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X-RAY EMISSION FROM ACCRETION ON TO WHITE DWARFS

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SUMMARY

We have extended calculations by Hoshi and by Aizu to produce a self-consistent model for X-radiation from accreting, possibly magnetized, white dwarfs. To generate keV X-rays the flow must be radial on to the stellar surface. We expect X-ray luminosities to be in the range 10^{32}–10^{36} erg s^{-1}, and the spectra to be quasi-bremsstrahlung with $kT \sim 30$–100 keV and with a substantial low energy cut-off. The optical (bolometric) luminosity should be comparable to that emitted in the X-rays. If the white dwarf is magnetized a comparable, or greater, amount can be radiated as cyclotron emission in the infrared or optical.

1. INTRODUCTION

The emission of X-radiation from accretion on to compact objects has been intensively studied over the past few years. X-ray temperatures ($T \gtrsim 10^7$ K) are reached if the kinetic energy due to infall of gas on to the surface of an object of solar mass, and radius $R \lesssim 3 \times 10^{10}$ cm, is converted via a shock into internal energy. White dwarfs, neutron stars and the possibly more massive black holes are therefore all candidates. The weaker gravitational field at the surface of a white dwarf, compared to a more compact object, means that a much higher mass flux ($\times 10^3$) is required to produce the same luminosity. The much larger surface area makes the blackbody temperatures of even luminous sources, $L_X \sim 10^{38}$ erg s^{-1}, less than 10^6 K, and so in order to radiate X-rays the infalling matter must pass through a shock and emit *optically thin* bremsstrahlung. Consequently, most theoretical work on the detected compact X-ray sources has concentrated on neutron star or black hole accretion.

We investigate X-ray emission from accretion on to white dwarfs without regard to specific observed sources, extending earlier work by Hoshi (1973) and Aizu (1973). Shapiro & Salpeter (1975) have recently discussed the analogous problem for neutron stars, though without including a magnetic field. Electron scattering and absorption in the infalling material are considered, as well as cyclotron emission in the cases where strong ($B \gtrsim 10^6$ G) magnetic fields are present. We find that the X-ray luminosities are unlikely to exceed $\sim 10^{36}$ erg s^{-1} and that the bremsstrahlung temperatures are comparatively high ($kT \sim 50$ keV). Accreting white dwarfs may produce X-ray sources that have not yet been detected.

In Section 2 we review and discuss the general properties of accretion shocks above the surface of white dwarfs when magnetic fields are negligible. The magnetic fields may, however, be strong enough to channel the inflow towards the magnetic polar caps, and in this situation *cyclotron cooling* is competitive with bremsstrahlung.

The influence of cyclotron emission on the location of the shock and on the expected spectrum is considered in Section 3; and in Section 4 we discuss how the magnetic field affects the accretion flow and calculate the ranges of parameters which permit 'self-consistent' X-ray emission from accreting white dwarfs. The details of Section 4 are summarized in Figs 2 and 3. Applications to various observed systems —e.g. DQ Her, Mira B and transient X-ray sources—are made in Section 5.

2. SHOCK STRUCTURE WITH BREMSSTRAHLUNG COOLING

Consider material accreting on to a white dwarf of mass M and radius R, at a rate F. Then the luminosity due to accretion alone is

$$L = \frac{FGM}{R} = 1\cdot 3 \times 10^{37} F_{20} \left(\frac{M}{M_\odot}\right) R_9 \text{ erg s}^{-1} \tag{1}$$

where R_9 is the stellar radius in units of 10^9 cm and F_{20} is the accretion rate in units of 10^{20} g s^{-1} = $1\cdot 5 \times 10^{-6} M_\odot$ yr^{-1}. From the mass–radius relation for white dwarfs given by Webbink (1975) and the assumption that the white dwarf is composed of pure helium, the luminosity per gram can be derived as a function of white dwarf mass (Fig. 1). For generality we define f as the fraction of the stellar surface area on to which the material accretes. In the spherically symmetric situation $f = 1$, but if the white dwarf has a strong enough magnetic field to affect the accretion flow then in general $f < 1$. The velocity of matter falling freely on to the

