

Wormholes, Time Machines, and the Weak Energy Condition

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It is argued that, if the laws of physics permit an advanced civilization to create and maintain a wormhole in space for interstellar travel, then that wormhole can be converted into a time machine with which causality might be violatable. Whether wormholes can be created and maintained entails deep, ill-understood issues about cosmic censorship, quantum gravity, and quantum field theory, including the question of whether field theory enforces an averaged version of the weak energy condition.

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Normally theoretical physicists ask, "What are the laws of physics?" and/or, "What do those laws predict about the Universe?" In this Letter we ask, instead, "What constraints do the laws of physics place on the activities of an arbitrarily advanced civilization?" This will lead to some intriguing queries about the laws themselves.

We begin by asking whether the laws of physics permit an arbitrarily advanced civilization to construct and maintain wormholes for interstellar travel. Such a wormhole is a short "handle" in the topology of space, which links widely separated regions of the Universe (Fig. 1). The Schwarzschild metric, with an appropriate choice of topology, describes such a wormhole.^{1,2} However, the Schwarzschild wormhole's horizon prevents two-way travel, and its throat pinches off so quickly that it cannot be traversed in even one direction.^{2,3} To prevent pinchoff (singularities) and horizons, one must thread the throat with nonzero stress and energy.⁴ One then faces two questions: (i) Does quantum field theory permit the kind of stress-energy tensor that is required to maintain a two-way-traversable wormhole? (ii) Do the laws of physics permit the creation of wormholes in a universe whose spatial sections initially are simply connected? These questions take on added importance when one recognizes (see below) that, if the laws of physics permit traversible wormholes, then they probably also permit such a wormhole to be transformed into a "time machine" with which causality might be violatable. In the remainder of this Letter we discuss in turn the creation of wormholes, their maintenance by quantum-field-theoretic stress-energy tensors, and their conversion

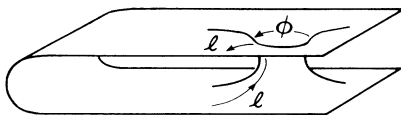


FIG. 1. A wormhole and the external universe at a specific moment of time, embedded in a fictitious higher-dimensional space.

into time machines.

Wormhole creation.—Wormhole creation, with such mild spacetime curvature that classical general relativity is everywhere valid, must be accompanied by closed timelike curves and/or a noncontinuous choice of the future light cone,⁵ and also by a violation of the "weak energy condition."⁶ Specific spacetimes with such wormhole creation are known.⁷ However, it is not known whether the stress-energy tensors required by the Einstein equations in those spacetimes are permitted by quantum field theory.

Wormhole creation accompanied by extremely large spacetime curvatures would be governed by the laws of quantum gravity. A seemingly plausible scenario entails quantum foam^{1,8} [finite probability amplitudes for a variety of topologies on length scales of order of the Planck-Wheeler length, $(G\hbar/c^3)^{1/2} = 1.3 \times 10^{-33}$ cm]: One can imagine an advanced civilization pulling a wormhole out of the quantum foam and enlarging it to classical size. This might be analyzable by techniques now being developed for computation of spontaneous wormhole production by quantum tunneling.⁹

Wormhole maintenance.—For any traversible wormhole a two-sphere surrounding one mouth (but well outside it where spacetime is nearly flat), as seen through the wormhole from the other mouth, is an outer trapped surface. This implies¹⁰ (since there is no event horizon) that the wormhole's stress-energy tensor $T_{\alpha\beta}$ must violate the averaged weak energy condition¹¹ (AWEC); i.e., passing through the wormhole there must be null geodesics, with tangent vectors $k^a = dx^a/d\zeta$, along which $\int_0^\infty T_{\alpha\beta} k^a k^\beta d\zeta < 0$. (Our conventions are those of Ref. 2.) Thus, if one could show that quantum field theory forbids violations of AWEC, one could rule out advanced civilizations maintaining traversible wormholes.

Specialize to a static, spherical wormhole, with spacetime metric⁴

$$ds^2 = -e^{2\Phi} dt^2 + dl^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

where Φ and r are functions of proper radial distance l . Set $l=0$ at the throat ($l < 0$ on the "left" side of the throat and $l > 0$ on the "right" side). Far from the

throat, $r \cong |l| - M \ln(|l|/r_0)$ and $\Phi \cong -2M/|l|$, where M is the wormhole's mass. As l increases from $-\infty$ to 0, r decreases monotonically to a minimum value r_0 , the throat's "radius"; and as l increases onward to $+\infty$, r increases monotonically. Φ is everywhere finite (no horizons). For such a wormhole, AWEC is violated on radial null geodesics, and its violation can be expressed as $\int_{l_1}^{\infty} (\tau - \rho) e^{-\Phi} dl > 0$ for any $l_1 < 0$. Here ρ and τ are the energy density and radial tension, $\rho = e^{2\Phi} \times T^{tt}$, $\tau = -T^{ll}$, and the affine parameter is $\zeta = \int e^{\Phi} dl$.

The following model explores the use of the "Casimir vacuum"¹² (a quantum state of the electromagnetic field that violates the unaveraged weak energy condition¹¹) to support a wormhole: At $l = \pm s/2$, we place two identical, perfectly conducting spherical plates with equal electric charges Q ; we require $s \ll r_0$; and we carry out our analysis only up to fractional errors of order s/r_0 . Between the plates the electromagnetic field is in the Casimir vacuum state for which ρ , τ , and the lateral pressure $p \equiv T^{\theta\theta} = T^{\phi\phi}$ are¹³

$$\tau = 3p = -3\rho = (3\pi^2/720)(\hbar/s^4) \equiv \tau_C.$$

Outside the plates is a classical, radial Coulomb field with $\tau = \rho = p = Q^2/8\pi r^4$, which produces a Reissner-Nordstrom spacetime geometry.² Force balance at the plates requires $Q = (8\pi r_0^4 \tau_C)^{1/2}$. The Einstein field equations can then be satisfied throughout if⁴ (i) $\tau_C = (8\pi r_0^2)^{-1}$, so that $s = (\pi^3/30)^{1/4} (r_0 \sqrt{\hbar})^{1/2}$ [$s \sim 10^{-10}$ cm for $r_0 =$ (distance from Sun to Earth)]; (ii) the wormhole's charge Q and mass M , as measured by distant observers, are both equal to r_0 (aside from fractional corrections $\lesssim s/r_0$); and (iii) the energy per unit area σ of each plate is such as to violate AWEC: $2\sigma < \frac{4}{3} \tau_C s$, or more precisely $\frac{4}{3} \tau_C s - 2\sigma > O(\tau_C s^2/r_0)$.

This violation of AWEC is compatible with a total nonnegative energy of plates plus Casimir field, $2\sigma + \rho s = 2\sigma - \frac{1}{3} \tau_C s \geq 0$. However, if quantum field theory requires that the plates' mass-to-charge ratios exceed that of an electron, $4\pi r_0^2 \sigma/Q > m/e$, then $2\sigma < \frac{4}{3} \tau_C s$ corresponds to a plate separation smaller than the electron Compton wavelength,

$$s < [(\pi^3/270)e^2/\hbar]^{1/2} \hbar/m = 0.029 \hbar/m,$$

which might well be forbidden. To determine whether $\sigma < \frac{4}{3} \tau_C s$ is allowed would require explicit study of quantum-field-theory models for the plates (a task we have not attempted) or a general theorem that quantum field theory forbids violations of AWEC.

Conversion of wormhole into time machine.—Figure 2 is a spacetime diagram for the conversion of a spherical, traversible wormhole into a time machine. Shown unstippled is the nearly flat spacetime outside the wormhole, with Lorentz coordinates T, Z (shown) and X, Y (suppressed). Shown stippled is the wormhole interior, i.e., the region of large spacetime curvature. The central lines of the stippled strips are the wormhole throat,

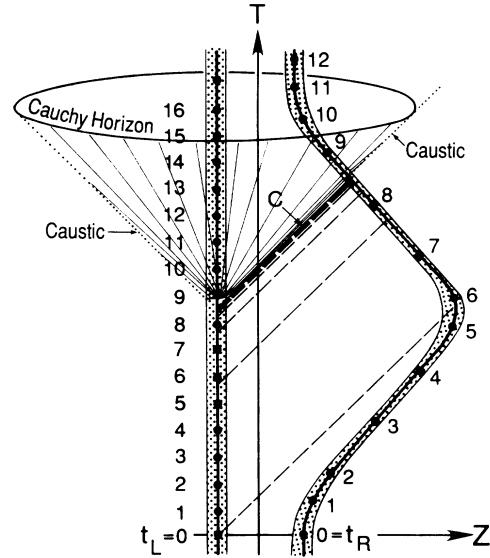


FIG. 2. Spacetime diagram for conversion of a wormhole into a time machine.

parametrized by a time coordinate t introduced below.

At $T=0$, the wormhole's mouths are at rest near each other. Subsequently, the left mouth remains at rest while the right mouth accelerates to near-light speed, then reverses its motion and returns to its original location. The advanced beings can produce this motion by pulling on the right mouth gravitationally or electrically. This motion causes the right mouth to "age" less than the left as seen from the exterior. Consequently, at late times by traversing the wormhole from right mouth to left, one can travel backward in time (i.e., one can traverse a closed timelike curve) and thereby, perhaps, violate causality.

The metric inside the accelerating wormhole and outside but near its mouths is

$$ds^2 = -(1 + g l F \cos\theta)^2 e^{2\Phi} dt^2 + dl^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

Here $\Phi = \Phi(l)$ and $r = r(l)$ are the same functions as for the original, static wormhole; $F = F(l)$ is a form factor that vanishes in the left half of the wormhole $l \leq 0$, and rises smoothly from 0 to 1 as one moves rightward from the throat to the right mouth; and $g = g(t)$ is the acceleration of the right mouth as measured in its own asymptotic rest frame. Just outside the right and left mouths the transformation from wormhole coordinates to external, Lorentz coordinates (with $ds^2 \cong -dT^2 + dX^2 + dY^2 + dZ^2$) is $T = T_R + v\gamma l \cos\theta$, $Z = Z_R + \gamma l \times \cos\theta$, $X = l \sin\theta \cos\phi$, $Y = l \sin\theta \sin\phi$; $T = t$, $Z = Z_L + l \times \cos\theta$, $X = l \sin\theta \cos\phi$, $Y = l \sin\theta \sin\phi$. Here Z_L is the time-independent Z location of the left mouth's center; $Z = Z_R(t)$, $T = T_R(t)$ is the world line of the right mouth's center with $dt^2 = dT_R^2 - dZ_R^2$; $v(t) \equiv dZ_R/dT_R$ is

the velocity of the right mouth; and $\gamma(t) \equiv (1 - v^2)^{-1/2}$. The acceleration appearing in the wormhole metric is $g(t) = \gamma^2 dv/dt$. The right mouth's maximum acceleration g_{\max} and the distance S through the wormhole from left mouth to right must satisfy $g_{\max} S \ll 1$, $g_{\max} |dg/dt|^{-1} \gg S$. This guarantees that with tiny fractional changes of T^{ab} , the wormhole's size and shape are held nearly constant throughout the acceleration.

The region of spacetime containing closed timelike curves is separated from that without such curves by a Cauchy horizon. One might have expected this Cauchy horizon to be unstable (in accord with strong cosmic censorship¹⁴). Indeed, in the analogous two-dimensional (2D) Misner space¹⁵ [obtainable from 2D Minkowski spacetime by identification of $(T, Z) = (\xi, 0)$ with $(T, Z) = (\gamma\xi, L - \gamma v\xi)$, where $L > 0$, $v > 0$, $\gamma = (1 - v^2)^{-1/2}$, and ξ runs from $-\infty$ to $L/\gamma v$] the Cauchy horizon H [located at $T - Z = L/(B - 1)$ where $B \equiv (1 + v)^{1/2}/(1 - v)^{1/2}$] is unstable. Rightward-propagating waves in Misner space get boosted in frequency by a factor B with each passage through the identification world line, and they pass through it infinitely many times as they approach the Cauchy horizon, H . As a result, the stress-energy tensor of such waves diverges at H —presumably thereby preventing the spacetime from evolving the closed timelike curves that it otherwise would have beyond H . [This is the same instability as occurs at the Misner hypersurface in Taub-NUT (Newman-Unti-Tamburino) space.^{10,16}]

In our 4D wormhole spacetime the Cauchy horizon H seems not to suffer this particular instability. There H possesses precisely one closed null generator¹⁷: the curve C in Fig. 2, which runs along the Z axis from left mouth to right mouth, through the throat, then along the Z axis again. The remainder of H consists of null geodesics (very thin lines in Fig. 2) which peel off C to form a future light cone of the wormhole's left mouth (with caustics, where future-directed generators leave the horizon, along the Z axis to the left mouth and right of the right mouth). The most likely place for the Misner-type instability is on C . Indeed, a light ray (dashed curve of Fig. 2), running along the Z axis before horizon formation, gets Doppler shifted by the factor B with each traversal through the wormhole; and it traverses the wormhole infinitely many times as it asymptotes to C . However, the wormhole's AWEC violation causes the throat to act like a diverging lens with focal length $f \approx r_0/2$. Correspondingly, if D is the Z distance between wormhole mouths as measured along C , waves propagating along the dashed curve get reduced in amplitude by $f/D \approx r_0/2D$ with each round trip from left mouth to right and through the wormhole. If the advanced beings arrange that $(f/D)B < 1$, the reduction in amplitude will dominate over the boost in frequency, thereby reducing the wave energy with each round trip and leaving the Cauchy horizon immune to this instability. We suspect,

in fact, that the Cauchy horizon is fully stable and thus constitutes a counterexample to the conjecture of strong cosmic censorship.¹⁴

For Misner space¹⁵ (as also for Taub-NUT space^{10,16}) the extension of spacetime through the Cauchy horizon H is not unique: In one extension all "leftward" causal geodesics (those with initial rightward velocities less than a critical value) are well behaved, while all "rightward" causal geodesics terminate at H after finite affine parameter; in another extension the rightward geodesics are well behaved and the leftward terminate. By analogy, one might expect there to exist other extensions (besides Fig. 2) of the 4D wormhole spacetime beyond H ; one might even hope that in the real universe such a wormhole would actually find and evolve into an extension (possibly nonanalytic) with no closed timelike curves. However, because the set of spacetime geodesics that terminate on H is of "measure zero" (it is a four-parameter set compared to six parameters for generic geodesics), we suspect (provided H is indeed stable) that the extension beyond H is uniquely that of Fig. 2. More generally, we speculate that whenever a spacetime has a fully stable Cauchy horizon, its evolution through that horizon is unique. Similarly, we speculate (as has been suggested to us by Friedman⁷) that in such a spacetime physical fields, both quantum and classical, evolve through and beyond H in unique ways.¹⁸ For example, if initial data for a classical scalar field ψ are specified at $T=0$ in the spacetime of Fig. 2, the resulting evolution via $\square\psi=0$ beyond H will exist and be unique. This is because the set of causal geodesics to the future of H , which do not extend back through $T=0$, is of measure zero (is only a four-parameter set); and such geodesics experience an infinity of "diverging-lens" wormhole traversals as one follows them backward in local time to points from which they could carry unspecified initial data. This makes us doubt that any "new" field ψ can propagate into the spacetime along them.

This wormhole spacetime may serve as a useful test bed for ideas about causality, "free will," and the quantum theory of measurement. As an infamous example, can an advanced being measure Schrödinger's cat to be alive at an event P (thereby "collapsing its wave function" onto a "live" state), then go backward in time via the wormhole and kill the cat (collapse its wave function onto a "dead" state) before it reaches P ?

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