E+07	-2.5000E+07	9.3289E+02	5.1920E+07	4.7609E+03	4.4650E-01
29	4 5830E-04	2 9143E+00	8.9332E+03	8 2145E+07	6 5478E+07	-2.5000E+07	9 2203E+02	1 3000E+07	4 7665E+03	4 4650E-01
30	5 0829E-04	2 3122E+00	8.9329E+03	7 7062E+07	6 0396E+07	-2.5000E+07	9 0549E+02	2 5756E+06	4 7661E+03	4 4650E-01
31	5 5828E-04	1 5845E+00	8.9323E+03	6 9108E+07	5 2442E+07	-2.5000E+07	9 0717E+02	3 1159E+06	4 7656E+03	4 4650E-01
32	6 0827E-04	1 4028E+00	8.9308E+03	4 5075E+07	2 8408E+07	-2.5000E+07	9 0838E+02	7 7427E+05	4 7640E+03	4 4650E-01
33	6 5827E-04	1 0401E+00	8.9303E+03	3 6829E+07	2 0162E+07	-2.5000E+07	9 1296E+02	1 5470E+06	4 7634E+03	4 4650E-01
34	7 0827E-04	1 2031E+00	8.9298E+03	2 8213E+07	1 3369E+07	-2.2265E+07	9 3827E+02	0 0000E+00	4 7629E+03	4 4650E-01
35	7 5827E-04	4 0329E-01	8.9300E+03	3 3272E+07	1 6605E+07	-2.5000E+07	9 5332E+02	3 4238E+06	4 7631E+03	4 4650E-01
36	8 0827E-04	8 7845E-01	8.9294E+03	1 9492E+07	8 1392E+06	-1 7029E+07	1 0024E+03	0 0000E+00	4 7626E+03	4 4650E-01
37	8 5828E-04	3 6227E-01	8.9290E+03	5 9524E+06	3 7665E+06	-3 2789E+06	1 0574E+03	2 2039E+06	4 7622E+03	4 4650E-01
38	9 0828E-04	5 5230E-01	8.9292E+03	1 0431E+07	8 9700E+06	-2 1920E+06	1 1657E+03	0 0000E+00	4 7626E+03	4 4650E-01
39	9 5829E-04	6 3805E-02	8.9285E+03	6 7797E+06	9 3323E+05	-8 7697E+06	1 3419E+03	2 0850E+06	4 7620E+03	4 4650E-01
40	1 0083E-03	5 2333E-01	8.9282E+03	5 8071E+06	2 7836E+06	-4 5331E+06	1 7573E+03	0 0000E+00	4 7620E+03	4 4650E-01
41	1 0583E-03	2 1843E-01	8.9286E+03	-3 5339E+06	2 3266E+05	5 6498E+06	2 7833E+03	1 2991E+06	4 7615E+03	4 4650E-01
-----FRACTURE-----										
42	1 1518E-03	2 0940E+02	8.9268E+03	-4 3686E+06	-1 7797E+06	6 8832E+06	2 7291E+03	6 2243E+05	4 7614E+03	4 4650E-01
43	1 2018E-03	2 0975E+02	8.9281E+03	4 5733E+06	1 0150E+06	-5 3375E+06	1 7289E+03	0 0000E+00	4 7619E+03	4 4650E-01
44	1 2518E-03	2 0929E+02	8.9285E+03	4 9130E+06	-6 9315E+05	-8 4092E+06	1 2917E+03	2 1347E+06	4 7619E+03	4 4650E-01
45	1 3018E-03	2 0937E+02	8.9294E+03	1 1048E+07	9 7704E+06	-1 9158E+06	1 0953E+03	0 0000E+00	4 7626E+03	4 4650E-01
46	1 3518E-03	2 0883E+02	8.9291E+03	8 9270E+06	3 2472E+06	-8 5197E+06	9 8485E+02	2 3033E+06	4 7627E+03	4 4650E-01
47	1 4018E-03	2 0922E+02	8.9297E+03	2 3659E+07	1 1391E+07	-1 8402E+07	9 1657E+02	0 0000E+00	4 7628E+03	4 4650E-01
48	1 4518E-03	2 0839E+02	8.9305E+03	4 0337E+07	2 3670E+07	-2 5000E+07	8 5345E+02	3 8279E+06	4 7637E+03	4 4650E-01
49	1 5018E-03	2 0806E+02	8.9312E+03	4 9818E+07	3 3151E+07	-2 5000E+07	8 1910E+02	1 1579E+06	4 7643E+03	4 4650E-01
50	1 5518E-03	2 0759E+02	8.9321E+03	6 2336E+07	4 5649E+07	-2 5000E+07	7 7458E+02	1 9808E+06	4 7651E+03	4 4650E-01
51	1 6017E-03	2 0735E+02	8.9328E+03	7 2957E+07	5 6291E+07	-2 5000E+07	7 5704E+02	1 0383E+06	4 7659E+03	4 4650E-01
52	1 6517E-03	2 0695E+02	8.9336E+03	8 5911E+07	6 8844E+07	-2 5000E+07	7 3208E+02	1 7280E+06	4 7660E+03	4 4650E-01

SAMPLE PROBLEM 3 - CU FLYER ON CU TARGET (210M/S)

CYCLE = 121      TIME = 1.00568E-06      TIME STEP = 8.74711E-09      LDT = 27

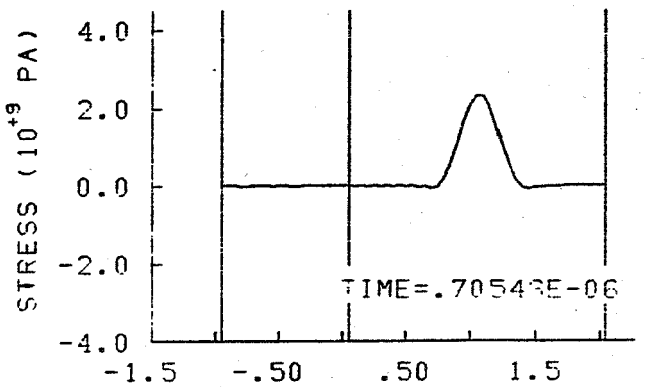
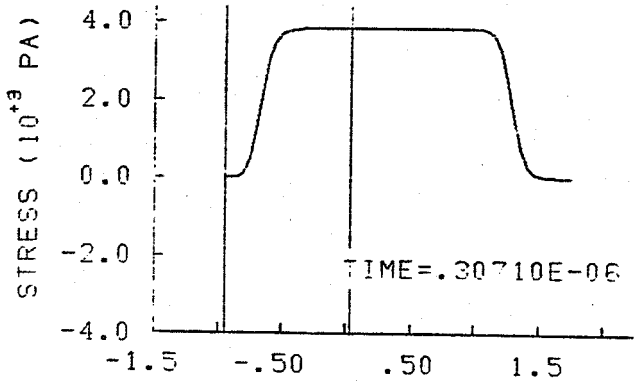
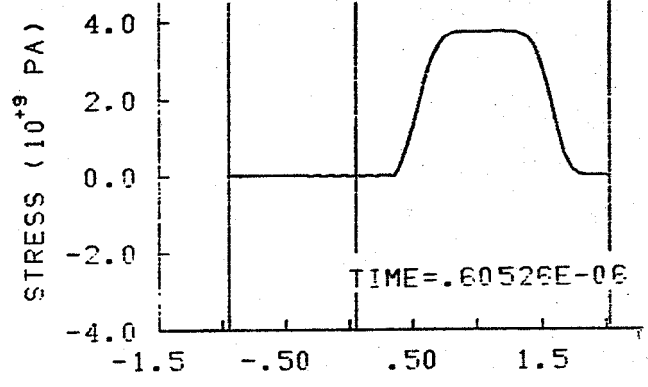
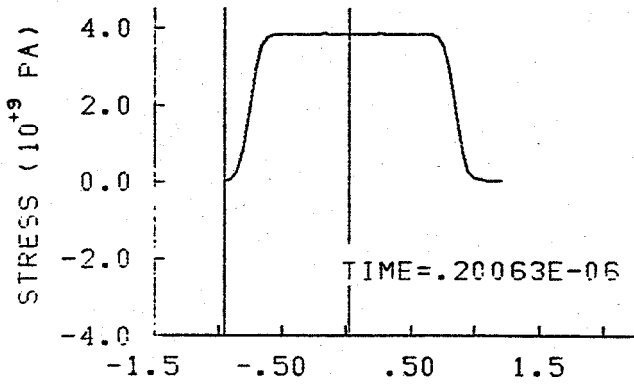
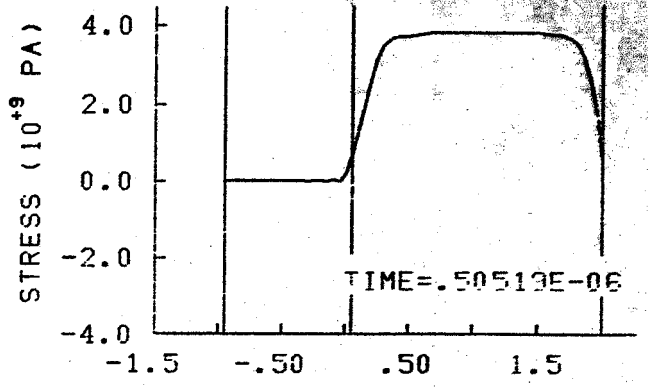
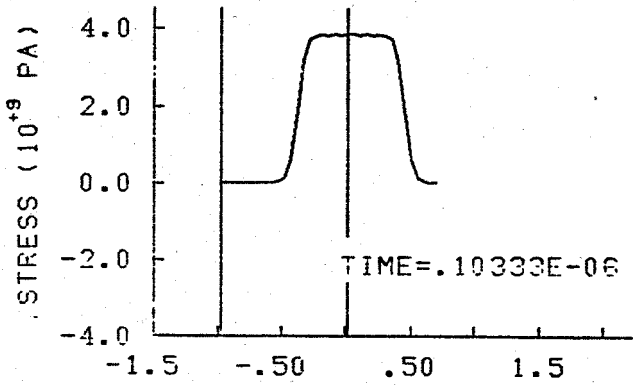
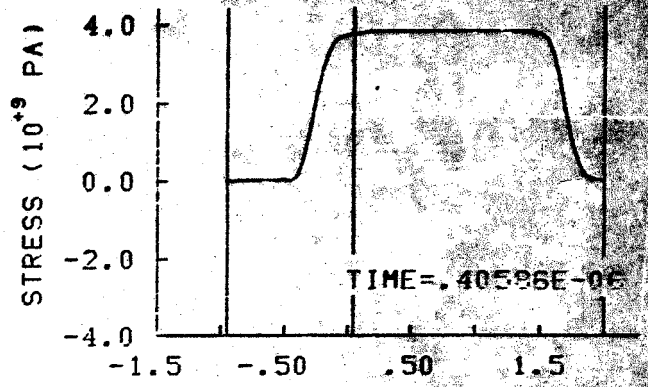
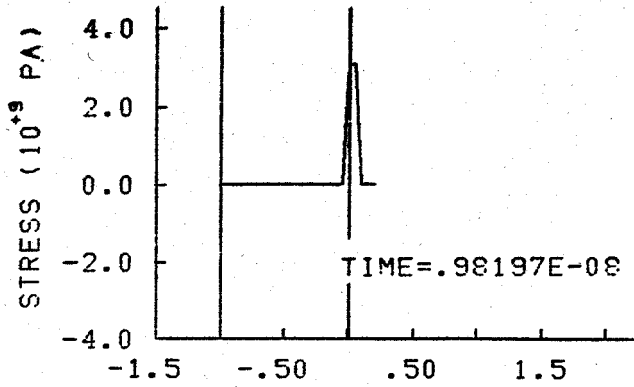
L	POSITION	VELOCITY	DENSITY	STRESS	PRESSURE	PHI	ENERGY	ART VISC	SND SPD	MASS
53	1.7017E-03	2.0639E+02	8.7337E+03	8.7600E+07	7.0934E+07	-2.5000E+07	7.1636E+02	2.3810E+06	4.7669E+03	4.4650E-01
54	1.7517E-03	2.0343E+02	8.7338E+03	8.7571E+07	7.0905E+07	-2.5000E+07	6.7304E+02	1.2888E+07	4.7669E+03	4.4650E-01
55	1.8017E-03	1.9218E+02	8.7284E+03	3.1534E+06	-1.3513E+07	-2.5000E+07	6.6733E+02	5.2104E+07	4.7612E+03	4.4650E-01
56	1.8518E-03	1.6712E+02	8.7057E+03	-3.4712E+08	-3.6379E+08	-2.5000E+07	6.9284E+02	1.2689E+08	4.7372E+03	4.4650E-01
57	1.9022E-03	1.3433E+02	8.8600E+03	-1.0442E+09	-1.0608E+09	-2.5000E+07	9.8890E+02	1.7231E+08	4.6887E+03	4.4650E-01
58	1.9529E-03	1.1054E+02	8.8184E+03	-1.6665E+09	-1.6831E+09	-2.5000E+07	1.5904E+03	1.1633E+08	4.6443E+03	4.4650E-01
59	2.0035E-03	1.0013E+02	8.8148E+03	-1.7214E+09	-1.7381E+09	-2.5000E+07	1.8768E+03	4.6230E+07	4.6403E+03	4.4650E-01
60	2.0540E-03	9.8734E+01	8.8489E+03	-1.2359E+09	-1.2339E+09	2.8778E+06	8.1791E+02	5.8329E+06	4.6765E+03	4.4650E-01
61	2.1041E-03	9.8397E+01	8.7903E+03	-4.2721E+08	-4.1425E+08	1.9450E+07	1.1195E+02	1.4246E+06	4.7339E+03	4.4650E-01

-----MATERIAL INTERFACE-----

WORK AT LBOUND	0.00000E+00	WORK AT RBOUND	0.00000E+00
INTERNAL ENERGY	0.27019E+05	KINETIC ENERGY	0.16812E+06
ADDED ENERGY	0.00000E+00	TOTAL MOMENTUM	0.18733E+04
REL. ENERGY ERROR	0.78386E-02	REL. MOMENTUM ERROR	0.35100E-07

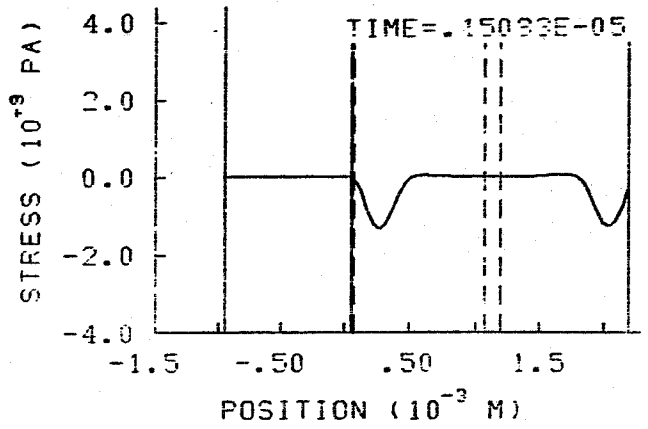
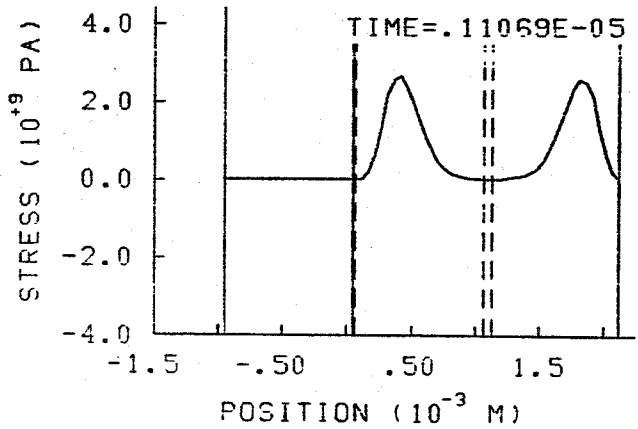
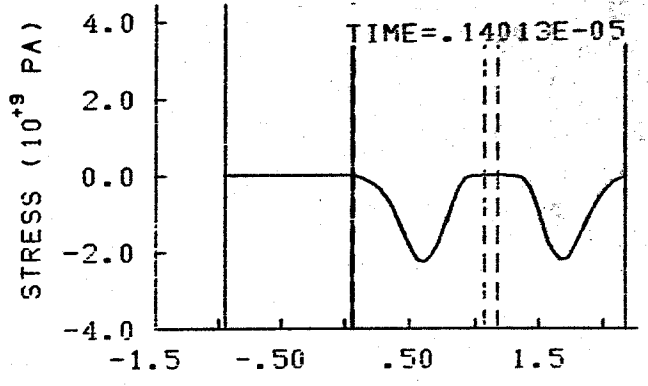
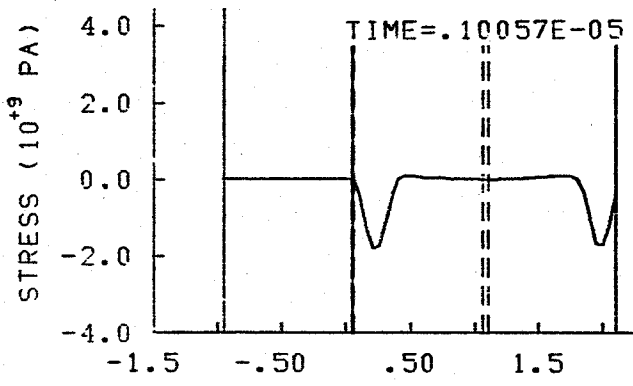
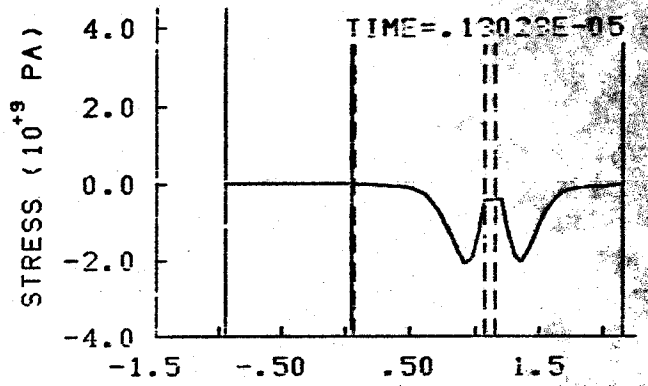
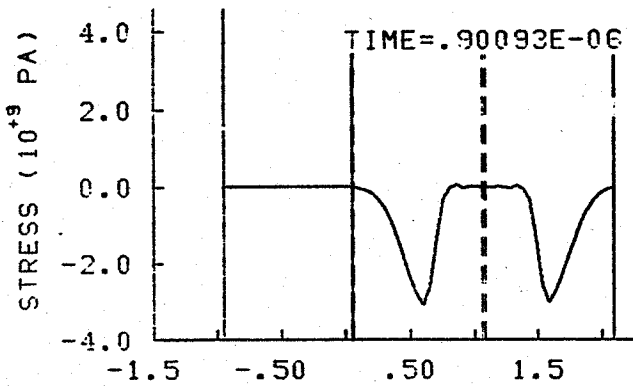
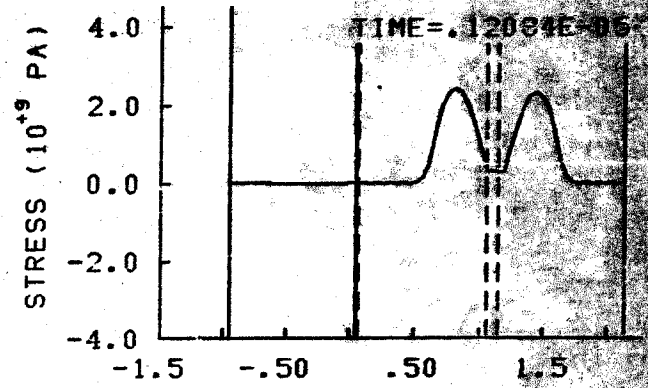
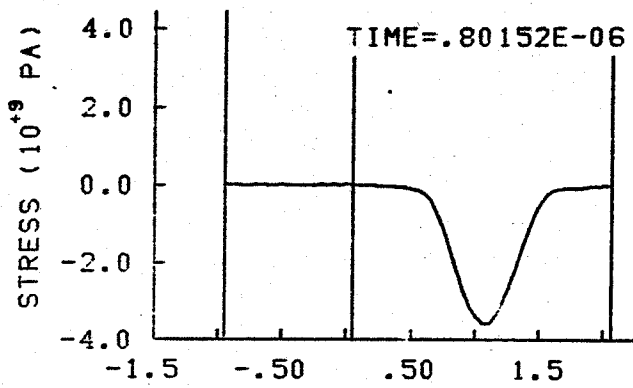
NORMAL EXIT ON CYCLE 179      AT TIME 1.50834E-06      COMPUTED TIME 3.02622E+00 SEC  
 PROBLEM RUNNING AT 1.204E+07 ZONE-CYCLES PER HOUR

SAMPLE PROBLEM 3 - CU FLYER ON CU TARGET (210M/S)



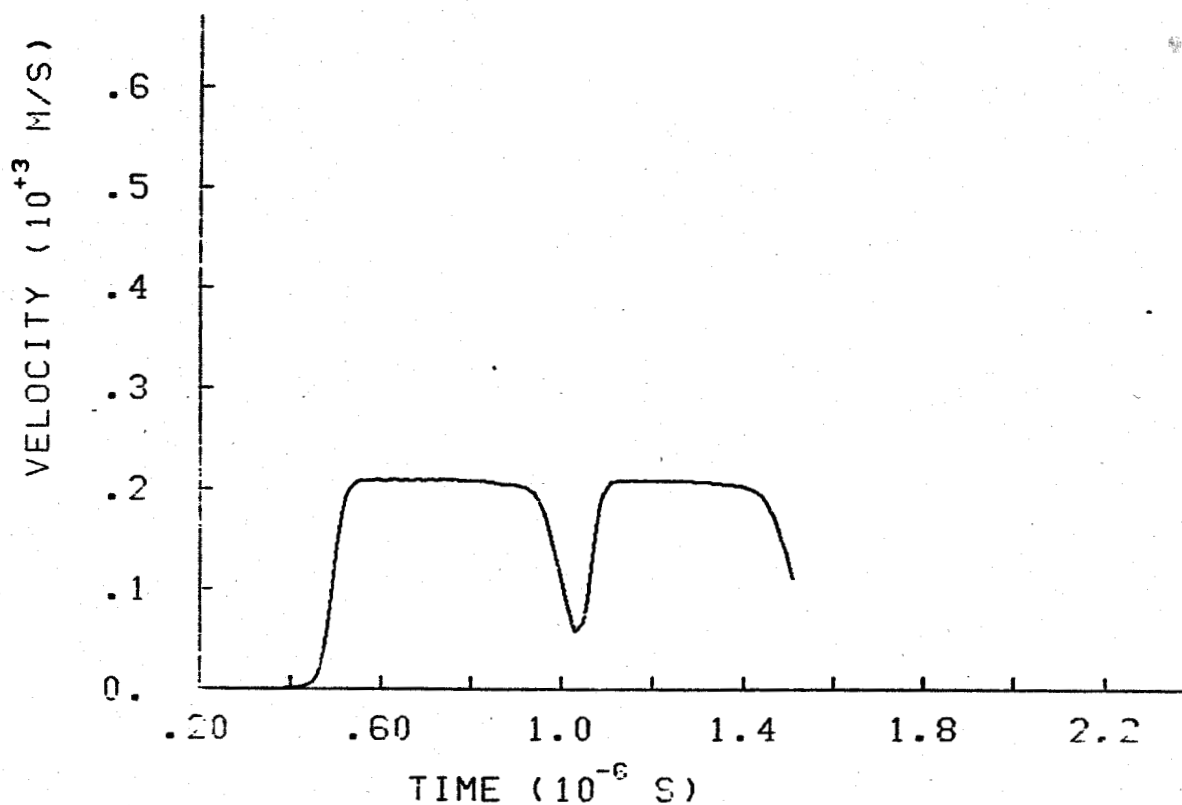
POSITION (10<sup>-3</sup> M)

POSITION (10<sup>-3</sup> M)



Rear free-surface particle velocity history:

INTERFACE 3 PLUS 0 ZONES



SAMPLE PROBLEM 3 - CU FLYER ON CU TARGET (210M/S)

## APPENDIX E

### WONDY V INPUT INSTRUCTIONS

In the subsequent record descriptions the following conventions apply:

1. The subscripts on a variable name indicate the columns in which it is to be located, e.g. 11VAR20 indicates VAR is to be located in columns 11 through 20, according to the specified format.
2. Columns 1 - 5 are reserved for a record identifying number, CARDNO. The number given should actually be located in the appropriate column(s). This allows a much freer form of input for many of the optional input records.
3. Columns 6 - 10 are reserved for additional information about the record, e.g. number of parameters to be read, storage location of data in the record, or plate number to which the data applies, etc.
4. All default values given apply to variables that are left blank. If no default value is listed, zero is assumed.
5. Numbers in parentheses following a variable description refer to page numbers in the WONDY V report where the variables are more fully described.
6. In the case when redundant record type identification appears, the data on the last one in the data sequence will be saved.

The input routines allow stacking of multiple problems in a single run. This option is particularly useful when a series of problems is desired to check the effects of varying some input parameter, say the yield strength. When multiple problems are desired, this fact must be specified by setting NJOB for the first problem (on Record 2) equal to the number of problems in the run. The following illustrates the input structure for single or multiple problems on a given run. If MORSTOR is not to be used, the MORSTOR portions of the file must not be present; this includes the MORSTOR data and its subsequent End-of-Record, <EOR>. Note that the data for the second and following problems need include only data which is different from the first problem; the unchanged data is appropriately carried over by the code. However, if one variable on a record is changed, all the others should contain appropriate values. Note also that the form specified for the End-of-Record <EOR> and End-of-File <EOF> have been chosen for CDC computers.

	}	Appropriate control stream (update and execute WONDY)
		7/8/9 <EOR>
1st		Data for initial WONDY problem
Problem		7/8/9 <EOR>
	}	MORSTOR data for initial problem
		7/8/9 <EOR>
	}	Changes to WONDY data for 2nd problem
2nd		7/8/9 <EOR>
Problem		Changes to MORSTOR data for 2nd problem
	}	7/8/9 <EOR>
		.
		.
		.



Last (NJOB <sup>th</sup> ) Problem	}	Changes to WONDY data for last (NJOB <sup>th</sup> ) problem
		7/8/9 <EOR>
	}	Changes to MORSTOR data for NJOB <sup>th</sup> problem
		6/7/8/9 <EOF>

If it is desired to execute WONDY using only as much storage as the specific problem at hand requires, then the following instructions are appropriate (p. 117) (CAUTION: If REZONE is active, the user must be certain that sufficient size is reserved.)

Appropriate control stream  
(update and execute PREPROCESSOR and WONDY)

7/8/9 <EOR>

7/8/9 <EOR>

1NOP<sub>5</sub> 6NVAR<sub>10</sub> 11NZON<sub>15</sub>      FORMAT(3I5)

(NOP = number of plots)

(NVAR = number of variables - normally 10)

(NZON = total number of zones in problem)

7/8/9 <EOR>

\*READ TAPE50

7/8/9 <EOR>

Data for WONDY problem

7/8/9 <EOR>

Normal Input Data:

RECORD 1       FORMAT (2I5, 7A10)

Record 1 provides identification information used in standard edit and output routines. Data may include user's name, run date, run number or any other identification.

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5	- 1
11TITLE80	- any string of BCD data.

RECORD 2       FORMAT (16I5)

<u>NAME</u>	<u>DESCRIPTION</u>	<u>DEFAULT</u>
1CARDNO5	- 2	
11LPHA15	- geometry coefficient. 1 - rectangular. 2 - cylindrical. 3 - spherical.	
15NOP20	- number of material layers (plates), maximum of 20.	
21NVAR25	- number of variables required to describe material state, normally 10. (p. 118)	
26LHBT30	- left boundary type. (p. 84) 1 - fixed. 2 - fixed minimum (maximum) position. (must specify LBCN (RBCN) in Record 3) 3 - free boundary. 4 - special boundary condition from subroutine BOUNDRY.	3

<u>NAME</u>	<u>DESCRIPTION</u>	<u>DEFAULT</u>
31RHBT35	- right boundary type, same as LHBT above.	3
36LACT40	- zone number of right-hand limit of initial activity. (p. 89)	LMAX
41NJOB45	- number of stacked problems in this job.	1
46NIL50	- Unused	
51NOL55	- initial conditions (cycle 0) printed if NOL $\neq$ 0.	
56NUL60	- last cycle before computer time termination will be printed if NUL $\neq$ 0. NUL is the number of CPU seconds remaining to cause termination.	
61MORE65	- set to 1 if MORSTOR is to be called, NVAR must be at least 11 if MORE $\neq$ 0. (p. 87)	
66JTAPE70	- for restart only, set equal to number of plates to be read from restart tape. (p. 91)	
71NSTART75	- for restart only, set equal to restart cycle number to be read from restart tape. (p. 91)	

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5	- 3
11TMAX20	- maximum problem time, problem has normal exit when T > TMAX.
21DELT(4)30	- maximum initial time step,* when zero initial time step is determined by GENERAT. (p. 79)
31SIGSEP40	- separation stress for previously fractured zone, a negative value exceeding in amplitude the expected numerical noise in stress. (p. 76)
41SIGACT50	- zones become active as stress exceeds SIGACT. (p. 89)
51XZERO60	- initial position of left boundary. (p. 81)
61LBCN70	- left boundary parameter. (p. 85) LBCN = XMIN for LHBT = 2, or can be used for communication with subroutine BOUNDRY.
71RBCN80	- right boundary parameter. (p. 85) RBCN = XMAX for RHBT = 2, or can be used to communicate with subroutine BOUNDRY.

\*A useful guide, for problems in which the first zone may be subjected to large strains by a boundary stress, is

$$\Delta t = \Delta x_1 \sqrt{\epsilon \rho_0 / 2 \sigma_{bdry}}$$

where  $\epsilon = \eta_2$  is the desired strain.

RECORD 4 is optional. Default values are assumed for any variable set to zero or left blank, or if RECORD 4 is absent.

<u>NAME</u>	<u>DESCRIPTION</u>	<u>DEFAULT</u>
1CARDNO5	- 4	
11KT120	- stability criterion constant. (p. 28-29)	0.95
21KT230	- maximum rate of time step increase. (p. 29)	1.05
31B140	- quadratic viscosity coefficient. (p. 26)	2.0
41B250	- linear viscosity coefficient. (p. 27)	0.1
51SIGMAX60	- problem terminates whenever stress exceeds SIGMAX. (p. 99-100)	1.0E100
61EMAX70	- problem terminates whenever energy error exceeds EMAX, warning is printed whenever error exceeds 10 percent EMAX, 1.0 equals 100 percent. (p. 99)	1.0
71HMAX80	- maximum allowable momentum error, problem terminates when this value is exceeded, normally the default value is used. (p. 97)	1.0E100

RECORD 5

FORMAT (2I5, 7E10)

RECORD 5 is optional and specifies the frequency of producing the standard edit. If absent, no standard edit is produced. (p. 93)

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5 - 5	
11S120	- Standard printed edit will start at time S1 and is produced at every $\Delta 1$ time interval until time S2, then at every $\Delta 2$ time interval until time S3, etc. If S3 = 0 edit will stop at S2, if S4 = 0 edit will stop at S3.
21 $\Delta$ 130	
31S240	
41 $\Delta$ 250	
51S360	
61 $\Delta$ 370	
71S480	

RECORD 6

FORMAT (2I5, 7E10)

RECORD 6 is optional and controls calling of subroutine OUTPL to write file for subsequent plotting of selected variables (see RECORD 23). When RECORD 6 is absent, OUTPL will not be called. (p. 95)

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5 - 6	
11S120	- see RECORD 5 for description of parameters.
.	
.	
.	
71S480	

RECORD 7

FORMAT (2I5, 7E10)

RECORD 7 is optional and controls calling of subroutine OUTN. If RECORD 7 is absent, OUTN will not be called.

(p. 95)

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5	- 7
11S120	- see RECORD 5 for description of parameters.
.	
.	
71S480	

RECORD 8

FORMAT (2I5, 7E10)

RECORD 8 is optional and is used to control the writing of a restart tape. If no restart tape is to be produced RECORD 8 is to be omitted. (p. 94)

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5	- 8
11S120	- see CARD 5 for description of parameters.
.	
.	
71S480	

RECORD 9

FORMAT (2I5,7E10)

RECORD 9 is optional and is used to specify additional data as may be required for subroutines OUTPL, OUTN, BOUNDRY etc. (p. 86, 93, 95)

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5	- 9
6I10	- position in the array ADDATA where data on this card is to be stored, I < 94.
11ADDATA(I)20	- specific use of the variables will depend upon the subroutines using them, maximum dimension of ADDATA is 100.
.	
.	
.	
71ADDATA(I+6)80	



RECORD 10 specifies equation of state and initial zoning for each plate. Must be present for each plate or material layer in the problem configuration.

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5	- 10
6PLATE10	- plate number for data in this record.
11STATE20	- equation of state indicator for this plate, $1.0 < STATE < 6.0$ , corresponding to desired EOS routine. If STATE = -IPLT, then EOS parameters, initial conditions, and fracture data for IPLT will be assigned to PLATE.
21NOMESH30	- number of zones in this plate (p. 80). If NOMESH = -IPLT, then zoning configuration and rezoning parameters for IPLT will be assigned to PLATE.
31THKNS40	- plate thickness. (p. 80)
41DELTAX50	- size of leftmost zone. (p. 80)
51DELTAX160	- size of rightmost zone. (p. 80)
61XRATIO70	- not used.
71XGAP80	- distance between right boundary of this plate and left boundary of the next, when XGAP = 0.0 plates are in contact. (p. 81)

RECOR 11 specifies rezone parameters. If RECORD 11 is absent for any plate, rezone will not occur in that plate.

<u>NAME</u>	<u>DESCRIPTION</u>	<u>DEFAULT</u>
1CARDNO5	- 11	
6PLATE10	- plate number for data in this record.	
11RZTIME20	- time to start rezoning this plate. (p. 103)	
21DXMIN30	- smallest allowable zone size for this plate ( $\neq 0$ ). (p. 104)	
31DXMAX40	- largest zone size allowed in this plate. (p. 106)	THKNS
41RCCOMB50	- zones to be combined must have a difference in density less than $RCCOMB \cdot \rho_0$ . (p. 106)	0.1
51RSCRIT60	- stress resolution constant. (p. 102)	$10^{-4} \cdot \rho_0 c_0^2$
61RZCO70	- stress resolution fraction. (p. 102)	0.01
71RZC180	- reference (ambient) stress level. (p. 102)	

RECORD 12 is optional, and specifies the initial conditions for a given plate. When absent default values are assumed. (p. 77-79)

<u>NAME</u>	<u>DESCRIPTION</u>	<u>DEFAULT</u>
1CARDNO5	- 12	
6PLATE10	- plate number for data in this record.	
11EZERO20	- initial energy in this plate.	0.0
21PZERO30	- initial pressure in this plate.	0.0
31RZERO40	- initial density in this plate.	$\rho_0$
41SZERO50	- initial stress in this plate.	0.0
51UZERO60	- initial velocity of this plate.	0.0
61UZEROI70	- initial velocity of right boundary of this plate.	0.0
71ZZERO80	- initial value of $\phi$ in this plate.	0.0

RECORD 13 is optional and specifies plate fracture criteria. Default values are assumed only when entire record is absent, in which case, no fractures will occur in the plate. (p. 72-76)

<u>NAME</u>	<u>DESCRIPTION</u>	<u>DEFAULT</u>
1CARDNO5	- 13	
6PLATE10	- plate number for data in this record.	
11FCRIT20	- fracture mechanism type.	2.0
	1. - stretching at minimum stress. (p. 72)	
	2. - maximum tensile stress failure. (p. 73)	
	3. - cumulative damage criterion (NVAR > 12). (p. 73)	
21SIGMAF30	- (FCRIT = 1) - minimum stretching stress, < 0.	
	(FCRIT = 2) - fracture stress, < 0.	-1.0E100
	(FCRIT = 3) - fracture integral, $K_{max}$ , > 0.	
31FCONST40	- (FCRIT = 1) - melt energy.	
	(FCRIT = 2) - not used.	
	(FCRIT = 3) - exponent, $\lambda$ , in fracture integral, > 0.	
41SIGMAO50	- (FCRIT = 1) - not used.	
	(FCRIT = 2) - not used.	
	(FCRIT = 3) - tensile threshold limit for fracture integral, $\sigma_0$ , < 0.	

RECORD 14 is optional and specifies fracture criteria for the right hand interface of a given plate. Default values are assumed only if RECORD 14 is absent, in which case, the specified interface will not fracture. Parameter definitions are the same as those for RECORD 13. (p. 72-76)

<u>NAME</u>	<u>DESCRIPTION</u>	<u>DEFAULT</u>
1CARDNO5	- 14	
6PLATE10	- plate number, data which follows applies to right hand interface of PLATE.	
11FCRITI20	- FCRIIT at right hand interface on PLATE.	2.0
21SIGMAIF30	- SIGMAI at right hand interface of PLATE.	-1.0E100
31FCONSTI40	- FCONST at right hand interface of PLATE.	
41SIGMAOI50	- SIGMAO at right hand interface of PLATE.	

RECORD 15

FORMAT (2I5, 7E10)

RECORDS 15-19 specify, by plate, the equation of state data which is read into the array CES(I,PLATE). In cases where the specified state routine (STAT2, STAT3, etc.) does not require certain of the 35 constants, the corresponding records may be omitted. All unspecified CES parameters are defaulted to 0. Definitions of the 35 constants appropriate for each of the available state routines are found at the end of this appendix:

STAT1 - p. ,  
STAT2 - p. ,  
STAT3 - p. ,  
STAT4 - p. ,  
STAT6 - p. .

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5	- 15
6PLATE10	- plate number for data in this record.
11CES(1,PLATE)20	- equation of state constants.
.	
.	
.	
71CES(7,PLATE)80	

RECORD 16      FORMAT (2I5, 7E10)

Same as RECORD 15 only for CES(8,PLATE) through  
CES(14,PLATE).    1CARDNO<sub>5</sub> = 16.

RECORD 17      FORMAT (2I5, 7E10)

Same as RECORD 15 only for CES(15,PLATE) through  
CES(21,PLATE).    1CARDNO<sub>5</sub> = 17.

RECORD 18      FORMAT(2I5, 7E10)

Same as RECORD 15 only for CES(22,PLATE) through  
CES(28,PLATE).    1CARDNO<sub>5</sub> = 18.

RECORD 19      FORMAT (2I5, 7E10)

Same as RECORD 15 only for CES(29,PLATE) through  
CES(35,PLATE).    1CARDNO<sub>5</sub> = 19.

RECORD 21

FORMAT (2I5, 3(E10,A10), E10)

RECORD 21 allows the user to replace any or all of three standard edit variables in edit columns 5-7 with other variables. This option is particularly useful when NVAR > 10, and the user desires edits containing these new variables. Default is the standard edit. (p. 94)

<u>NAME</u>	<u>DESCRIPTION</u>	<u>DEFAULT</u>
1CARDNO5	- 21	
11EX120	- variable number for edit column 5.	4
21LABA30	- heading for edit column 5.	PRESSURE
31EX240	- variable number for edit column 6.	10
41LABB50	- heading for edit column 6.	PHI
51EX360	- variable number for edit column 7.	2
61LABC70	- heading for edit column 7.	ENERGY



RECORD 22      FORMAT (2I5, 3(E10,A10), E10)

RECORD 22 has the same function as RECORD 21, except  
for edit columns 8-10.

<u>NAME</u>	<u>DESCRIPTION</u>	<u>DEFAULT</u>
1CARDNO5	- 22	
11EX420	- variable number for edit column 8	5
21LABD30	- heading for edit column 8	VISCOSITY
31EX540	- variable number for edit column 9	1
41LABE50	- heading for edit column 9	SND SPD
51EX660	- variable number for edit column 10	3
61LABF70	- heading for edit column 10	MASS

RECORD 23      FORMAT (2I5, 7E10)

RECORD 23 allows the user to replace any or all of  
the five variables written onto the plotting output file,  
TAPE23. (Note: RECORD 6 must be present to obtain data  
for plotting). (p. 95)

<u>NAME</u>	<u>DESCRIPTION</u>	<u>DEFAULT</u>
1CARDNO5	- 23	
11EPL120	- variable number for first plot output variable	9 (x)
21EPL230	- variable number for second plot output variable	8 (u)
31EPL340	- variable number for third plot output variable	6 ( $\rho$ )
41EPL450	- variable number for fourth plot output variable	7 ( $\sigma_x$ )
51EPL560	- variable number for fifth plot output variable	2 ( $\epsilon$ )

Special Input Cards Read by MORSTOR:

This version of MORSTOR allows for the input to WONDY of an arbitrary energy density deposition profile. An <EOR> must appear between the above WONDY data and the following MORSTOR data. (p. 87-89)

RECORD 31        FORMAT (2I5, 2E10)

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5	- 31
11TDEP20	- deposition time (P. 88)
21ENOR30	- energy per unit surface area deposited, set ENOR = 0. to avoid normalization of deposition profile to this value (p. 87-88)

RECORD 38        FORMAT (2I5, 6E10/(8E10))

RECORD 38 is a multiple record set which specifies the unnormalized arbitrary energy density profile. At least two points should be used for each material layer, even if their energy density values are zero. No points should appear in gaps which may exist between plates. (p. 87)

<u>NAME</u>	<u>DESCRIPTION</u>
1CARDNO5	- 38
6NXY10	- number of points specifying the arbitrary energy density profile, < 100
11XE(1)20	- Eulerian coordinate for first point in profile

<u>NAME</u>	<u>DESCRIPTION</u>
21YE(1)30	- unnormalized energy per unit mass at first point in profile
.	
.	
.	

Use as many records as required, in the specified format; note that the data is in the order XE(1), YE(1), XE(2), YE(2),..., XE(NXY), YE(NXY)

## Equation of State Constants for STAT1

This equation of state routine is used for rate independent initially solid materials. (p. 33-48)

1.  $\rho_0$  - reference density of uncompressed material (p. 34)
  2.  $c_0$  - bulk sound speed of uncompressed material (p. 34)
  3.  $\mathcal{E}_s$  - sublimation energy density (used only in vapor EOS; if blank or zero,  $\mathcal{E}_s$  is defaulted to  $1.0 \times 10^{30}$ ) (p. 37)
  4.  $\mathcal{E}_m$  - melt energy density (used only in tensile stress limitation with vapor EOS; if blank or zero,  $\mathcal{E}_m$  is defaulted to  $1.0 \times 10^{30}$ ) (p. 40)
  5.  $\sigma_{\min}^0$  - maximum tensile stress allowable with vapor EOS (a negative quantity; if blank or zero,  $\sigma_{\min}^0$  is defaulted to  $-1.0 \times 10^{30}$ ) (p. 40)
  6.  $\nu$  - Poisson's ratio (used only when  $\text{NOY} > 0$ ) (p. 44-45)
  7.  $H = \gamma - 1$  - where  $\gamma$  = ratio of specific heats for expanded vapor (set  $H = 0.$  to bypass vapor EOS) (p. 37)
- 
8.  $\text{NOK}^*$  - number of K constants, including  $K_0$  (p. 34-35)
  9.  $K_0$  - ambient bulk modulus (computed internally from  $\rho_0, c_0$ ) (p. 35)
  10.  $k_1$  - if  $\text{NOK} = 0.$  the  $U_s/u_p$  slope  $s$  appears here; otherwise,  $k_1 - k_5$  are non-dimensional parameters in polynomial
  11.  $k_2$  for K. (p. 35)
  12.  $k_3$
  13.  $k_4$
  14.  $k_5$
- 

\* For linear elastic material set  $\text{NOK} = \text{NOH} = 1., \text{NOG} = \Gamma_0 = 0., \text{NOY} = 2.$

15.  $\text{NOH}^*$  - number of  $\Gamma$  constants, including  $\Gamma_0$  (p. 33, 35-36)
16.  $\Gamma_0^*$  - ambient Gruneisen parameter (if  $\text{NOH} = 1.$ ,  $\Gamma = \Gamma_0$ ) (p. 35)
17.  $h_1$  - if  $h_1 = -1.$ , other  $h$ 's zero,  $\Gamma_p = \Gamma_0 \rho_0$  (p. 36)
18.  $h_2$
19.  $h_3$
20.  $h_4$
21.  $h_5$
- 
22.  $\text{NOG}^*$  - number of  $G$  constants, including  $G_0$  (if  $\text{NOG} = 0.$ ,  $G$  is computed from  $K$  and  $\nu$  internally) (p. 41-45)
23.  $G_0$  - ambient shear modulus (computed internally from  $c_0$ ,  $\nu$ ) (p. 45)
24.  $g_1$  -
25.  $g_2$  -
26.  $g_3$  - non-dimensional parameters in polynomial for  $G$ . (p. 45)
27.  $g_4$  -
28.  $g_5$  -
- 
29.  $\text{NOY}^*$  - yield strength indicator (p. 40-48)  
 $\text{NOY} = 0.$  - material is treated hydrodynamically (p. 40)  
 $\text{NOY} = 1.$  -  $Y$  is constant at  $Y_0$  (p. 45)  
 $\text{NOY} = 2.$  -  $Y$  is infinite (p. 45)  
 $\text{NOY} = 3.$  -  $Y$  varies with energy and strain (p. 46)  
 $\text{NOY} = 4.$  - linear isotropic work hardening (p. 47)  
 $\text{NOY} = 5.$  - power law isotropic work hardening (p. 47)
30.  $Y_0$  - initial yield strength
31.  $Y_1$  - coefficient in variable yield models
32.  $Y_2$  - coefficient in variable yield models
33.  $Y_3$  - coefficient in variable yield models
34. - not used
35. - not used

## Equation of State Constants for STAT2

This equation of state routine is used for detonating high explosives.

The three models available are ideal gas, JWL, and BKW. (p. 48-53)

1.  $\rho_0$  - density of undetonated explosive
2.  $c_0$  - initial sound speed (generally D is used)
3.  $\gamma$  - detonation products form:  
 $\gamma > 0$ , ratio of specific heats for ideal gas model (p. 50)  
-  $\gamma = 0$ , selects JWL model (p. 52)  
-  $\gamma < 0$ , selects BKW model (p. 52)
4. D - detonation wave velocity (p. 49)
5.  $X_D$  - detonation point (problem dependent constant) (p. 48-49)
6.  $B_5$  - wave width constant, general  $\sim 2.5$  (p. 49)
7. DT - initiation delay time for given HE layer (p. 49)
- Ideal Gas - ( $\gamma > 0$ )
  8. - 35. - not used
- JWL - ( $\gamma = 0$ )
  8. A -  
dimensional constants
  9. B -
  10.  $R_1$  -
  11.  $R_2$  - non-dimensional constants.
  12.  $\omega$  -
  13. EDET - initial specific energy of HE
  14. - 35. - not used.
- BKW - ( $\gamma < 0$ )
  8. A -

- 9. B -
- 10. C -
- 11. D -
- 12. E -
- 13. EDET - initial specific energy of HE
- 14. - not used
- 15. K -
- 16. L -
- 17. M -
- 18. N -
- 19. O -
- 20.  $\epsilon_a$  - specific energy adjustment
- 21. - not used
- 22. R -
- 23. S -
- 24. T -
- 25. U -
- 26. - 35. - not used

### Equation of State Constants for STAT3

This equation of state routine is used for perfect gasses\*. (p. 53-54)

1.  $\rho_0$  - initial density
2.  $c_0$  - initial sound speed
3.  $\gamma$  - ratio of specific heats
4. - 35. - not used

---

\*When using this equation of state, initial pressure, density, and energy must be prescribed with RECORD 12.



### Equation of State Constants for STAT4

This equation of state routine is used for rate dependent and strain hardening initially solid materials. Superscript letters refer to notes immediately following. (p. 54-65)

1.  $\rho_0$  - reference density of uncompressed material (p. 34)
  2.  $c_0$  - bulk sound speed of uncompressed material (p. 34)
  3.  $\mathcal{E}_s$  - sublimation energy density (used only by vapor EOS; if blank or zero,  $\mathcal{E}_s$  is defaulted to  $1.0 \times 10^{30}$ ) (p. 37)
  4.  $\mathcal{E}_m$  - melt energy density (used in  $k(\mathcal{E})^{(a)}$ ; if blank or zero,  $\mathcal{E}_m$  is defaulted to  $1.0 \times 10^{30}$ ) (p. 59)
  5.  $\sigma_{min}^0$  - maximum tensile stress allowable with vapor EOS<sup>(b)</sup> (a negative quantity; if blank or zero,  $\sigma_{min}^0$  is defaulted to  $-1.0 \times 10^{30}$ ) (p. 40)
  6.  $\nu$  - Poisson's ratio (p. 65)
  7.  $H = \gamma - 1$  - where  $\gamma$  = ratio of specific heats for expanded vapor (set  $H = 0$  to bypass vapor EOS) (p. 37)
- 
8.  $K_0$  - ambient bulk modulus (computed internally from  $\rho_0, c_0$ ) (p. 64)
  9.  $s$  - slope of shock velocity - particle velocity Hugoniot (p. 54)
  10. NOH - Gruneisen parameter indicator (p. 64)  
NOH = 1. -  $\Gamma = \Gamma_0$   
NOH = 2. -  $\Gamma\rho = \Gamma_0\rho_0$
  11.  $\Gamma_0$  - ambient Gruneisen parameter (p. 64)
  12.  $k_0$  - constant in  $k(\mathcal{E})^{(a)}$  (if blank or zero  $k_0$  is defaulted to -2.) (p. 59)
  13. AK - strain rate dependence indicator (p. 60-65)  
AK = 0. - no rate dependence  
AK = 1. - rate dependence

14. NOY(h) - yield strength indicator (P. 60)
- NOY = 0. - hydrodynamic (rate independent) (p. 60)
  - NOY = 1. - constant yield stress,  $Y = Y_0$  (p. 60)
  - NOY = 2. - infinite yield stress (use STAT1 for this option (p. 45)
  - NOY = 3. - variable yield stress<sup>(c)</sup> (p.60)
  - NOY = 4. - linear isotropic work hardening<sup>(d)</sup> (p. 60)
  - NOY = 5. - power law isotropic work hardening<sup>(e)</sup> (p. 60)
  - NOY < 0. - anisotropic work hardening with |NOY| components (maximum of 7) (p. 60)

		<u>NOY &gt; 0.</u>	<u>NOY &lt; 0.</u>
15.	$Y_0$	- initial yield stress	$Y_1$ -
16.	$Y_1$	- used for NOY > 3.	$Y_2$ -
17.	$Y_2$	- used for NOY > 4.	$Y_3$ -
18.	$Y_3$	- computed internally	$Y_4$ -
19.	$Y_4$	- computed internally	$Y_5$ -
20.		- not used	$Y_6$ -
21.		- not used	$Y_7$ -

} individual components of anisotropic yield stress<sup>(f)</sup> (p. 57-58)

22.	$a_1$	}	weights of $Y_1$ for NOY < 0. (p. 57-58)
23.	$a_2$		
24.	$a_3$		
25.	$a_4$		
26.	$a_5$		
27.	$a_6$		
28.	$a_7$		

29.  $\delta$  - maximum allowable fractional error in rate dependence (if blank or zero,  $\delta$  is defaulted to .01) (p. 61)
30. - not used

31. A1 -  
 32. A2 -  
 33. A3 -  
 34. A4 -  
 35. - not used
- } constants for use in relaxation function<sup>(g)</sup> (p. 63)

Notes:

$$(a) \quad k(\ ) = 1 - (k_0 + 2) \frac{\mathcal{E}}{\mathcal{E}_m} + (k_0 + 1) \left( \frac{\mathcal{E}}{\mathcal{E}_m} \right)^2 > 0 \quad (p. 59)$$

$$-2 < k_0 < -1$$

(b) If a given zone-cycle passes through the vapor EOS, then

$$\sigma > \sigma_{\min} = \sigma_{\min}^0 \quad k(\mathcal{E}) < 0 \quad (p. 65)$$

$$(c) \quad Y = Y_0(1 + y_1 \eta) \quad k(\mathcal{E}) > 0 \quad (p. 60)$$

$$(d) \quad Y = E \varepsilon, \quad Y < Y_0$$

$$Y = Y_0 + E^*(\varepsilon - \varepsilon_0), \quad Y > Y_0$$

where  $\varepsilon$  is the strain in uniaxial stress,  $E$  is Young's modulus,  $E^*$  is the tangent modulus, and  $Y_0 = E\varepsilon_0$ .  $E^*$  is input as  $y_2$  and  $E$  is input as  $y_1$ . If  $y_1$  is left blank it is calculated as

$$y_1 = E = 3(1 - 2\nu)K_0$$

$$(e) \quad Y = E \varepsilon, \quad Y < Y_0$$

$$Y = Y_0 + \beta(\varepsilon - Y/E)^\alpha, \quad Y > Y_0$$

where  $\epsilon$  is the strain in uniaxial stress, and  $\alpha$  and  $\beta$  are material constants input as  $y_1$  and  $y_2$  respectively (E is not explicitly required as input to this model)

(f) If the vapor EOS is employed for a given material layer, then the individual yield components are reduced by

$$Y_i = Y_i k(\mathcal{E}) > 0$$

(g) The currently programmed relaxation function is

$$g = \frac{\sigma^d - \sigma_e^d}{G \cdot A1}$$

where  $\sigma^d$  and  $\sigma_e^d$  are the total and equilibrium stress deviators respectively, G is the shear modulus, and A1 is the material relaxation time. Other functions can easily be inserted, and the input constants A2 through A4 are provided for this purpose. (p. 63)

(h) NOY = 0: NVAR must be at least 10

1. < NOY < 3.: NVAR must be at least 13

NOY = 4., 5.: NVAR must be at least 14

-1. > NOY > -7.: NVAR > 13 + |NOY|

Equation of State Constants for STAT6

This equation of state routine is used for initially distended or porous materials. (p. 65-71)

THE INITIAL DENSITY OF THE FOAM MATERIAL MUST APPEAR ON RECORD 12 AS RZERO (PLATE).

1.  $\rho_{SO}$  - density of uncompressed reference solid material (p.67).
  2.  $c_0$  - bulk sound speed of uncompressed reference solid material (p. 68)
  3.  $\epsilon_s$  - sublimation energy density (used only with vapor EOS; if blank or zero,  $\epsilon_s$  is defaulted to  $1.0 \times 10^{30}$ ) (p. 37)
  4.  $\epsilon_m$  - melt energy density (used in tensile stress limitation with vapor EOS; if blank or zero,  $\epsilon_m$  is defaulted to  $1.0 \times 10^{30}$ ) (p. 40)
  5.  $\sigma_{min}^0$  - maximum allowable tensile stress with vapor EOS (a negative quantity; if blank or zero,  $\sigma_{min}$  is defaulted to  $-1.0 \times 10^{30}$ ) (p. 40)
  6. - not used
  7. H - vapor  $\gamma - 1$  (set H = 0. to bypass the vapor EOS) (p. 37)
- 
8. NOK - number of K constants, including  $K_0$  (p. 34-35)
  9.  $K_0$  - ambient bulk modulus for reference solid material (calculated internally from  $\rho_{SO}, c_0$ ) (p. 35)
  10.  $k_1$  - if NOK = 0., the  $U_s/u_p$  slope s appears here (p. 35)
  11.  $k_2$
  12.  $k_3$
  13.  $k_4$
  14.  $k_5$
-

15. NOH - number of  $\Gamma$  constants, including  $\Gamma_0$  (p. 33, 35-36)
16.  $\Gamma_0$  - ambient Gruneisen parameter for reference solid (if NOH = 1.,  $\Gamma = \Gamma_0$ ) (p. 35)
17.  $h_1$  - if  $h_1 = -1.$ , other h's zero,  $\Gamma_{\rho_s} = \Gamma_0 \rho_{so}$  (p. 36)
18.  $h_2$
19.  $h_3$
20.  $h_4$
21.  $h_5$
- 
22.  $c_e$  - bulk sound speed in uncompressed foam material (p. 68)
23.  $\alpha_0$  - initial distention ratio =  $\frac{\rho_{so}}{\rho_{fo}}$ , where  $\rho_{fo}$  is the initial density of the foam (p. 65)
24.  $P_{e0}$  - elastic limit pressure of unheated foam material (p. 67, 69)
25.  $P_{s0}$  - crushup pressure for unheated material (p. 67, 69)
26.  $k_0$  - constant used in energy dependent model (usually  $-1. > k_0 > -2.$ ; if blank or zero,  $k_0$  is defaulted to  $-2.$ ) (p. 69)
27.  $\epsilon_0$  - energy density at which foam strength becomes negligible (if  $\alpha_0 = 0.$ , energy independent model is used) (p. 69)
28. - not used
- 
29.  $B_3$  - additional quadratic viscosity coefficient (try 3.0) (p. 71)
30.  $B_4$  - additional linear viscosity coefficient (try 0.7) (p. 71)
31. N - calculated internal constant  $C_0^2 / \Gamma_0 \epsilon_s$
32. a - calculated internally (p. 68)
33.  $(k_0+2)/\epsilon_0$  - calculated internally (p. 69)
34.  $(k_0+1)/\epsilon_0^2$  - calculated internally (p. 69)
35. - not used

## APPENDIX F

### PLOTTING INPUT INSTRUCTIONS FOR WONDPLT - A GENERAL PURPOSE PLOT ROUTINE

It is frequently of interest to have a visual display of the simulation calculated by WONDY, and the plotting routine WONDPLT provides this capability. Two kinds of plots are available - history of a variable at a fixed Lagrangian point, and spatial dependence of a variable at a fixed time. The latter may be used to generate movies of the material response, which are often indispensable in obtaining a good overall view of a given problem. The former is a useful tool when cross-plotting variables. History plots may be used in conjunction with digitized experimental data to merge on the same plot both the numerical solution and the experimental results.

#### TAPE UNITS WONDPLT

10	plot output
23	input from WONDY
50	Data for merging with History plots

In order to obtain plot output from WONDY, RECORD 6 must be included in the WONDY input. Input to the WONDY code (RECORD 23) may be used to define output for other than the listed variables, and the WONDPLT code may be updated to plot in color, and to plot multiple sets of information on a single frame (i.e. suppress film advance).

Current plotting limitations are set at ten (10) TIMEPLOTS (histories), and the number of points on any given plot may not exceed 1200 without causing plotting truncation.





RECORD 3a      FORMAT (10X, 7A10)

(1-6)            "XLABEL"

(11-80)          Desired Hollerith label for x-axis  
(Center of scale at col. 40)

---

RECORD 4a      FORMAT (15X, 3F5.0, 2E10.0)

(1-5)            "YDATA"

(16-20)          Variable number (see RECORD 2a)

(21-25)          0                    linear scale  
                  1                    logarithmic scale\*

(26-30)          0                    automatic scaling\*  
                  1                    scaling supplied by user

(31-40)          YMIN                if (26-30) non-zero

(41-50)          YMAX                if (26-30) non-zero

---

RECORD 5a      FORMAT (10X, 7A10)

(1-6)            "YLABEL"

(11-80)          Desired Hollerith label for y-axis  
(center of scale at col. 40 for hardcopy, col. 30 for MOV.)

---

RECORD set 1a-5a may be repeated as necessary for other variable combinations.

If a movie is desired, RECORD 1a should be preceded by a RECORD with "MOVIE" in col. (1-5).

---

\*See footnote on previous page.



RECORD 4b      FORMAT (15X, 3F5.0, 2E10.0, 3F5.0)

(1-5)	"YDATA"	
(16-20)	Variable number (see RECORD 2a)	
(21-25)	0	linear scale
	1	logarithmic scale
(26-30)	0	automatic scaling
	1	scaling supplied by user
(31-40)	YMIN	if (26-30) non-zero
(41-50)	YMAX	if (26-30) non-zero
(51-55)	0	no merge plot
	1	merge plot
		merge tape format (TAPE50): NPOINTS, (X(I), Y(I), I = 1, NPOINTS)
(56-60)	0	points not connected on merge data
	1	points connected on merge data
(61-65)		character code for merge data (see Table E-1)

---

RECORD 5b      FORMAT (10X, 7A10.0)

(1-6)	"YLABEL"
(11-80)	Desired Hollerith label for y-axis

---

RECORD set 1b-5b may be repeated up to a limit of ten (10) TIMEPLOT's, in spatial order. The TIMEPLOT card need not be repeated to obtain histories of other variables at the same point.

---

Both XPLOT's and TIMEPLOT's may appear in the same plotting data file.

The final record in the data file must be a record with "END" in col. (1-3)

TABLE E-1  
Standard Scientific Characters

<u>Character</u>	<u>Decimal Code</u>	<u>Character</u>	<u>Decimal Code</u>
0	0	W	54
1	1	X	55
2	2	Y	56
3	3	Z	57
4	4	=	11
5	5	+	16
6	6	-	32
7	7	•	44
8	8	/	49
9	9	.	27
A	17	,	59
B	18	[	60
C	19	]	28
D	20	\$	43
E	21	(Blank)	48
F	22	∂	10
G	23	"	12
H	24	'	13
I	25	δ	14
J	33	α	15
K	34	?	31
L	35	β	29
M	36	±	30
N	37	π	26
O	38	•	42
P	39	γ	45
Q	40	~	46
R	41	d	47
S	50	0	58
T	51	f	61
U	52	Σ	62
V	53	□	63

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