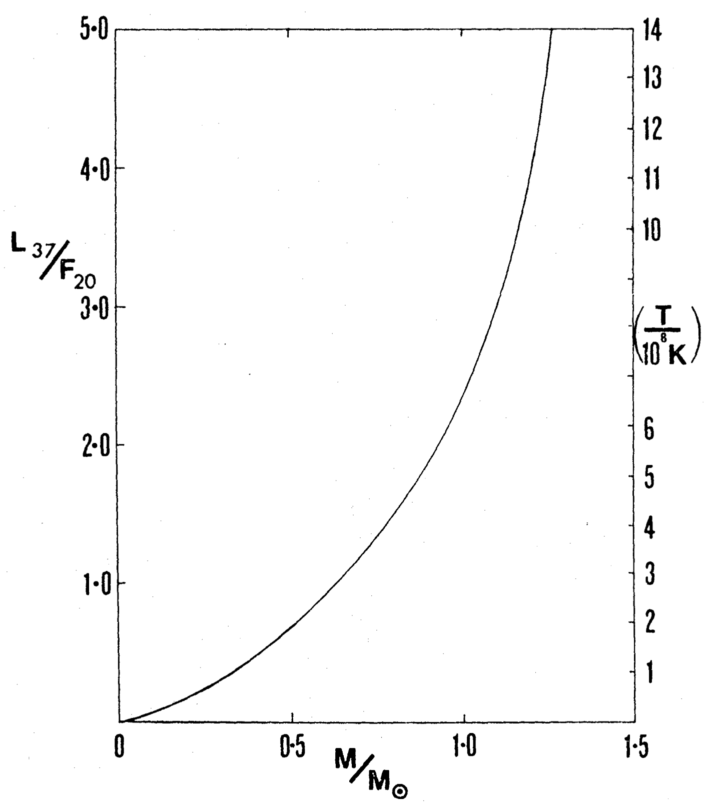

FIG. 1. *The ratio of luminosity to mass flow rate, L_{37}/F_{20} (left-hand scale) and shock temperature, T_s, in units of 10^8 K (right-hand scale) plotted as a function of white dwarf mass. The mass–radius relation for (cold) white dwarfs of Webbink (1975) has been assumed.*

star is

$$V = \left(\frac{2GM}{R}\right)^{1/2} = 5\times 10^8 \left(\frac{M}{M_\odot}\right)^{1/2} R_9^{-1/2}\ \mathrm{cm\ s^{-1}} \quad (2)$$

and, if the mass flux is uniform and radial over the accreting area, the density is

$$\rho_1 = \frac{F}{4\pi R^2.f.V} = 1{\cdot}5\times 10^{-8} F_{20} f^{-1} \left(\frac{M}{M_\odot}\right)^{-1/2} R_9^{-3/2}\ \mathrm{g\ cm^{-3}}. \quad (3)$$

At a distance D above the surface of the star, a standing shock forms (see, e.g. Hoshi 1973; Aizu 1973). Before encountering the shock, the gas will in general be so cool that its inflow velocity is highly supersonic. The shock is therefore ‘ strong ’, causing the density to increase by 4, and the inward velocity to decrease by the same factor. This is so provided that the optical depth through the shock thickness is less than unity. Otherwise there is also a solution in which the accretion energy is thermalized and re-radiated by the white dwarf atmosphere. If we take the shock thickness to correspond to $\sim 10^{-2}\ \mathrm{g\ cm^{-2}}$ then this has an optical depth of unity if the atmospheric temperature $\sim T_{\mathrm{cr}} = 2{\cdot}5\times 10^4\ \mathrm{K}$ (Allen 1973). Thus for a standing shock and possible X-ray emission we require $T_{\mathrm{cr}} < T_{\mathrm{b}}$, where T_{b} is defined by equation (7), i.e.

$$F_{20} > 2{\cdot}1\times 10^{-5} f (M/M_\odot)^{-1} R_9^3.$$

The value of D, and the density and velocity profile in the subsonic region, are determined by the condition that the gas should have time to cool as it ‘ settles ’ on to the stellar surface. D varies inversely with ρ, and for most cases of interest Ff^{-1} will be high enough to ensure that $D \ll R$. The density of the shocked region is then $\rho_2 = 4\rho_1$, where ρ_1 is given by (3) for $R = R_*$. Note, however, that even if $D \gtrsim R$ most of the energy is still released at radii between R and $2R$, and the temperature and density structure in this region is hardly changed even if $D \gg R$ (see Appendix).

When the dominant radiation process for the shocked material is bremsstrahlung it is possible to find an approximate analytic profile for the flow below the shock (Aizu 1973). For our purposes, however, it is adequate to assume that the material in the region between the shock and the stellar surface has uniform density and temperature. We take the temperature to be given by the shock temperature (see Fig. 1):

$$T_{\mathrm{s}} = \frac{3}{8}\frac{GMm_{\mathrm{p}}\mu}{kR} = 3{\cdot}7\times 10^8 \left(\frac{M}{M_\odot}\right) R_9^{-1}\ \mathrm{K} \quad (4)$$

where m_{p} is the proton mass, k is Boltzmann's constant, and we have taken the mean molecular weight $\mu = 0{\cdot}615$ corresponding to a hydrogen mass fraction of $X = 0{\cdot}7$ in the accreting material. When bremsstrahlung is the dominant post-shock cooling mechanism, the distance D_{ff} is given by

$$L = f.4\pi R^2.D_{\mathrm{ff}}\epsilon_{\mathrm{ff}} \quad (5)$$

where ϵ_{ff} is the bremsstrahlung cooling rate. Hence

$$D_{\mathrm{ff}} = 3{\cdot}3\times 10^7 F_{20}^{-1} f (M/M_\odot)^{3/2} R_9^{1/2}\ \mathrm{cm}. \quad (6)$$

This agrees well with the more detailed calculation of Aizu (1973).

The bremsstrahlung cooling time is *longer* than the time scale for equalizing electron and proton temperatures via coulomb encounters. Thus we are here justified in assuming that the electron temperature does indeed attain a value T_s, and that the shock thickness is $\ll D_{ff}$, even if there is no enhanced coupling between protons and electrons due to plasma oscillations. This is not necessarily still true, however, if cyclotron or Compton cooling exceed bremsstrahlung, and we discuss this further in the next section.

Thermal conduction of energy from the shocked region into the star can be neglected. The fraction of radiated luminosity lost to thermal conduction is approximately

$$\frac{L_c}{L_x} = f.4\pi R^2.K.TD^{-1}L_x^{-1}$$
$$= 2\cdot8\times10^{-2}\, F_{20}{}^{7/2}f^{-7/2}\left(\frac{M}{M_\odot}\right)^{5/2} R_9{}^{-5/2},$$

where $K = 1\cdot0\times10^{-6}\, T^{5/2}$ is the thermal conductivity (Allen 1973). This is much less than unity for the cases of interest. There remains the possibility, however, that there is another *self-consistent* solution with D much smaller than above in which the dominant energy loss from the shocked region is due to thermal conduction down into the star. Presumably this energy would emerge as thermalized radiation from the stellar surface. If the radiation released were radiated as a black body, the temperature of the radiating region would be

$$T_b = \frac{L}{f.4\pi R^2.\sigma} = 3\cdot5\times10^5\, F_{20}{}^{1/4}f^{-1/4}\left(\frac{M}{M_\odot}\right)^{1/4} R_9{}^{-3/4}\,\mathrm{K} \tag{7}$$

where σ is Stefan's constant. Thus, for the cases we shall be interested in ($F_{20} \lesssim 1$), $T_b \ll T_s$ and the emitting region is optically thin to bremsstrahlung. However, as a number of authors (de Gregoria 1974; Hayakawa 1974) have pointed out, the spectrum of the emerging radiation can be seriously affected by its outward passage through the accreting material. The optical depth vertically through the emitting region due to electron scattering is

$$\tau_{es}(D) = 0\cdot2(1+X)\,\rho_2 D_{ff}$$
$$= 0\cdot7(M/M_\odot)\,R_9{}^{-1} \tag{8}$$

independent of the accretion rate. We may therefore neglect the effect of electron scattering in the radiating region. The value to infinity along a radius is

$$\tau_{es}(\infty) = \int_R^\infty 0\cdot2(1+X)\,\rho_1(r)\,dr$$
$$= 10F_{20}f^{-1}(M/M_\odot)^{-1/2}\,R_9{}^{-1/2}. \tag{9}$$

As the calculations of de Gregoria (1974) show, when $\tau_{es}(\infty) > 1$, inelastic Compton scattering degrades all photons with energies

$$E \gtrsim m_e c^2/\tau_{es}{}^2(\infty).$$

For $f < 1$, the photons can escape sideways through the accretion column. The radius of the column $d \approx \sqrt{2f}R$ and then the optical depth sideways through the column is

$$\tau_{es}(d) = 7\cdot4F_{20}f^{-1/2}(M/M_\odot)^{-1/2}\,R_9{}^{-1/2}. \tag{10}$$

When $D \ll d$, the photons must all escape through the cool unshocked material and the inelastic scattering criterion can be applied using $\tau_{es}(d)$. (In general $D \ll d$ for $\tau_{es}(d) \gtrsim 1$.) Usually, however, the X-ray flux is insufficient to maintain the heavy elements in the incoming material ionized (Hayakawa 1974) and photo-electric absorption cannot be ignored. We discuss this process quantitatively in Section 5.4.

3. INFLUENCE OF MAGNETIC FIELD

We now consider in more detail the case ($f < 1$) when the accretion flow is funnelled down the magnetic field on to the magnetic poles of a white dwarf. We take the strength of the surface field at the poles to be $B = 10^6 B_6$ gauss and initially treat f and B as independent parameters. We estimate later (Section 4) how f and B can be related. As noted by Ichimaru & Nakano (1973), cyclotron losses may now dominate the emission process. Inoue & Hoshi (1975) considered the case $f < 1$, but did not take account of the cyclotron process. The cyclotron emission rate is

$$\epsilon_c = n_e \frac{B^2}{8\pi} 2\sigma_T c \left(\frac{v}{c}\right)^2 \tag{11}$$

where σ_T is the Thomson cross-section and $v = 6 \times 10^5 \, T_e^{1/2}$ cm s^{-1} is the rms electron velocity. We may then define a dimensionless ratio

$$\gamma = \frac{\epsilon_c}{\epsilon_{ff}} = 1.9 \times 10^2 \, F_{20}^{-1} f \left(\frac{M}{M_\odot}\right) R_9 B_6^2. \tag{12}$$

If the emitting region is optically thin to cyclotron radiation, cyclotron losses may be ignored only if $\gamma < 1$, that is if

$$F_{20} > 1.9 \times 10^2 f(M/M_\odot) \, R_9 B_6^2. \tag{13}$$

The optical depth to cyclotron emission is not easy to assess, since the details of the radiative transfer are difficult to calculate. For our present purposes, however, we may obtain a rough estimate in the following manner. Define a cyclotron emission 'temperature' T_c by

$$T_c = h\nu_c/k = 1.3 \times 10^2 \, B_6 \, \mathrm{K}$$

where $\nu_c = 2.8 \times 10^{12} \, B_6$ Hz is the cyclotron frequency. Then the requirement that the intensity of radiation at frequency ν_c be less than that given by a Planck spectrum with temperature T_c yields an approximate optical depth†

$$\tau_c = T_b^4 / T_e T_c^3. \tag{14}$$

When $D \gtrsim d$, we should take account of the fact that the effective surface area of the emitting region is increased by a factor $\phi = (1 + 2D/d)$, and modify (7) accordingly by including an extra factor $\phi^{-1/4}$. The opacity to cyclotron radiation then exceeds unity (that is $\tau_c > 1$) if

$$F_{20} > 4.8 \times 10^{-8} f R_9^2 B_6^3 \phi. \tag{15}$$

† This estimate for τ_c depends on the shape and breadth of the cyclotron emission spectrum. The spread of magnetic field strength in the emitting region, and thermal doppler broadening, widen the cyclotron line such that $\Delta\nu_c/\nu_c \gtrsim 0.3$. Thus the approximation of the cyclotron emission spectrum by a Planck curve with temperature T_c is probably not unreasonable.

When $\tau_c > 1$, the effective ratio of cyclotron to free–free luminosity is

$$\gamma_1 = \gamma \tau_c^{-1}$$
$$= 9 \cdot 3 \times 10^{-6} f^2 F_{20}^{-2} (M/M_\odot)\, R_9^3 \phi.$$

For X-ray emission to dominate, we require $\gamma_1 < 1$, that is

$$F_{20} > 3 \times 10^{-3} f (M/M_\odot)^{1/2}\, R_9^{3/2} B_6^{5/2} \phi^{1/2}. \tag{16}$$

We must now check our assumption that $T_e = T_s$. It is, for example, conceivable that cyclotron emission is so efficient that the ion and electron temperatures can never reach equality after the shock. By comparing the relevant time scales we find that $T_e = T_s$ provided that

$$F_{20} > 8 \times 10^{-4} f (M/M_\odot)\, R_9 B_6^{5/2} \phi^{1/2}. \tag{17}$$

When $\gamma_1 = 1$, $T_e = T_s$ provided that

$$(M/M_\odot)\, R_9 > 6 \cdot 3 \times 10^{-2}, \tag{18}$$

which is satisfied for white dwarfs.

When $T_e < T_s$ the efficiency of cyclotron emission ($\propto T_e$) relative to bremsstrahlung emission ($\propto T_e^{1/2}$) is reduced, although, for the parameters we consider, the cyclotron is always optically thick. The electron temperature T_e is calculated by balancing the (optically thick) cyclotron emission loss with the rate at which the ions heat the electrons by collisional heating. We find

$$T_e = 2 \cdot 2 \times 10^{10} F_{20}^{4/7} f^{-4/7} (M/M_\odot)^{3/7}\, R_9^{-4/7} B_6^{-10/7} \phi^{-2/7}. \tag{19}$$

In this situation the ratio of cyclotron to bremsstrahlung emission is given by

$$\gamma_2 = \gamma_1 \left(\frac{T_e}{T_s}\right)^{1/2}$$
$$= 4 \cdot 4 \times 10^{-3}\, F_{20}^{-8/7} f^{8/7} \left(\frac{M}{M_\odot}\right)^{1/7} R_9^{15/7} B_6^{20/7} \phi^{6/7}. \tag{20}$$

Note that when $T_e = T_s$ (and $\phi = 1$), $\gamma_1 = \gamma_2 = 15 (M/M_\odot)^{-1} R_9$.

Electron energy loss due to scattering of the cyclotron photons can be ignored for the values of the parameters we consider here. About half of the bremsstrahlung radiation is emitted downwards and is absorbed in, and re-radiated thermally from, the stellar surface. Thus we expect the X-ray luminosity to be roughly $L_x = \frac{1}{2}\Gamma^{-1} L$ where

$$\Gamma = \begin{cases} 1 & \gamma_1 < 1 \\ \gamma_1 & \gamma_1 > 1 \text{ and } T_e = T_s \\ \gamma_2 & T_e < T_s \text{ (in this case the X-rays have a softer spectrum).} \end{cases}$$

Similarly there is an additional contribution to the luminosity from the stellar magnetic poles at a temperature $T_* = (2\Gamma)^{-1/4}\, T_b$, provided $T_* > T_c$.

We now estimate the parameter $\phi = 1 + 2D/d$. When bremsstrahlung is the dominant emission mechanism behind the shock, the height of the shock above the stellar surface $D = D_{ff}$ (defined in equation (6)). However, when cyclotron emission dominates, we have $D = D_{ff}\Gamma^{-1}$. It should be noted that for $\Gamma > 1$, Γ and hence D is a function of ϕ, and the equation $\phi = 1 + 2D(\phi)/d$ must be solved properly to obtain ϕ.

For self-consistency we must now check that, even when bremsstrahlung dominates over cyclotron emission, it also dominates over Compton scattering losses with ambient photons. The time scale for energy loss by an electron of energy E to the Compton process is approximately $t_c = E/\langle \dot{E} \rangle$ where

$$\langle \dot{E} \rangle \simeq -\frac{4}{3}\sigma_T \frac{2c\rho_r}{m_e c^2} E. \tag{21}$$

Here ρ_r is the radiation energy density. This is to be compared with t_b, the time scale for energy loss to bremsstrahlung, and we find

$$\frac{t_c}{t_b} \simeq \frac{7{\cdot}5 \times 10^{-5}\, n_e}{\rho_r T^{1/2}}. \tag{22}$$

We may neglect Comptonization if $t_c \ll t_b$.

The dominant source of ambient photons is the hot surface of the star where about one-half of the emitted X-ray photons are absorbed and thermalized. In this case

$$\rho_r \simeq \frac{L}{2f.4\pi R^2.c}$$

and we find $t_c/t_b \simeq 2{\cdot}4(M/M_\odot)^{-2}$, where we have assumed that the luminosity L is *solely* due to accretion. This contrasts with the neutron star case (*cf.* Shapiro & Salpeter 1975) where L/F and T are higher than for white dwarf accretion. Thus we are justified in neglecting Comptonization for our purposes but it should not be ignored in more detailed calculations. If, however, the dwarf is intrinsically highly luminous and, in particular, if the accreted nuclear fuel is burnt in a continuous fashion, Comptonization is the *dominant* mechanism for electron energy loss (see also, Katz & Salpeter 1974) and the resulting hard X-ray flux is considerably reduced. We assume throughout, therefore, that nuclear burning of accreted matter is not taking place continuously (for a fuller discussion of the plausibility of this assumption see Bath *et al.* 1974).

4. ESTIMATE OF ALFVÉN RADIUS AND SELF-CONSISTENT MODELS

We now show how f and B may be related. In common with previous work on accretion by magnetized stars (for example, Lamb, Pethick & Pines 1973) we assume that the stellar field is dipolar and that it controls the accretion flow within the Alfvén radius R_M. If inside R_M material flows only along those field lines that are open at R_M, it is easy to show that $d/R \approx (R/R_M)^{1/2}$ and thus $f \approx R/2R_M$. The precise way in which infalling matter distorts the stellar field and diffuses across field lines is exceedingly complex, although Arons & Lea (1976) find that the above relationships are valid in at least an order of magnitude sense. If the accretion is spherically symmetric outside R_M we can obtain an estimate for R_M by balancing the magnetic pressure and the ram pressure of infall at that radius.

This yields

$$\frac{R_M}{R} = 2{\cdot}3\, F_{20}^{-2/7}\left(\frac{M}{M_\odot}\right)^{-1/7} R_9^{5/7} B_6^{4/7}$$

or

$$f_{sph} = 0{\cdot}2\, F_{20}^{2/7}\left(\frac{M}{M_\odot}\right)^{1/7} R_9^{-5/7} B_6^{-4/7}.$$

If, however, the flow outside R_M is in the form of an accretion disc, this is probably an overestimate of R_M. Using the estimate of Bath, Evans & Pringle (1974) for this case we find

$$f_{\text{disc}} = 0{\cdot}2\, F_{20}{}^{7/27}(M/M_\odot)^{7/27}\, R_9{}^{-7/9} B_6{}^{-16/27}. \tag{23}$$

In addition, if the magnetized white dwarf rotates with a period P (min), the 'corotation radius' R_Ω at which the centrifugal force of matter attached to field lines balances the gravitational pull of the white dwarf is given by

$$\frac{R_\Omega}{R} = 2{\cdot}3\, P^{2/3} \left(\frac{M}{M_\odot}\right)^{1/3} R_9{}^{-1}. \tag{24}$$

Roughly speaking, accretion may only take place if $R_M < R_\Omega$ (see Section 5.3). We define

$$f_\Omega = \frac{R}{2R_\Omega} = 0{\cdot}2\, P^{-2/3} \left(\frac{M}{M_\odot}\right)^{-1/3} R_9. \tag{25}$$

Thus the condition that a white dwarf accretes material and is a pulsing source can be written as $f_\Omega < f < 1$. This is possible if

$$P \gtrsim 0{\cdot}4 (M/M_\odot)^{-1/2}\, R_9{}^{3/2} \qquad (f = f_{\text{disc}}) \tag{26}$$

or

$$P \gtrsim 0{\cdot}1 (M/M_\odot)^{-1/2}\, R_9{}^{3/2} \qquad (f = f_{\text{sph}}). \tag{27}$$

We can now use the above estimates of f to construct some models of accreting magnetized white dwarfs, and the radiation they emit, which are at least self-consistent in the context of our simplifying assumptions. The results are summarized in Figs 2 and 3.

(a) *Spherically symmetric flow outside R_M*

The conditions derived in this section are illustrated in Fig. 2 for a $1\, M_\odot$ white dwarf. The condition $\gamma_1 = 1$, that cyclotron and bremsstrahlung emission balance, is

$$F_{20} = 3{\cdot}5 \times 10^{-5} (M/M_\odot)^{9/10} R_9{}^{11/10} B_6{}^{27/10}, \tag{28}$$

provided that $\phi = 1$. The regions in which $T_e = T_s$ and $T_e < T_s$ are separated by the line $\gamma_1 = \gamma_2$ which (for $\phi = 1$) is

$$F_{20} = 5 \times 10^{-6} (M/M_\odot)^{6/5}\, R_9{}^{2/7} B_6{}^{27/10}. \tag{29}$$

On both these lines $\phi = 2$ when $D_{ff} = 0{\cdot}5$ d, that is when

$$F_{20} = 1{\cdot}1 \times 10^{-2} (M/M_\odot)^{11/6}\, R_9{}^{-17/12} B_6{}^{-1/3}. \tag{30}$$

When $\phi \gg 1$, $\gamma_1 =$ constant implies that $F \propto B^{25/16}$, and $\gamma_1 = \gamma_2$ implies that $F \propto B^{1123/488}$.

In the region $T_e < T_s$, the lines $\gamma_2 =$ constant are given (for $\phi = 1$) by

$$F_{20} = 1{\cdot}5 \times 10^{-4} (M/M_\odot)^{3/8}\, R_9{}^{13/8} B_6{}^{27/10} \gamma_2{}^{-49/40}. \tag{31}$$

On these lines $\phi = 2$ when $D_{ff} = 0{\cdot}5\gamma_2$ d which implies $F \propto B^{-61}$. For $\phi \gg 1$ (and $T_e < T_s$) $\gamma_2 =$ constant implies $F_2 \propto B^{144/121}$. Note that, in the region $\phi \gg 1$, lines of constant cyclotron to bremsstrahlung ratio can switch between $\gamma_1 =$ constant and $\gamma_2 =$ constant by crossing the line $\gamma_1 = \gamma_2$.

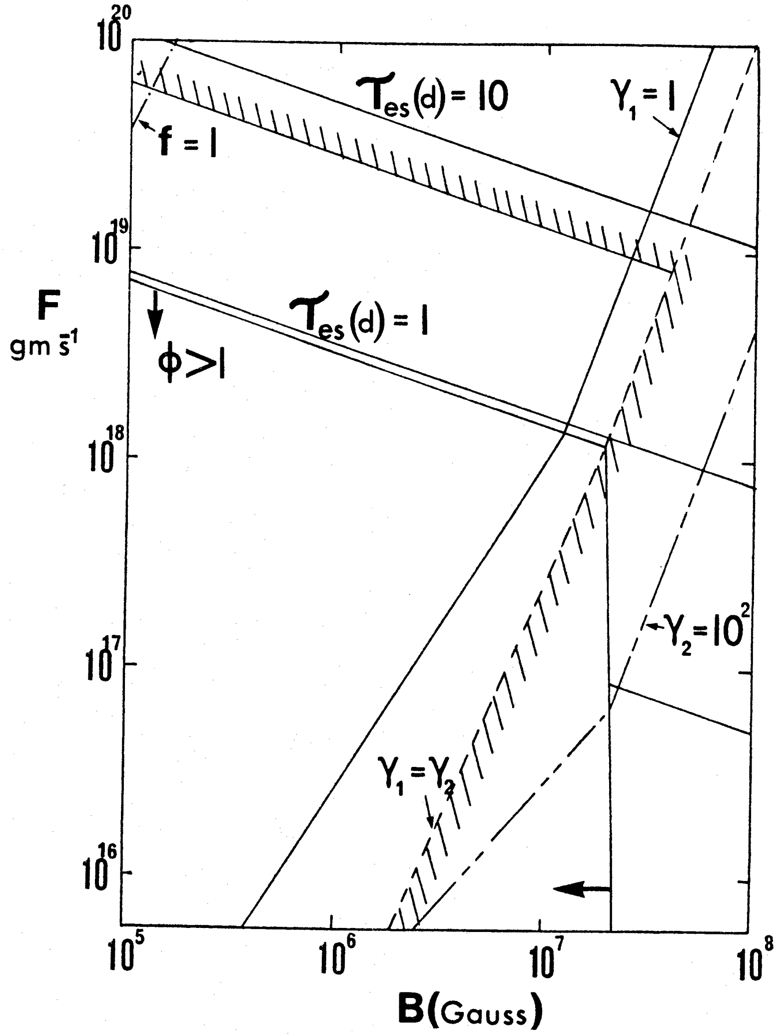

FIG. 2. *The mass-flow-rate F, plotted as a function of surface magnetic field, for various criteria (see Section (4)). The accretion flow beyond the Alfvén radius is assumed spherically symmetrical, and the white dwarf is taken to have mass $M_\odot$ and to be non-rotating. The magnetic field is dynamically important at the surface (a necessary condition for* pulsed *emission) to the right of the $f = 1$ line (equation (33)). X-rays above 10 keV are degraded above the $\tau_{es}(d) = 7$ line (equation (32)). In the region marked $\phi > 1$, the shock in the accretion column occurs at a height $\gtrsim f^{1/2}R$ above the surface, so the radiation escapes predominantly through the sides of the shocked region. To the right of the line $\gamma_1 = 1$ (equations (16) and (28)) cyclotron cooling dominates bremsstrahlung, and the efficiency of hard X-ray emission is $\sim \frac{1}{2}\gamma_1^{-1}$. To the right of the $\gamma_1 = \gamma_2$ line (equation (29)) cyclotron cooling is so efficient that it prevents the electrons from ever attaining the shock temperature T_s and thus suppresses hard X-ray emission (unless collective effects enhance the coupling between electrons and ions). On the line $\gamma_2 = 100$, the ratio of cyclotron and bremsstrahlung cooling, calculated for the 'self-consistent' electron temperature $T_e < T_s$, is 100:1. These considerations lead us to expect hard X-ray emission only when F and B lie within the hatched region on the diagram. We emphasize, however, that the equations derived in this paper undoubtedly oversimplify the real situation, so the lines in this figure (and in Fig. 3) are of an approximate character.*

The lines $\tau_{es}(d) = \tau =$ constant are given by

$$F_{20} = 4 \times 10^{-2}(M/M_\odot)^{2/3} R_9^{1/6} B_6^{-1/3} \tau^{7/6}. \tag{32}$$

These are only relevant if $\phi \approx 1$: if $\phi \gg 1$ the radiation escapes predominantly from the sides of the shocked region.

For accretion to be appreciably channelled by the field we require $f_{\mathrm{sph}} < 1$, that is

$$F_{20} < 2 \times 10^2 (M/M_\odot)^{-2} R_9^{5/2} B_6^2. \tag{33}$$

If the white dwarf rotates, the rough condition that accretion takes place, $f_{\mathrm{sph}} > f_\Omega$, is

$$F_{20} > 1 \cdot 1 (M/M_\odot)^{-5/3} R_9^6 B_6^2 P^{-7/3}. \tag{34}$$

At the edge of this region, $f_{\mathrm{sph}} = f_\Omega = 0 \cdot 2 (M/M_\odot)^{-1/3} R_9 P^{-2/3}$.

(b) *Accretion disc flow outside R_M*

The conditions derived in this section are illustrated in Fig. 3 for a white dwarf of 1 $M_\odot$. The condition $\gamma_1 = 1$ ($\phi = 1$) is

$$F_{20} = 2 \times 10^{-5} (M/M_\odot) R_9 B^{18/7}. \tag{35}$$

The line $\gamma_1 = \gamma_2$ ($\phi = 1$) is

$$F_{20} = 3 \times 10^{-5} (M/M_\odot)^{5/3} R_9^{1/3} B^{18/7}. \tag{36}$$

On both these lines $\phi = 2$ when $D_{\mathrm{ff}} = 0 \cdot 5$ d, that is when

$$F_{20} = 2 \times 10^{-2} (M/M_\odot)^{13/7} R_9^{-1} B_6^{-16/49}. \tag{37}$$

When $\phi \gg 1$, $\gamma_1 =$ constant implies $F \propto B^{200/133}$ and $\gamma_1 = \gamma_2$ implies $F \propto B^{9784/4485}$.

In the region $T_{\mathrm{e}} < T_{\mathrm{s}}$ the lines $\gamma_2 =$ constant ($\phi = 1$) are given by

$$F_{20} = 8 \times 10^{-4} (M/M_\odot)^{1/2} R_9^{3/2} B_6^{18/7} \gamma_2^{-7/6}. \tag{38}$$

On these lines $\phi = 2$ implies $F \propto B^{-976/7}$. For $\phi \gg 1$ (and $T_{\mathrm{e}} < T_{\mathrm{s}}$) $\gamma_2 =$ constant implies $F \propto B^{2304/2037}$.

The lines $\tau_{\mathrm{es}}(\mathrm{d}) = \tau =$ constant are given by

$$F_{20} = 7 \times 10^{-2} (M/M_\odot)^{5/7} R_9^{1/7} B_6^{-16/49} \tau^{8/7} \tag{39}$$

and are, again, only relevant for $\phi \approx 1$.

The condition $f_{\mathrm{disc}} < 1$ is

$$F_{20} < 10 (M/M_\odot)^{-1} R_9^3 B_6^{16/7} \tag{40}$$

and the condition $f_{\mathrm{disc}} > f_\Omega$ is

$$F_{20} > 0 \cdot 15 (M/M_\odot)^{-7/3} R_9^7 B_6^{16/7} P^{-8/3}.$$

5. THE MASS TRANSFER PROCESS

In the previous sections we have considered the processes occurring at the surface of the white dwarf, where most of the primary emission takes place. The gas flowing in the binary system can, however, have severe effects on the observable properties of the accretion energy, for example by reflection (including absorption and re-emission) and by absorption, particularly at non-X-ray wavelengths. In this section we consider the process of mass transfer and gas flow and estimate some of its observable consequences.

The matter accreted is generally considered to be transferred from the normal (non-compact) companion either by Roche lobe overflow or by a stellar wind. In

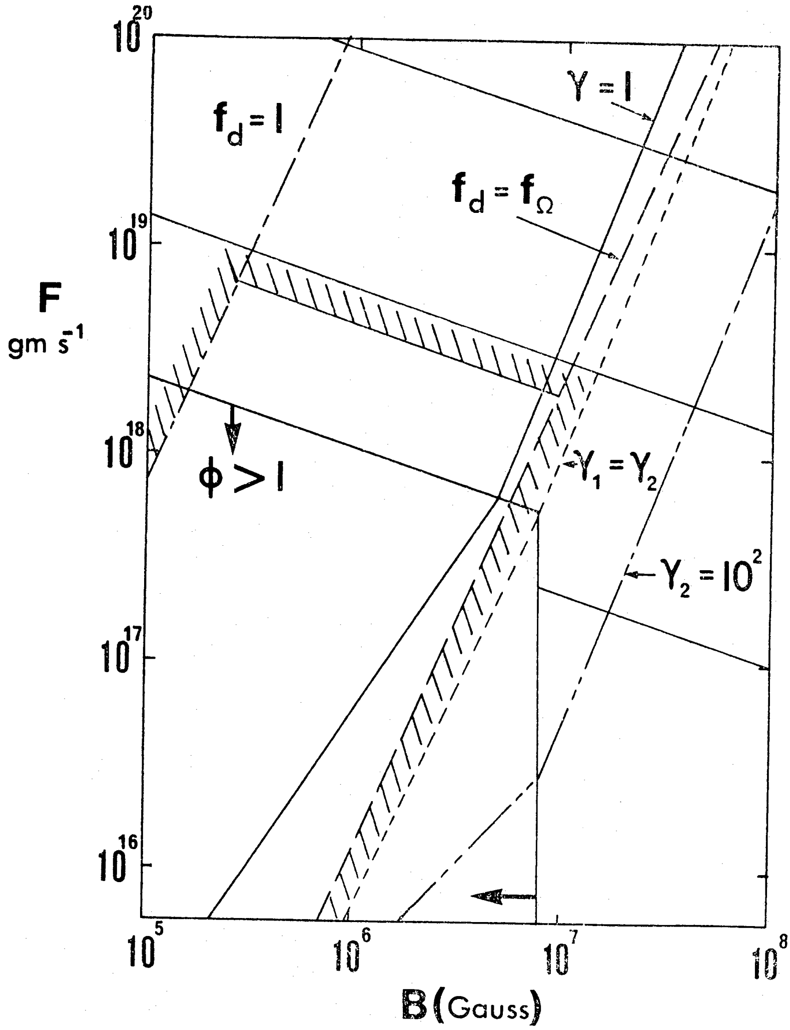

FIG. 3. *The same criteria as in Fig. 2 are here plotted for a system such as DQ Her where the accreted material is expected to form a disc outside the Alfvén radius (case (b) in Section 4). Note that a somewhat different value of B is now needed to ensure $f < 1$ (cf. equations (33) and (40)). This is a necessary condition for hard X-ray emission, because otherwise the disc extends right down to the stellar surface and no shock can form. For a white dwarf of mass $M_\odot$ spinning with a period $2\pi/\Omega = 142$ s (the appropriate value for DQ Her), the maximum value of B compatible with accretion is given by the line $f_d = f_\Omega$. For DQ Her (where the corresponding value of f is 0·062) it is this constraint, rather than the condition $\gamma_1 = \gamma_2$, which determines the maximum magnetic field compatible with hard X-ray emission. Hard X-rays (necessarily pulsed) are therefore expected only within the hatched area of the B–F plane.*

reality, of course, these are two extremes of a spectrum of possibilities and the transfer process may well be caused by a mixture of both. For simplicity, we consider each extreme possibility separately.

5.1 *Roche lobe overflow*

We use the term ' Roche lobe ' loosely in as much as we make no distinction as to whether or not the mass-losing component corotates with the binary period, and do not necessarily demand that the orbit should be circular. In other words, we just demand in this case that some parts of the outer layers of the mass-transferring star are ' sufficiently ' attracted to the accreting star for a ' large enough ' fraction of the orbital period. There are, however, strong reasons for supposing that if mass transfer takes place the orbit becomes circular very rapidly (Heggie & Pringle, in preparation). In this case the initial velocity of the mass transfer flow in the direction of the accreting star is approximately the speed of sound in the outer layers of the mass-losing component, which is in general much less than the orbital or rotational velocities. For this reason we expect accretion to take place by means of an accretion disc. If the disc extends down to the surface of the accreting star, the accretion energy is radiated in two approximately equal components. First, the radiation emitted in the disc itself, with a maximum temperature of about $2 \times 10^5 F_{20}^{1/4}(M/M_\odot)^{1/4} R_9^{-1/4}$ K for an optically thick disc, and second the radiation emitted from the boundary layer formed where the disc grazes the stellar surface, which is likely to be emitted at a higher temperature (Lynden-Bell & Pringle 1974), possibly in the soft (~ 0.1 keV) X-ray range. Hard X-rays are not likely to be emitted from such a disc even for luminosities approaching the Eddington limit.

A dwarf nova is a binary system in which Roche lobe transfer on to a white dwarf is taking place at a rate of $\sim 10^{16}$–10^{18} g s^{-1} and in which the optical radiation from the system is usually dominated by the accretion energy. No hard ($\gtrsim 2$ keV) X-rays have been observed from these systems, although there are reports of soft X-ray emission detected during outburst when the accretion rate is greatest (Rappaport *et al.* 1974; Heise *et al.* 1975).

If, however, the accreting white dwarf has a surface magnetic field strong enough to disrupt the disc out to say, radii $\gtrsim 3$ times the stellar radius, then accretion can take place along the field lines in a more or less radial fashion. A shock can form above the stellar surface and the accretion energy can be emitted as hard X-rays.

5.2 *Application to DQ Herculis*

The old nova (1934) DQ Herculis is known to be a binary system which contains an accreting magnetized white dwarf (Bath, Evans & Pringle 1974; Katz 1975; Lamb 1974; Kemp, