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## Research Article

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# WRELM based CO-OFDM system with improved performance, spectral efficiency and computational complexity

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**Abstract** — Coherent Optical Orthogonal Frequency Division Multiplexing (CO-OFDM) system with Pilot-free phase noise compensator was introduced in order to accomplish the need of high spectral efficiency in the optical communication. In CO-OFDM system Discrete wavelet packet transforms (DWPTs) in place of Fast Fourier Transforms (FFTs) had attracted more attention since it has removed the need of cyclic prefix used to compensate fiber dispersion. In this paper, a DWPT based CO-OFDM system with Wilcoxon Robust Extreme Learning Machine based pilot-free phase noise compensator using multi-level QPSK partitioning of 16-QAM has been proposed. From the results of this work it has been seen that the percentage improvement in performance (in terms of Q-Factor) and spectral efficiency over traditional pilot-aided techniques is approximately 6 and 21 respectively. Moreover, this proposed work will comparatively reduce the overall system complexity.

**Keywords:** Coherent Optical Orthogonal Frequency Division Multiplexing (CO-OFDM); Conventional Phase Estimation (CPE); Discrete Wavelet Packet Transform (DWPT); Extreme Learning Machine (ELM); Multi-level QPSK partitioning of 16-QAM; Phase noise Compensator (PNC); Wilcoxon Robust Extreme Learning Machine (WRELM).

## 1. Introduction

High-speed processing of signals is increasingly becoming a vital factor for several applications such as 5G Internet and cloud computing. The growing requirement of various services of the Internet are producing the need for the development of high spectral efficiency optical communication networks. Coherent Optical Orthogonal Frequency Division Multiplexing (CO-OFDM) system was introduced in order to achieve high spectral efficiency both in flexible passive optical networks and long-haul optical communication systems [1-2]. Laser phase noise caused due to longer symbol duration has become a critical issue in CO-OFDM systems since it results in a degradation in overall system performance [3]. Various compensation techniques such as Pilot-aided (PA) and RF-pilot (RFP) was introduced for the mitigation of phase noise in a CO-OFDM system and verified for the compensation capacity [4-7]. A key problem with pilot-based compensation techniques is the reduction in spectral efficiency because used pilot subcarriers require extra bandwidth. Therefore, various non-pilot aided CPE techniques [8-11] were introduced in order to overcome the problem of spectral efficiency decline. The main downside of these methods is computational expensiveness since the phase noise estimation is performed using the information of all the subcarriers. Various other techniques such as adaptive EKF Based estimator [12] and Gaussian Wavelet Basis Expansion based phase noise compensator were proposed [13].

In this paper, CP- and pilot-free Wilcoxon Robust Extreme Learning Machine (WRELM) based Phase noise compensator has been proposed. In this technique, prior knowledge of channel statistics in the form of pilot subcarriers is not needed, since the received training symbols will be used in the compensator network. It has been shown in results that the proposed technique in comparison to the PA technique improves approximately 4 spans of the transmission reach. Further, it has been noticed from the results that at optimum launch power of 4dBm the improvement in performance is approximately 0.6dB (in terms of Q-Factor). The most noteworthy observation received from this work is that by using the WRELM phase noise compensation technique the results will be more robust and are 21% high spectral efficient as compared to the PA technique.

This paper is divided into six sections. The first section gives an introduction. The second section explains the simulation setup with mathematical modeling of the CO-OFDM system. In the third section, mathematical modeling of proposed work is presented. The fourth section compares the spectral efficiency and overall system complexity of the proposed work with previous techniques. The fifth section discusses various simulation results. The main conclusions are drawn in the final section.

## 2. Simulation setup and mathematical modeling of a CO-OFDM system

### 2.1. Simulation setup

The transceiver design of a basic CO-OFDM system used for simulation work has been shown in Figure 1. Before applying the input data at the serial-to-parallel converter of the transmitter side, 16-QAM modulation using multi-level QPSK partitioning with sliding window [14] has been performed mapping of data. After that for generating the subcarriers, inverse discrete wavelet packet transform (IDWPT) has been used [15]. In our simulation model, a total number of subcarriers is equal to data subcarriers i.e., 128. Since with the

use DWPT in place of FFT, chromatic dispersion and polarization mode dispersion has been mitigated [15-17]. Therefore the OFDM frame consists of a preamble (training symbols for channel equalization) and data symbols only (means there is no need for cyclic prefix symbols). These OFDM frames has been applied to a digital to analog converter of clipping ratio 4dB. And low pass filter has been used to avoid aliasing.

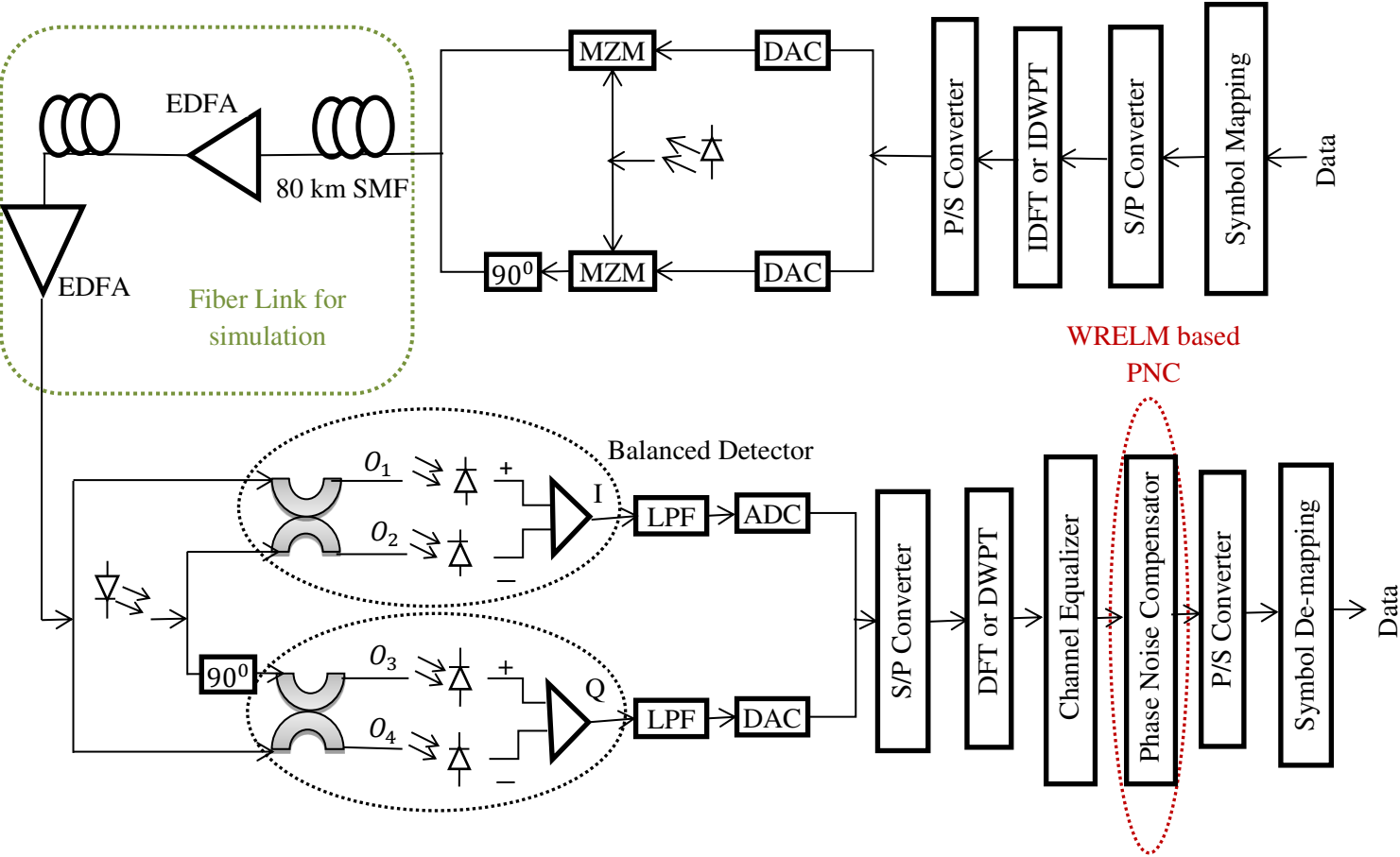


Figure 1 Simulation setup of a basic CO-OFDM system

Table 1. Simulation Parameters	
Modulation	16-QAM using level QPSK partitioning with sliding window
Number of subcarriers	128
Length of cyclic prefix $N_{CP}$	3
Clipping ratio	4dB
Operating wavelength	1550
Sampling rate	6.3Gsamples/s
Single channel data rate	200Gbps
Number of channels	5
Nonlinear fiber coefficient	1.32/W/Km
Noise Figure	4dB
Amplifier gain	16dB
Fiber dispersion	17ps/Km/nm
Fiber loss	0.2dB/Km
Differential group delay	5ps/ $\sqrt{\text{Km}}$
Power	0 to 6dBm
Fiber Length	800Km to 1600Km
Laser Linewidth	200KHz, 500KHz, 1000KHz

For the electrical to optical conversion, a pair of Mach-Zehnder modulators (MZMs) with a phase shift of 90 degrees has been used, to provide smooth frequency response over a range of multiple GHZ. The optical carrier line-width used in the IQ modulator is 1550nm. Then this modulated signal has been transmitted over multiple spans of single-mode fiber (one span is of 80km). Erbium-doped fiber

amplifiers has been used after each span for the compensation of link losses. The total amplifier gain  $G$  for an EDFA of length  $L$  is obtained using equation;

$$G = \Gamma_s \exp \left[ \int_0^L (\sigma_s^e N_2 - \sigma_s^a N_1) dz \right] \dots \dots \dots (i)$$

Here  $N_1 = N_t - N_2$ ,  $N_2 = -\frac{T_1}{a_d h \nu_s} \frac{\partial P_s}{\partial z} - \frac{s T_1}{a_d h \nu_p} \frac{\partial P_p}{\partial z}$ , and  $a_d = \Gamma_s a_s = \Gamma_p a_p$

$N_t$  is the total ion density,  $a_d$  is the cross-sectional area of the doped portion of the fiber core,  $P_s$  and  $P_p$  are input and pump signal powers,  $T_1$  is the spontaneous lifetime of the excited state (10ms for EDFAs),  $\nu_s$  and  $\nu_p$  are input and pump signal frequency,  $\sigma_s^e$  and  $\sigma_s^a$  are the emission and absorption cross section at the angular frequency of  $\omega_s$ , and  $\Gamma_s$  is the input confinement factor.

At the receiver side, coherent detection in place of direct detection is used since it has more spectral efficiency. Analog to digital converter with a sampling rate of 6.3 GSamples/s has been used. In this work, our main focus is on phase noise compensator block. Various simulation parameters used for this work is presented in Table 1.

## 2.2 Mathematical modeling

One of the main aims of this work is to improve the spectral efficiency of a CO-OFDM system. This is possible by avoiding the Cyclic prefix (CP). It has been seen in the previous studies that the need of CP can be eliminated by replacing FFT/IFFT based CO-OFDM systems with wavelet packet transform (WPT) based CO-OFDM systems [15-17]. The length of sinusoids functions of FFT in time domain is infinite while the length of wavelets both in time and frequency domain is finite. Moreover, the speed of computation with WPT is faster than FFT since we can use effective computational algorithm. In this paper, we have used IDWPT/ DWPT are at the transmitter and receiver end, respectively. The transmitted CO-OFDM signal after using Jones vector of IDWPT is expressed as follows:

$$s(n) = \sum_{i=-\infty}^{\infty} \sum_{k=-\frac{1}{2}N_{sc}+1}^{\frac{1}{2}N_{sc}} \bar{c}(k, i) \Pi(t - i T_s) \exp(j 2 \pi f_k (t - i T_s)) \dots \dots \dots (ii)$$

$$s(n) = \begin{pmatrix} s_x \\ s_y \end{pmatrix}, \quad \bar{c}(k, i) = \begin{pmatrix} c(k, i)_x \\ c(k, i)_y \end{pmatrix} \dots \dots \dots (iii)$$

$$f_k = \frac{k - 1}{t_s} \dots \dots \dots (iv)$$

$$\Pi(t) = \begin{cases} 1, & (-\Delta_G < t < t_s) \\ 0, & (t \leq -\Delta_G, t > t_s) \end{cases} \dots \dots \dots (v)$$

The time domain polarization components of  $s(n)$  has been represented by  $s_x$  and  $s_y$ . Jones vector of transmitted  $i^{th}$  OFDM information symbol for  $k^{th}$  subcarrier has been denoted by  $\bar{c}(k, i)$  with the polarization components  $c(k, i)_x$  and  $c(k, i)_y$ . Here,  $f_k$  is the  $k^{th}$  subcarrier frequency,  $T_s$  denotes a period of OFDM symbol,  $t_s$  is the observation period,  $\Delta_G$  is the length of Guard interval, and  $N_{sc}$  is the number of subcarriers i.e., 128.

For handling the fiber dispersion i.e., CD and PMD, the length of a guard interval has been selected by using the following marginal timing condition;

$$\frac{c}{f^2} |D_t| \cdot N_{sc} \cdot \Delta f + DGD_{max} \leq \Delta_G \dots \dots \dots (vi)$$

Here  $c$  is the speed of light,  $f$  is the optical carrier frequency,  $D_t$  is the total accumulated chromatic dispersion,  $\Delta f$  is the spacing between subcarrier channels, and  $DGD_{max}$  is the maximum value of differential group delay. For generating a sufficient margin, the chosen value of  $DGD_{max}$  should be equal to 3.5 times the mean value of PMD.

For the efficient computation of DWPT, FIR filter bank i.e., quadrature mirror filter (QMFs) has been used. The received CO-OFDM signal in the form of wavelet packet coefficients, can be expressed as follows:

$$\bar{r}(k, i) = e^{j\phi(k, i)} \cdot e^{j\phi_D(f_k) T(k)} \bar{c}(k, i) + \bar{n}(k, i) \dots \dots \dots (vii)$$

where  $\bar{r}(k, i) = (r(k, i)_x \ r(k, i)_y)^t$  represents the transpose of Jones vector of received  $i^{th}$  OFDM information symbol for  $k^{th}$  subcarrier. The inline optical amplifiers based ASE noise with polarization components has been represented by  $\bar{n}(k, i) = (n(k, i)_x \ n(k, i)_y)^t$ .

The optical fiber link Jones matrix of the channel has been denoted by  $T(k)$  which can be expressed as follows:

$$T(k) = \prod_{l=1}^N \exp \left\{ \left( -\frac{1}{2} j \beta_l f_k - \frac{1}{2} \alpha_l \right) \sigma \right\} \dots \dots \dots (viii)$$

where  $N$  is a number of spans or cascading elements,  $\beta_l$  is the birefringence vector, and  $\alpha_l$  is the Polarization dependent loss (PDL) vector,  $\sigma$  is the Pauli matrix.

The chromatic dispersion based phase change  $\phi_D(f_k)$  can be easily estimated as follows:

$$\phi_D(f_k) = \pi c D_t f_k^2 / f_{LD}^2 \dots \dots \dots (ix)$$

where  $f_{LD}$  is the laser frequency.

$\phi(k, i)$  represents the phase noise in  $i^{th}$  OFDM symbol and  $k^{th}$  subcarrier due to phase noises generated at both transmitter and receiver lasers and local oscillators. The estimated phase drift of the  $i^{th}$  OFDM symbol can be written as;

$$\phi(k, i) = \frac{1}{N_{sc}} \sum_{k=1}^{N_{sc}} \{\arg(r_{ik}) - \arg(\hat{s}_{ik})\} \dots \dots \dots (x)$$

Where  $\arg(\cdot)$  is the phase angle of the information symbol,  $\hat{s}_{ik}$  is the known transmitted training symbol, and  $r_{ik}$  is received symbol,  $N_{sc}$  the number of OFDM subcarriers. Moreover, laser phase noises are more dominating in  $\phi(k, i)$ . The focus of this work is to estimate and compensate the effects of laser phase noise. The  $I(i)$  is the Inter-carrier Interference noise in  $i^{th}$  OFDM symbol generated from phase noise and can be calculated as [18];

$$I(i) = \sum_{n=1, n \neq k}^{N_{sc}} \bar{c}(n, i) T(k) \eta_{n-k} \dots \dots \dots (xi)$$

$$\eta_m = \frac{1}{N_{sc}} \frac{\sin(\pi(m+e))}{\sin(\pi(m+e)/N_{sc})} \exp\left(j\pi(m+e)\left(1 - \frac{1}{N_{sc}}\right)\right) \dots \dots \dots (xii)$$

ICI coefficient is represented by  $\eta_m$  with  $m$  difference between two subcarriers; the value of normalized RFO  $\ll 1$ .

The frequency domain representation of subcarriers has been illustrated in Figure 2 (a) and (b) without any phase noise and with phase noise respectively. In Figure 2(b), it has been shown that due to the effect of inter-carrier interference generated from laser phase noise the orthogonality between the subcarriers of the OFDM system is lost. The main purpose of this work is to propose an improved phase noise mitigation technique and compare it with existing techniques.

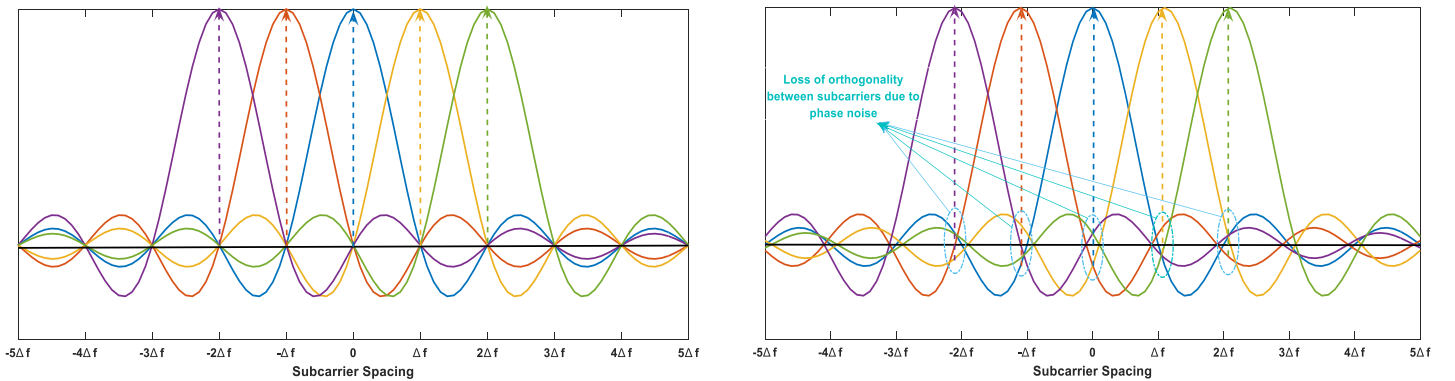
The variance of the interference  $I(i)$  can be calculated as [18];

$$\sigma_{ICI}^2 = \sigma_c^2 \sum_{n=1, n \neq k}^N |\eta_{n-k}|^2 \dots \dots \dots (xiii)$$

The effective signal to noise ratio ( $SNR'$ ) has been used as performance metric in this work and can be calculated as [19];

$$SNR' = \frac{\sigma_c^2}{\sigma_{ICI}^2 + \sigma_w^2} = \left( \sum_{n=1, n \neq k}^N |\eta_{n-k}|^2 + SNR^{-1} \right)^{-1} \dots \dots \dots (xiv)$$

where  $SNR = \frac{\sigma_c^2}{\sigma_w^2}$  is the original channel SNR without the effect of inter-carrier interference.  $\sigma_w^2$  is the variances of the ASE noise.



**Figure 2** Frequency domain picture of subcarriers (a) without phase noise corruption (b) with phase noise corruption

### 3. Proposed phase noise estimation technique

Machine learning is attracting much attention over the last several decades [19]-[21]. It has been noticed that the convergence rate of Wilcoxon robust extreme learning machine (WRELM) algorithm is very fast as compared to other available robust machine learning algorithms [22]. A real-complex extreme learning machine (RC-ELM) based phase noise compensation technique was proposed which has used the pilot subcarriers as a training set for learning [23]. But, this technique has many drawbacks such as difficult to implement, less robustness, and less spectral efficiency. For removing these drawbacks, WRELM based algorithm for phase noise mitigation has

been used in this work. Since we are using channel equalizer training symbols instead of pilot subcarriers, therefore, the proposed work has high spectral efficiency than previously proposed RC-ELM technique. In this section, the detailed mathematical modeling for proposed technique has been presented.

### 3.1 Basic concepts of Wilcoxon norm

Before discussing the proposed work some basic concepts of Wilcoxon norm have been presented in this section. In statistics, due to its robustness against outliers, Wilcoxon learning over linear regression methods is attracting widespread interest [24]. In Wilcoxon learning, as the main function Wilcoxon norm is calculated for a vector [20]. The Wilcoxon norm of a score function  $\varphi: [0,1] \rightarrow \mathbb{R}$  (which is a non-decreasing function) can be given by

$$\int_0^1 \varphi^2(u) du < \infty \dots \dots \dots (xv)$$

The value of this score function ( $\varphi$ ) has been calculated as [24];

$$a(i) = \varphi\left(\frac{i}{l+1}\right), \quad i \in l \dots \dots \dots (xvi)$$

where  $l$  denotes fixed value positive integer. To get the Wilcoxon norm of any function, a pseudo-norm (semi-norm) is utilized [19]-[24] which is calculated as follows;

$$\|x\|_w = \sum_{i=1}^l a(R(x_i))x_i \dots \dots \dots (xvii)$$

where  $x = [x_1 \ x_2 \ \dots \ x_l]^T \in \mathbb{R}^l$  while  $R(x_i)$  represents the Rank value of  $x_i$ .

### 3.2 Mathematical modeling of the proposed work

In this work, the phase noise for  $i^{th}$  OFDM symbol has been modeled as wiener process;

$$\phi(k, i) = \phi(k, i-1) + \Delta\phi(k, i) \dots \dots \dots (xviii)$$

where  $\Delta\phi(k, i)$  is a Gaussian random variable. It has been represented in matrix  $H$  and Penrose-Moore pseudo inverse  $H^+$  for  $k$  number of subcarriers and  $i$  number of OFDM symbols. The matrix  $H$  has been calculated using following equation [25];

$$H = \begin{bmatrix} G(a_1, v_1, \phi_1) & \dots & G(a_k, v_k, \phi_1) \\ \vdots & \dots & \vdots \\ G(a_1, v_1, \phi_i) & \dots & G(a_k, v_k, \phi_i) \end{bmatrix} \dots \dots \dots (xix)$$

where  $a_j$  and  $v_j$  are the mean and variance. In phase noise estimation, the value of  $a_j$  and  $v_j$  is zero and  $2\pi\Delta\nu T_s$  respectively. In this work, Radial Basis Function (RBF) network based Gaussian activation function i.e.,  $G(a_j, v_j, \phi)$  has been calculated using following equation [26];

$$G(a_j, v_j, \phi) = g\left(\frac{\|\phi - a_j\|}{2v_j^2}\right), \quad v_j \in \mathbb{R} \dots \dots \dots (xx)$$

The equation used for the calculation of gaussian activation function is;

$$g(.) = \frac{1}{\sqrt{2\pi}v_j} \exp\left[-\frac{\sum_{i=1}^O a[R(\phi_j(i) - a_j)]}{2v_j^2}\right] \dots \dots \dots (xxi)$$

where  $i$  is the OFDM symbol number and it varies from 1 to  $o$ . In this work, phase noise has been predicted by using training symbols (preamble) utilized for channel equalization. The value of common phase noise error has been estimated by using the Kalman filtering (Weiner process) based recursive formulae presented below:

$$\phi(k, i+1|i) = \phi(k, i|i-1) + K(k, i)(\theta(k, i)^\omega - \phi(k, i|i-1)) \dots \dots \dots (xxii)$$

where the initial value of apriori and the aposteriori phase change i.e.,  $\phi(k, i|i-1)$  and  $\phi(k, i)$  has been estimated by using training symbol and zero respectively.  $K(k, i)$  denotes the Kalman gain. The formula used to calculate the value of  $\theta(k, i)^\omega$  mean rotation angle is expressed as follows:

$$\theta(k, i)^\omega = w\theta(k, i)^{out} + (1-w)\theta(k, i)^{in} + \phi(k, i|i-1) \dots \dots \dots (xxiii)$$

In this technique, for preventing blind estimation based phase slip all symbols has been first rotated by apriori phase value. Then, these symbols has been raised to the power of four for estimating the  $\theta_m^{in}$  and  $\theta_m^{out}$  mean rotation angles. e.g., weight value  $w = 0.2$ , Rotation angle of outer constellation points i.e.,  $\theta(k, i)^{out}$  is  $40^\circ$ , Rotation angle of outer constellation points i.e.,  $\theta(k, i)^{in}$  is  $10^\circ$  and apriori phase change  $\phi(k, i|i-1)$  of training OFDM symbol is  $5^\circ$ . So the mean rotation angle is  $21^\circ$ . For a large number of subcarriers, the estimation of the rotation angle is more accurate. Since a number of points required for the inner and outer layer of constellation diagram has been accurately calculate for QPSK partitioning of QAM modulation format.

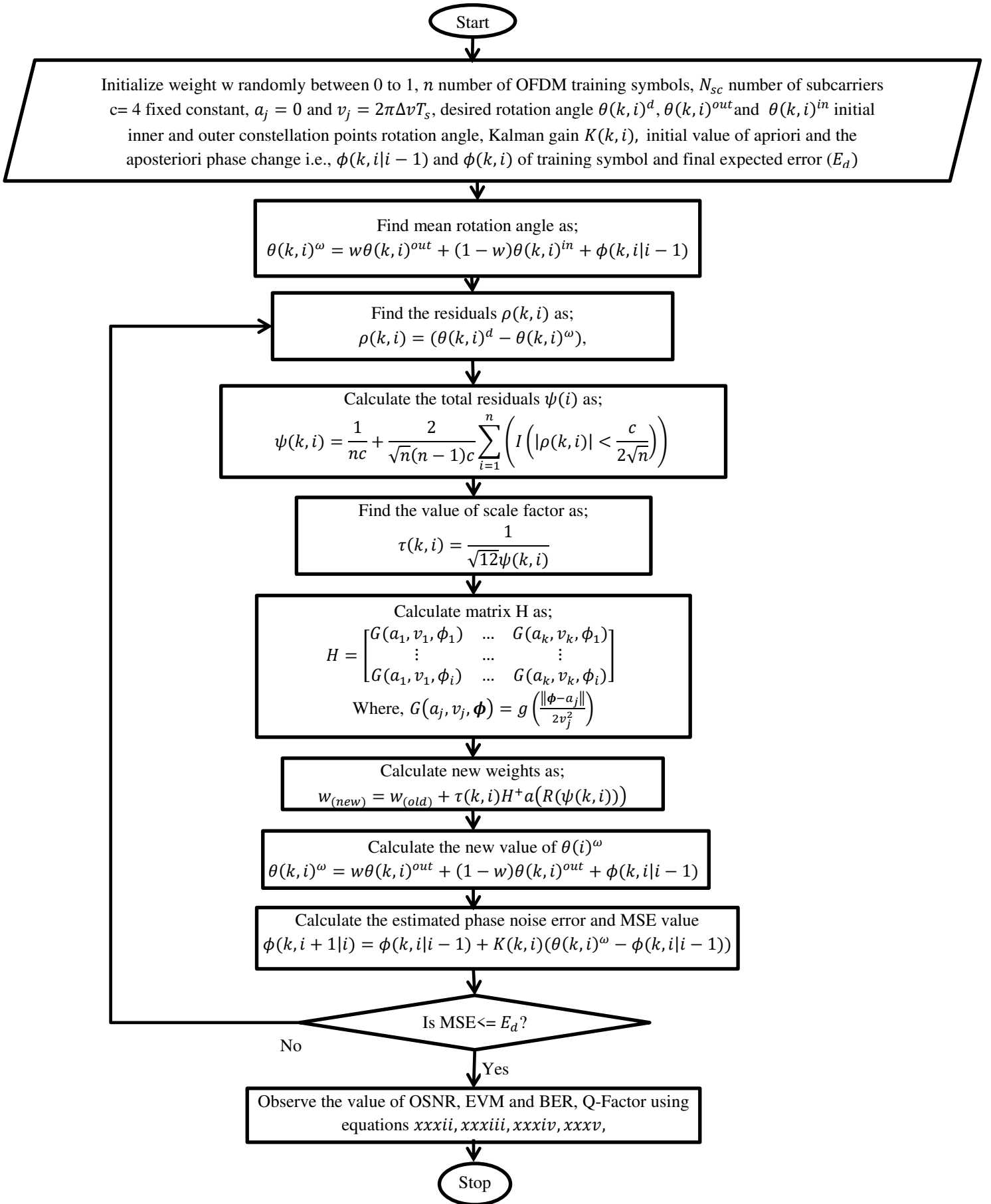


Figure 3 Flow chart of the proposed work

The weight  $w$  has been initially selected randomly between 0 to 1 and updated using following formula [22];

$$w_{(new)} = w_{(old)} + \tau(k, i) H^+ a(R(\psi(k, i))) \dots \dots \dots (xxiv)$$

the value of scale factor  $\tau(i)$  was calculated;

$$\tau(k, i) = \frac{1}{\sqrt{12}\psi(k, i)} \dots \dots \dots (xxv)$$

The Residual values( $\psi(i)$ ) has been calculated using following equation;

$$\psi(k, i) = \frac{1}{nc} + \frac{2}{\sqrt{n}(n-1)c} \sum_{i=1}^n \left( I \left( |\rho(k, i)| < \frac{c}{2\sqrt{n}} \right) \right) \dots \dots \dots (xxvi)$$

where  $\rho(k, i) = (\theta(k, i)^d - \theta(k, i)^\omega)$ , here  $\theta(k, i)^d$  is the desired rotation angle,  $\theta(k, i)^\omega$  is the actual rotation angle, and  $R(\rho(k, i))$  denotes the rank of the residual  $\rho(k, i)$ . Moreover,  $c$  is a fixed constant value and  $I(f_k)$  is the Fisher information, which has been calculated from the applied density function  $f_k$  as [22];

$$I(f_k) = \int_{-\infty}^{+\infty} (f'_k(x))^2 / f_k(x) dx < \infty \dots \dots \dots (xxvii)$$

This procedure of angle rotation and phase estimation has been repeated for the initially selected number of epochs. At the end, the response of WRELM algorithm for phase noise compensation in the CO-OFDM system has been checked. The number of epochs can be increased according to the required set SD-FEC limit of BER. The proposed compensator is working on many equations. For better clarification of the proposed work one flow chart is added in paper as Figure 3.

#### 4. Comparison of spectral efficiency and complexity

##### 4.1 Spectral efficiency calculation

The spectral efficiency (amount of information transmitted per unit bandwidth) of a traditional CO-OFDM system is;

$$SE \text{ (bits/sec/Hz)} = 2 \cdot \frac{R}{BW_{Occupied}} \dots \dots \dots (xxviii)$$

The spectral efficiency of a proposed CO-OFDM system is;

$$SE \text{ (bits/sec/Hz)} = \frac{N_{filled}}{N_{SC}} \times \frac{32G}{BW_{Occupied}} \times b \dots \dots \dots (xxix)$$

The data subcarriers by using 16-QAM and QPSK samples with PRBS-15 bit stream has been loaded in  $N_{filled}$  subcarriers. For the generation of 128 number of subcarriers the value of  $N_{SC}$  has been selected to 128.  $N_{CP}$  i.e, length of cyclic prefix has been set at 0 since the use of DWPT has removed the need of CP to compensate dispersion (linear impairments). One frame of OFDM has been comprised of the preamble (training symbols) used for channel estimation followed by data symbols. In this study, 32GBd CO-OFDM system has been considered. The symbol  $b$  denotes the number of bits per symbol.

To calculate the total occupied bandwidth (BW) following formula has been used:

$$BW_{occupied} = \frac{2}{T_s} + \frac{N_{sc} - 1}{t_s} \dots \dots \dots (xxx)$$

where  $t_s$  is the observation period and  $T_s$  is the OFDM symbol period. And Total symbol rate  $R$  is;

$$R = \frac{N_{SC}}{T_s} \dots \dots \dots (xxxi)$$

$t_s$  is the observation period and  $T_s$  is the OFDM symbol period. For the generation of 128 number of subcarriers the value of  $N_{SC}$  has been selected to 128. The % improvement in SE with the proposed technique is approximately 21 as compared to conventional pilot aided technique which had reported equal to 18 [11] in Pilot-free K4P technique by L. N. Venkatasubramani and co-authors.

##### 4.2 Overall Complexity Comparison

The step by step comparison of system complexity of the proposed work and other techniques has been presented in Table 2. The complexity with proposed work to compensate linear impairments such as CD and PMD is  $N_{sc}^2$  which was equal to  $4N_{sc}^2$  with conventional PA and Pilot free K4P technique.

The complexity for splitting the band into a number of subcarriers using a transform with proposed work is equal to  $(N_{sc} - 1) \cdot L$ , which was  $N_{sc} \cdot (\log_2 N_{sc})$  in conventional PA and pilot-free K4P technique. Here,  $L$  is a length of the QMF used for DWPT.

In the conventional Pilot-Aided phase noise compensation technique, the complexity was dependent on the number of pilots ( $N_p$ ) used. The complexity in Conventional Pilot-Aided technique and Pilot-free K4P algorithm for the compensation of phase noise was equal to  $9 \times N_p$  and  $(12 \wedge + 1)N_{sc}$ , which with the proposed technique is equal to  $((12 \wedge + 1)N_{sc}L) - 3L$ . Here,  $\wedge$  is the ratio of the outermost circle constellation points to the total number of constellation points of QPSK. The value of  $\wedge$  is equal to 0.25 for 16-QAM. From this Table 2 it has been observed that the overall system complexity will reduce with the proposed work.



Table 2 Overall system complexity comparison of the proposed technique with previous works in terms of complex multiplications

DSP Step	Conventional Pilot-Aided [6]	Pilot-free K4P [11]	Proposed work
Linear Impairments (CD and PMD) Compensation	$4N_{sc}^2$	$4N_{sc}^2$	$N_{sc}^2$
For splitting the band into a number of subcarriers using transform	$N_{sc} \cdot (\log_2 N_{sc})$	$N_{sc} \cdot (\log_2 N_{sc})$	$(N_{sc} - 1) \cdot L$
Phase noise compensation	$9 \times N_p$	$(12 \wedge + 1)N_{sc}$	$((12 \wedge + 1)N_{sc}L) - 3L$

## 5 Simulation results and discussions

The simulation results obtained using various phase noise compensation techniques to improve the performance of the CO-OFDM system have been summarized in Figure 4 to Figure 11. In this section various parameters used for performance measurement such as optical signal to noise ratio (OSNR), error vector magnitude (EVM), Bit error rate (BER) and Q-Factor has been presented.

The relationship between the optical SNR (OSNR) and effective SNR presented has been given by following equation;

$$OSNR(dB) = 10 \log_{10}[SNR] + 10 \log_{10} \frac{B_f}{R_s} \dots \dots \dots (xxxii)$$

where  $B_f$  denotes the central bandwidth while  $R_s$  denotes the symbol rate.

In this work, the multi-level QPSK partitioning of 16-QAM modulation format has been used. Here in 16-QAM modulation format, outermost and innermost constellations points has been calculated at a phase angle of  $(2n + 1)\pi/4$  using QPSK modulation technique, where  $n$  has been varied from 0 to 3.

For five-channel CO-OFDM system, at a combined laser line-width of 200 kHz, the received constellation diagram with the proposed work has been presented in Figure 4.

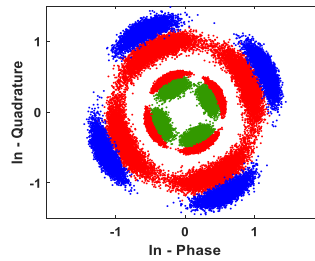


Figure 4 Received constellation diagram for proposed work

To calculate the numerical value of CO-OFDM system performance from the spreading of points in the received constellation diagram, error vector magnitude (EVM) has been used. The formula to calculate the EVM has been presented below [27]:

$$EVM_{rms} = \sqrt{\frac{\frac{1}{N} \sum_{k=1}^N |y_k - x_k|^2}{S_{max}^2}} \dots \dots \dots (xxxiii)$$

where  $x_k$  and  $y_k$  represents the ideal and received constellation points, respectively. And  $S_{max}$  denotes the maximum number of points in the constellation diagram over which the EVM has been calculated.

The formula used for the calculation of BER from the EVM in M-ary square QAM format has been presented below [28-29]:

$$BER = \frac{(1 - M^{-1/2})}{\frac{1}{2} \log_2 M} \operatorname{erfc} \left[ \sqrt{\frac{3/2}{(M - 1)EVM_{rms}^2}} \right] \dots \dots \dots (xxxiv)$$

Now let us discuss the simulation results of the proposed work. First of all, the effect of variation in normalized line-width on B2B performance of the CO-OFDM system has been studied. For the measurement of performance at each line-width, the value of BER has been calculated from OSNR. The normalized line width versus OSNR penalty, at BER of value  $1 \times 10^{-3}$  using proposed technique as compared to previously reported techniques in back to back transmission has been presented in Figure 5. It has been observed in this plot, that by increasing in a number of pilots is reducing the OSNR penalty. But it has reduced the spectral efficiency. Moreover, it has been seen in the results of Figure 5, that the proposed technique is significantly reduce the OSNR penalty at smaller values of  $\Delta\nu T_s$ .

The improvement in OSNR penalty (dB) and spectral efficiency (%) at high value of normalized line-width i.e.,  $2 \times 10^{-3}$  has been presented in Table 3. From this Table 3, two main observations have been concluded. First observation is that the proposed work is better in terms of spectral efficiency as compared to previously reported techniques for phase noise compensation with respect to the pilot-aided technique. Second observation from this Table 3 is that proposed technique is better in terms of OSNR penalty as compared to earlier techniques.

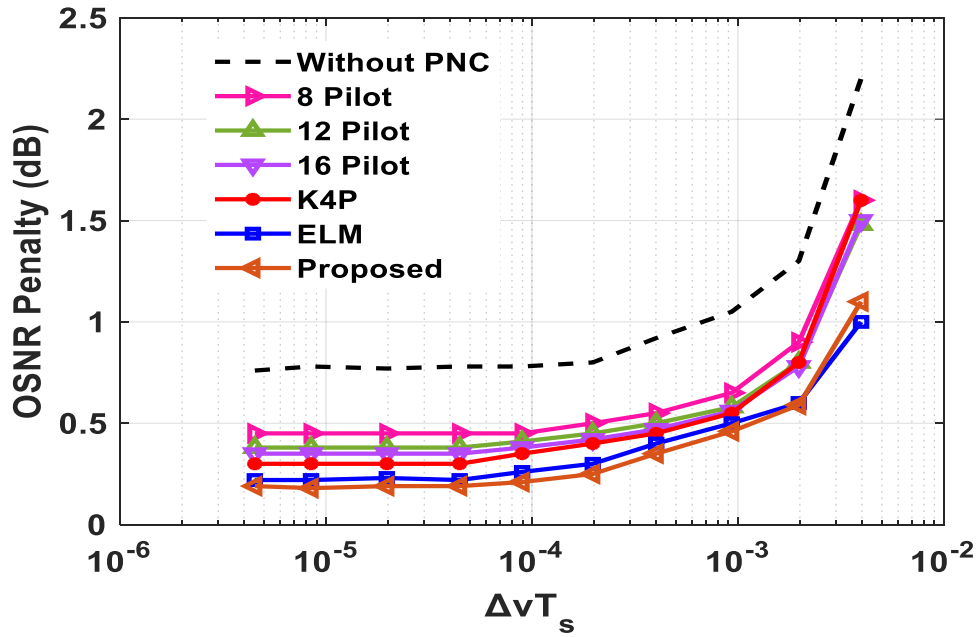


Figure 5 Comparison of normalized line width versus OSNR penalty at BER of  $1 \times 10^{-3}$  for a CO-OFDM system (PA – 8: Pilot aided technique with 8 pilots; PA - 12: Pilot aided technique with 12 pilots; PA - 16: Pilot aided technique with 16 pilots; K4P: Kalman filtering with raise to power four technique; RC-ELM: Real complex extreme learning machine; Proposed technique)

Table 3 Comparison of improvement in OSNR Penalty (dB) and approximate Spectral efficiency w.r.t. PA technique using Figure 4						
PA	K4P		RC – ELM		Proposed work	
Number of pilots	OSNR Penalty (dB)	Spectral efficiency (%)	OSNR Penalty (dB)	Spectral efficiency (%)	OSNR Penalty (dB)	Spectral efficiency (%)
8	0.1	8	0.15	4	0.2	9.5
12	0.03	13.5	0.08	6	0.12	15
16	0.01	18	0.06	9	0.1	21

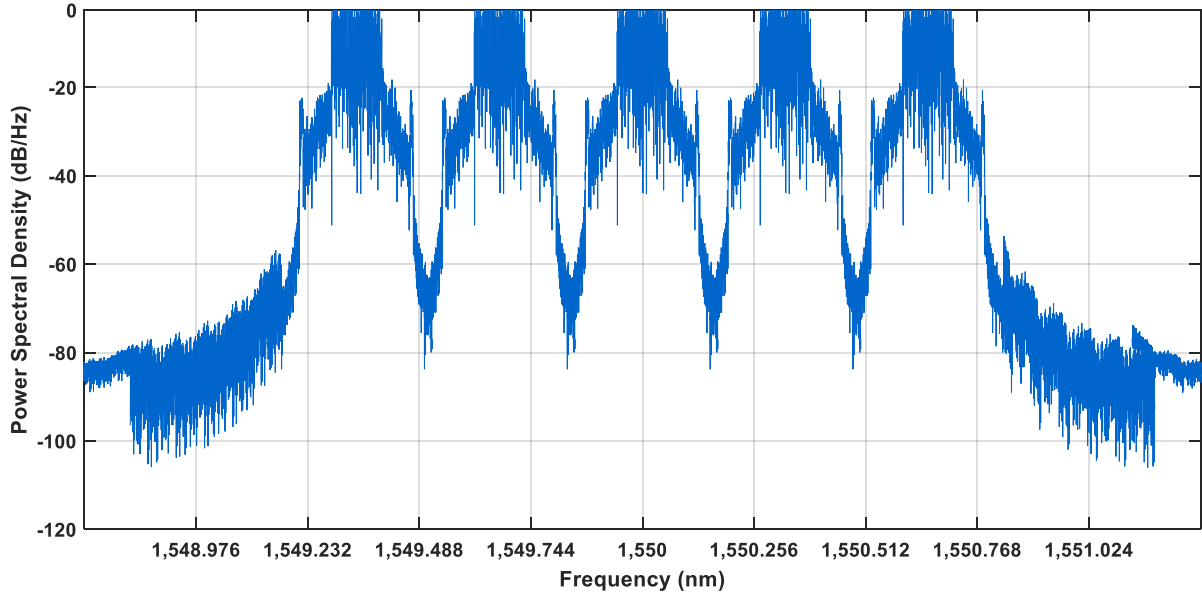


Figure 6 Multi-channel CO-OFDM system Frequency Spectrum of the received signal with phase noise

For the simulation of the multi-channel CO-OFDM system, the split-step Fourier method of nonlinear Schrödinger equation has been used to solve the propagation of the optical signal through the optical fiber channel. This signal has been passed through multiple spans of fiber with optical amplifiers for the compensation of loss from each span. For choosing the desired band, the received signal has been passed from the filter and applied to a coherent receiver. In the end, the received signal has been applied to the phase compensation

module and it has been assumed that the received signal is perfectly synchronized in time and frequency using Schmidl-Cox based synchronization algorithm.

The received modulated spectrum in terms of power spectral density for x-polarization with 5 channel CO-OFDM system is shown in Figure 6. The center carrier frequency has been set at 1550nm. One channel has been comprising bitrate of 200Gbps, and the net data rate for the multichannel CO-OFDM system has been set at 1 Tbps.

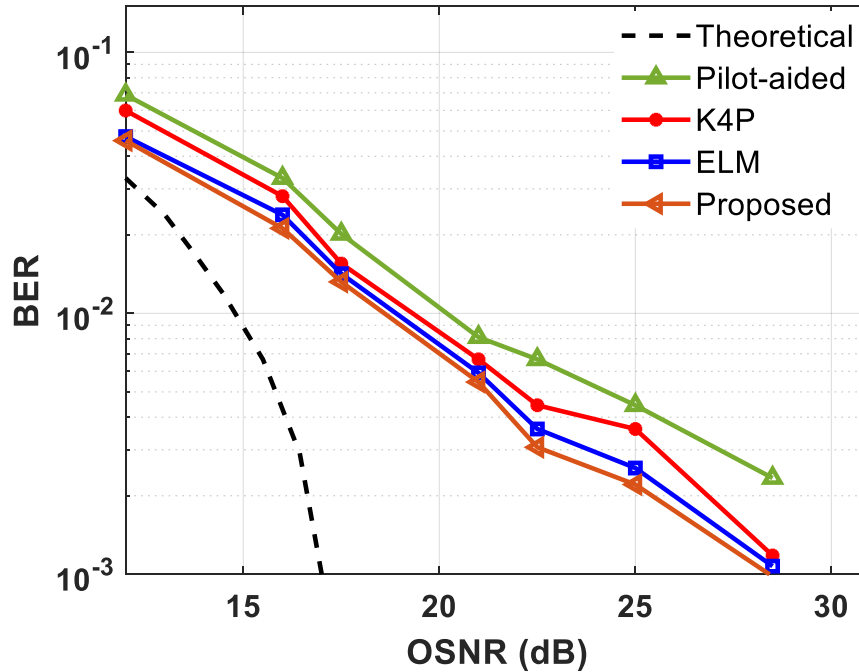


Figure 7 Comparison of OSNR (dB) versus BER for a CO-OFDM system (Theoretical; PA - 12: Pilot aided technique with 12 pilots; K4P: Kalman filtering with raise to power four technique; RC-ELM: Real complex extreme learning machine; Proposed technique)

OSNR (dB) versus BER using proposed technique as compared to theoretical results and previously reported techniques in 5 channel CO-OFDM system has been presented in Figure 7. It has been observed in this plot, that by using proposed technique the obtained results are close to the theoretical results as compared to all the previously used methods.

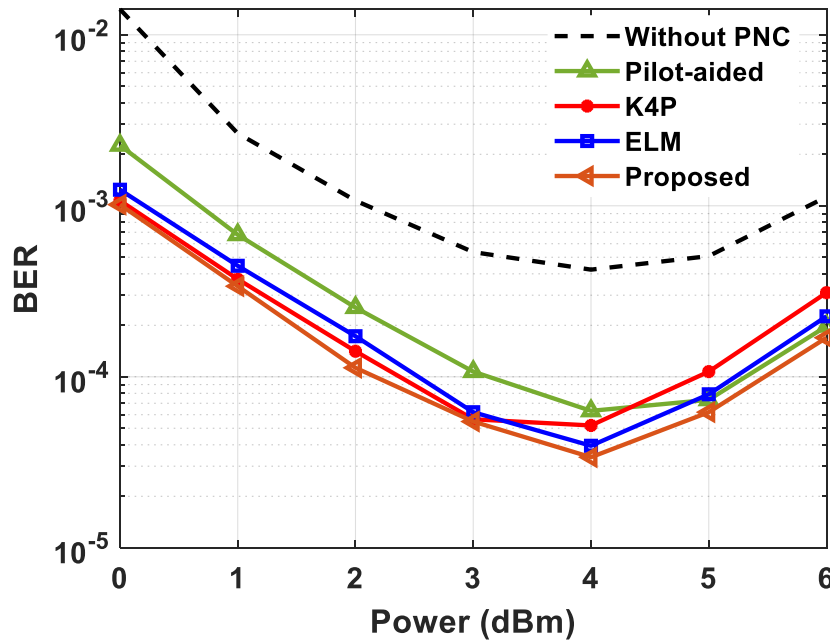
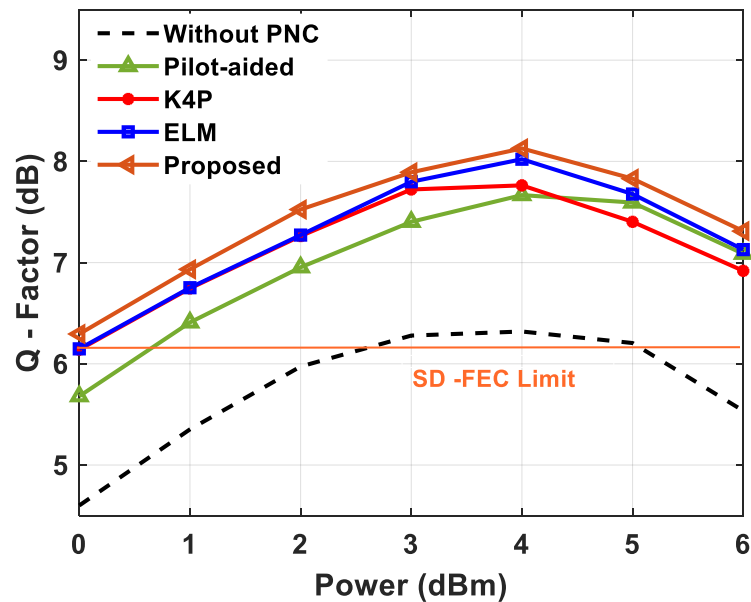


Figure 8 Comparison of launched power versus BER for 5 channel CO-OFDM systems using different algorithms under consideration with 200kHz combined laser linewidth (without PNC; PA: Pilot aided technique with 12 pilots; K4P: Kalman filtering with raise to power four technique; RC-ELM: Real complex extreme learning machine; Proposed technique)

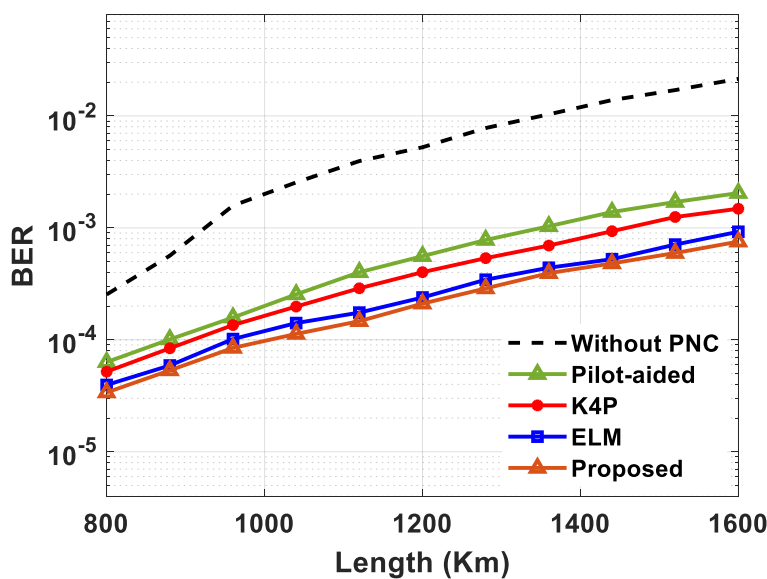
The comparison of launched power versus BER for 5 channel CO-OFDM system without phase noise compensator (PNC) and with different techniques has been presented in Figure 8. From this Figure, it has been observed that the performance is improving by increasing in launch power up to the optimum value of launch power (~4dBm). The BER performance, beyond this value of optimum launch power has been degrading which is due to the nonlinear effects of fiber. It has been seen in this plot that in the linear regime the performance with proposed work has been better as compared to other pilot-aided or non-pilot techniques.



**Figure 9 Comparison of launched power versus Q-Factor for 5 channel CO-OFDM systems using different algorithms (PA: Pilot aided technique with 12 pilots; K4P: Kalman filtering with raise to power four technique; RC-ELM: Real complex extreme learning machine; Proposed technique)**  
Till now, in most of the studies, the Q-Factor was the main factor under consideration for the measurement of CO-OFDM system performance. Therefore in this work, Q-Factor has also been calculated for comparison of proposed work with the previous techniques of phase noise compensation. The value of the Q-factor can be calculated by the formula given as [30];

$$Q = 20 \log_{10}(\sqrt{2} \operatorname{erfc}^{-1}(2 \operatorname{BER})) \dots \dots \dots (xxxv)$$

For this work, the margin value of Q-factor has been chosen to 6.2dB which has been obtained with 25% FEC soft-decision (SD) on the standard value of BER (i.e., approximately  $10^{-3}$ ). The plot of launch power versus Q-Factor for five-channel CO-OFDM, without PNC and with different techniques such as PA-12 pilots, K4P, RC-ELM, and proposed work, has been presented in Figure 9. At optimal launch power of 4dBm, the percent improvement in Q – Factor as compared to PA-12 pilots with K4P, RC-ELM, and proposed work, has been equal to is 1.25, 4.61, and 6.013 respectively.



**Figure 10 Length versus BER performance plot (PA: Pilot aided technique with 12 pilots; K4P: Kalman filtering with raise to power four technique; RC-ELM: Real complex extreme learning machine; Proposed technique)**

At 200 kHz combined laser line-width, the performance in BER versus fiber length of 800Km to 1600Km (10 to 20 number of spans) at a bitrate of 1Tbps for multi-channel CO-OFDM system has been presented in Figure 10. At the pre-FEC limit of BER value of  $1 \times 10^3$ , it has been observed that the improvement in transmission reach using the proposed technique is approximately 300 Km more in comparison to PA-12 pilots, while K4P and ELM technique was equal to 100Km and 240Km respectively. The performance in BER versus fiber length of 800Km to 1600Km at a bitrate of 1Tbps for different values of linewidths i.e., 200KHz, 500KHz, and 1000KHz using proposed technique in multi-channel CO-OFDM system has been presented in Figure 11. From this Figure it has been noticed that there is an improvement in BER at lower values of laser linewidths.

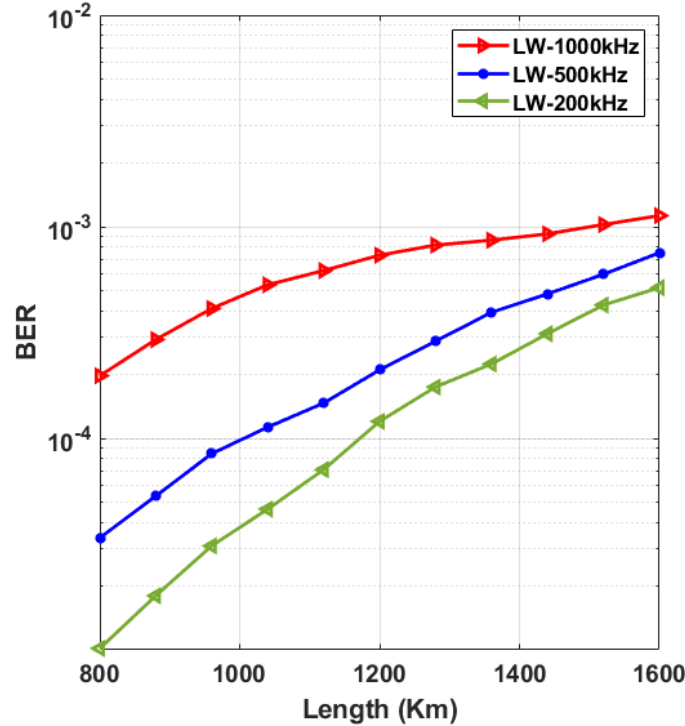


Figure 11 Length versus BER performance with different laser linewidths of 200KHz, 500KHz, and 1000KHz

## 6 Conclusion

A DWPT based CO-OFDM system with Wilcoxon Robust Extreme Learning Machine based pilot-free phase noise compensator using multi-level QPSK partitioning of 16-QAM has been proposed in this paper. From the results of this work it has been seen that there is an improvement in performance (in terms of Q-Factor), OSNR penalty and spectral efficiency over traditional phase noise compensation techniques. Moreover, this proposed work comparatively reduced the overall system complexity since DWPT has been used in place of FFT that removed the need of cyclic prefix. Wilcoxon Extreme learning machine has been used for phase compensation that offers an extremely fast speed of training and improved generalization performance with less number of user interventions with more robust results.

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# Figures

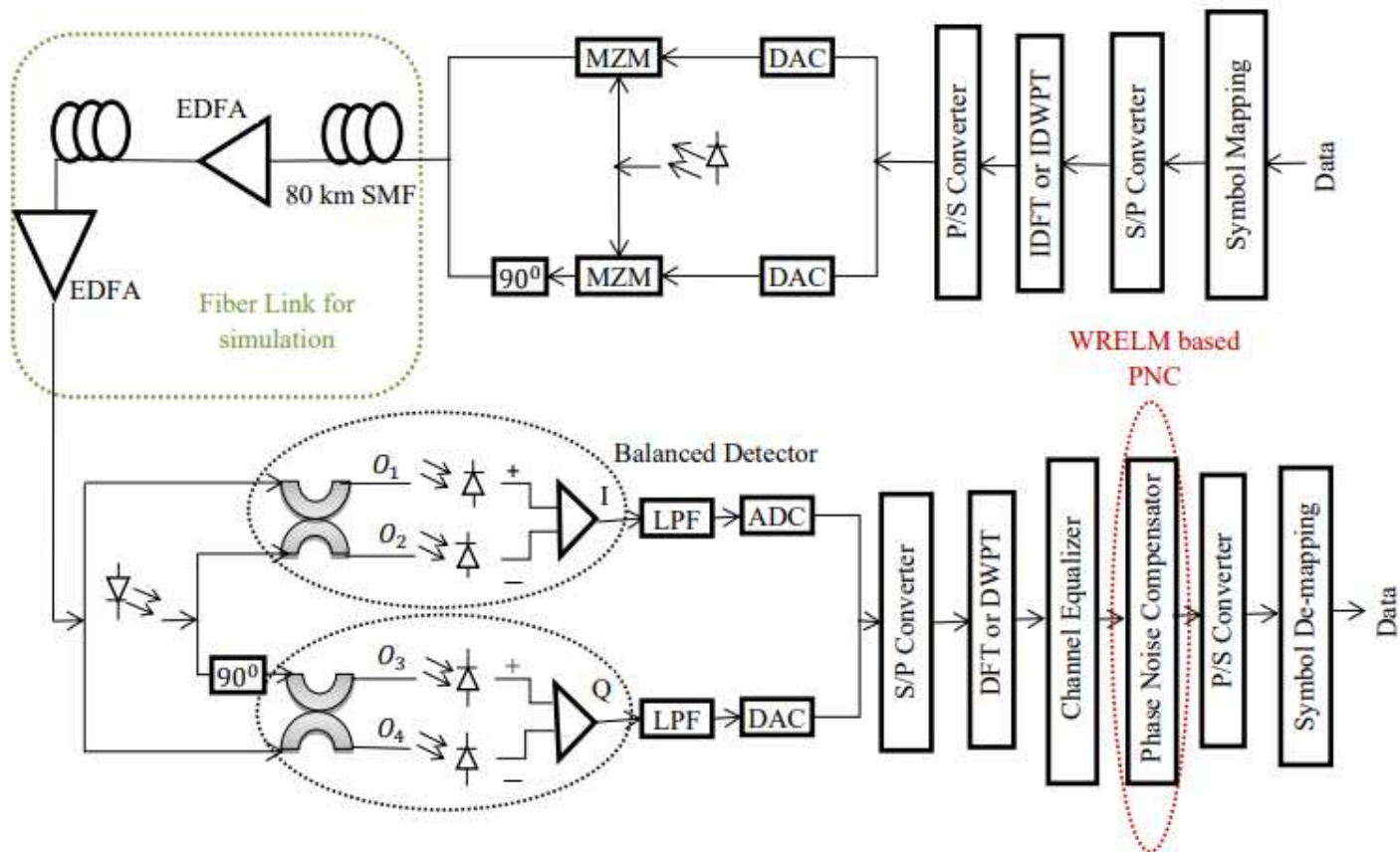


Figure 1

Simulation setup of a basic CO-OFDM system.

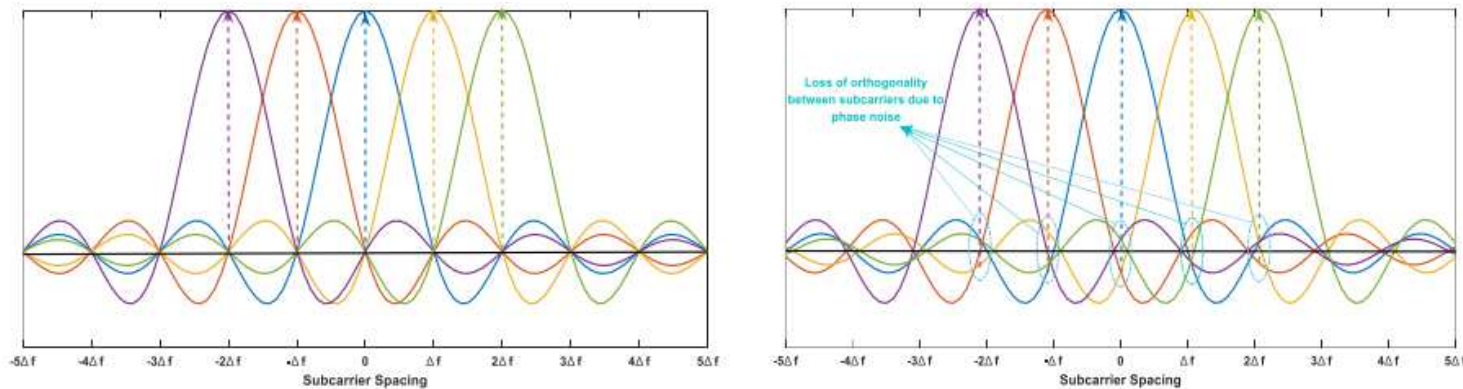


Figure 2

Frequency domain picture of subcarriers (a) without phase noise corruption (b) with phase noise corruption.



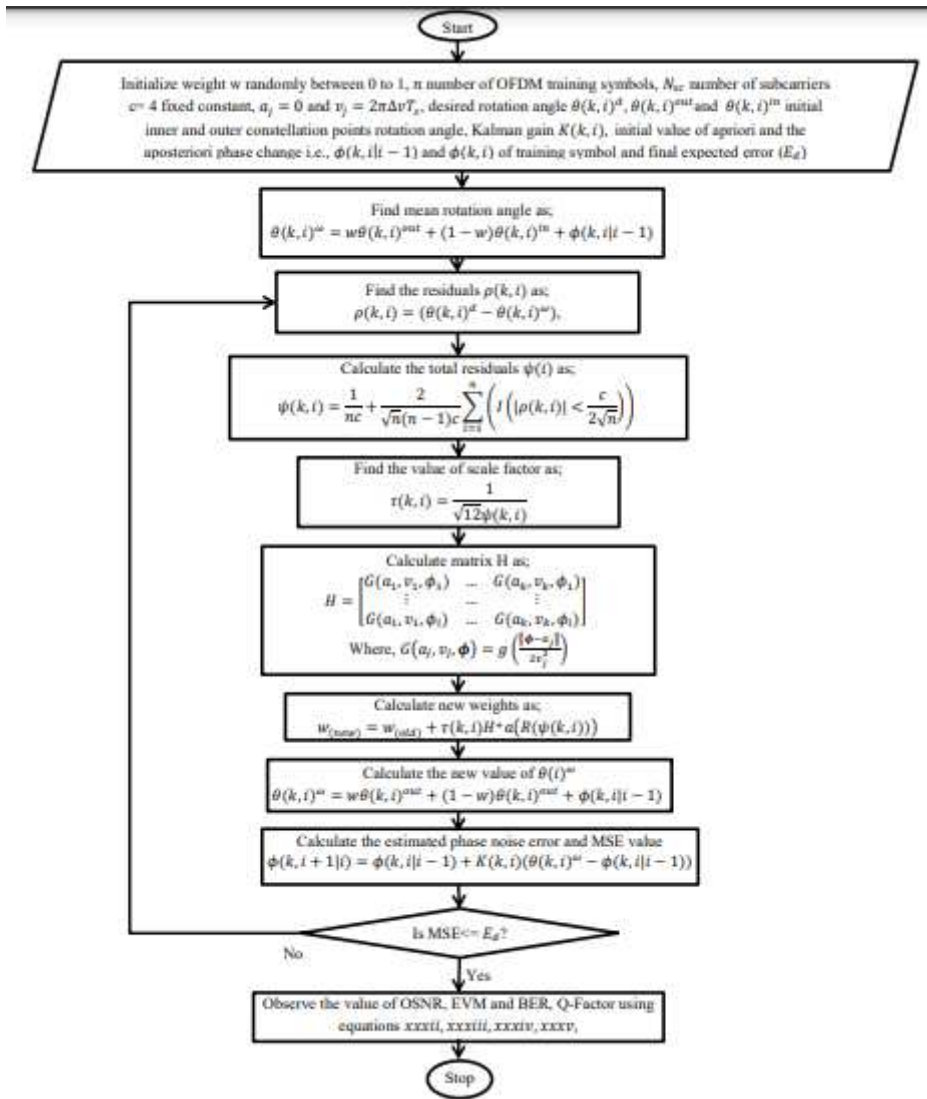


Figure 3

Flow chart of the proposed work

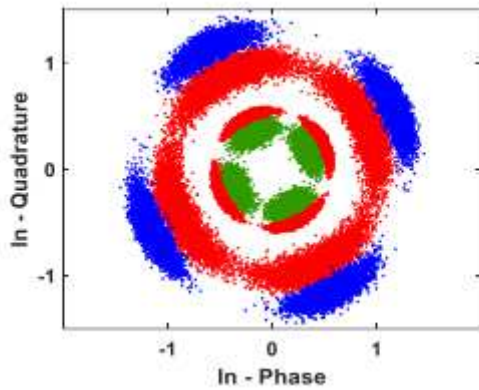


Figure 4

Received constellation diagram for proposed work



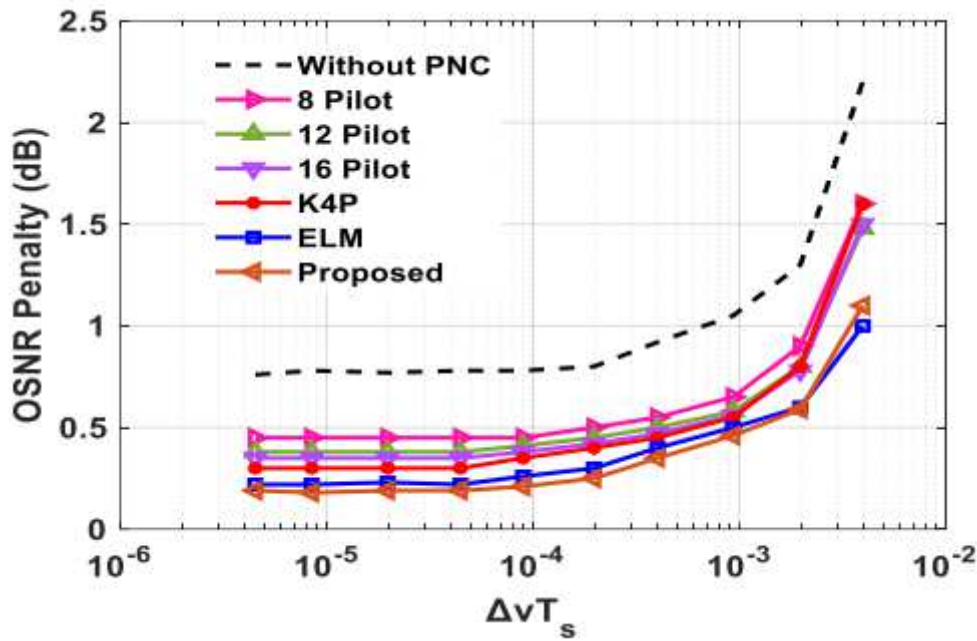


Figure 5

Comparison of normalized line width versus OSNR penalty at BER of  $1 \times 10^{-3}$  for a CO-OFDM system (PA - 8: Pilot aided technique with 8 pilots; PA - 12: Pilot aided technique with 12 pilots; PA - 16: Pilot aided technique with 16 pilots; K4P: Kalman filtering with raise to power four technique; RC-ELM: Real complex extreme learning machine; Proposed technique)

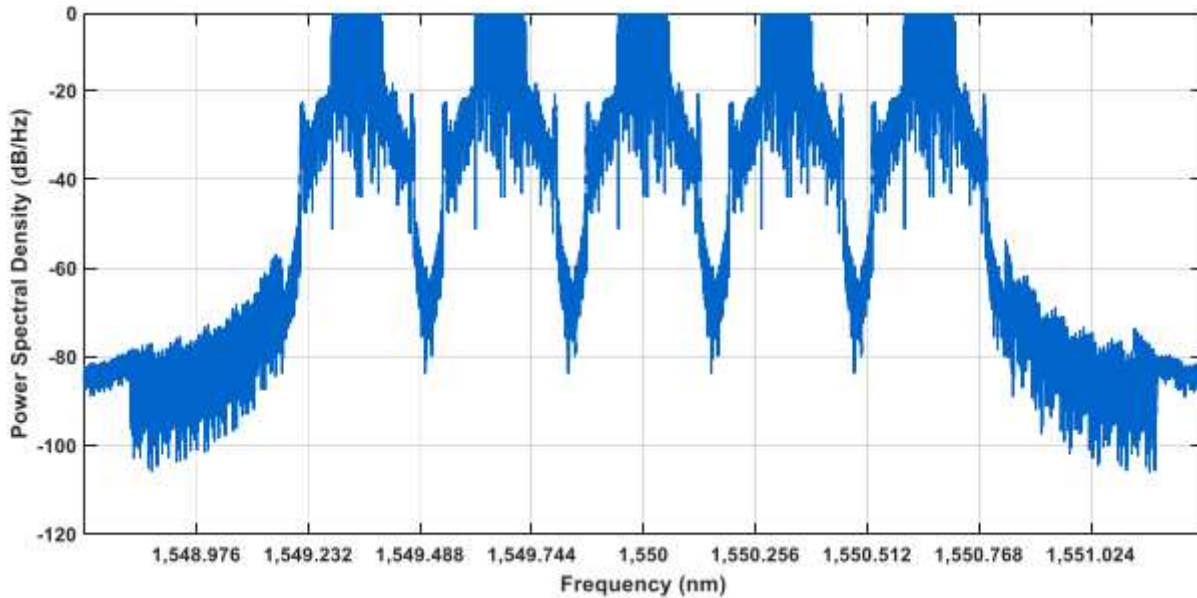


Figure 6

Multi-channel CO-OFDM system Frequency Spectrum of the received signal with phase noise

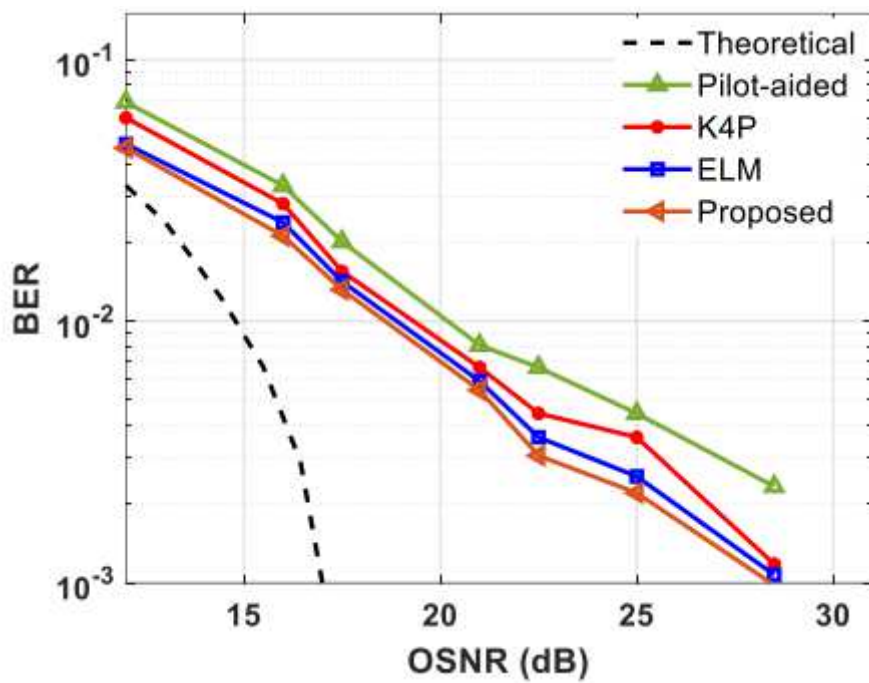


Figure 7

Comparison of OSNR (dB) versus BER for a CO-OFDM system (Theoretical; PA - 12: Pilot aided technique with 12 pilots; K4P: Kalman filtering with raise to power four technique; RC-ELM: Real complex extreme learning machine; Proposed technique)

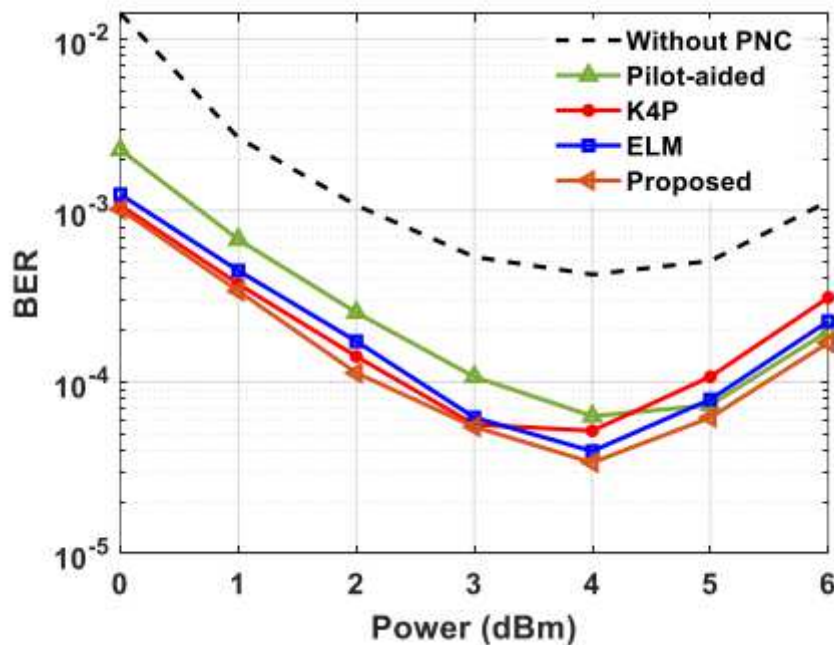


Figure 8

Comparison of launched power versus BER for 5 channel CO-OFDM systems using different algorithms under consideration with 200kHz combined laser linewidth (without PNC; PA: Pilot aided technique with

12 pilots; K4P: Kalman filtering with raise to power four technique; RCELM: Real complex extreme learning machine; Proposed technique)

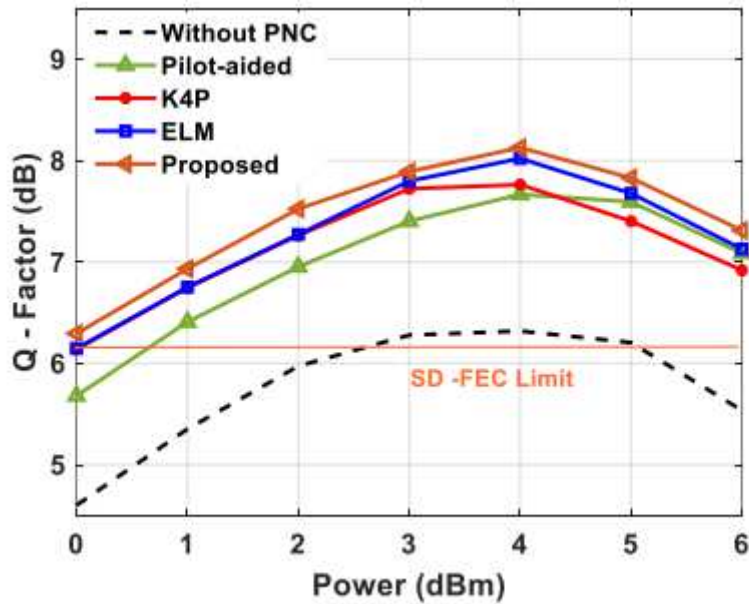


Figure 9

Comparison of launched power versus Q-Factor for 5 channel CO-OFDM systems using different algorithms (PA: Pilot aided technique with 12 pilots; K4P: Kalman filtering with raise to power four technique; RC-ELM: Real complex extreme learning machine; Proposed technique)

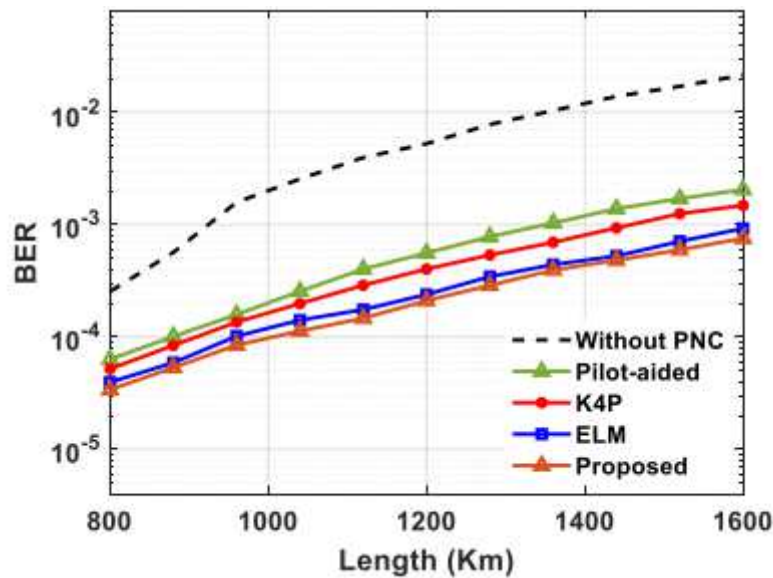


Figure 10

Length versus BER performance plot (PA: Pilot aided technique with 12 pilots; K4P: Kalman filtering with raise to power four technique; RCELM: Real complex extreme learning machine; Proposed technique)

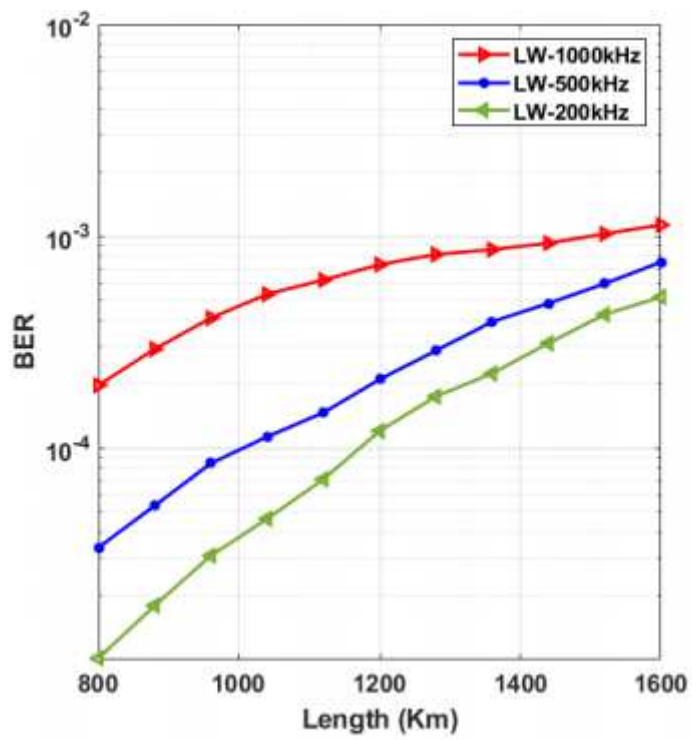


Figure 11

Length versus BER performance with different laser linewidths of 200KHz, 500KHz, and 1000KHz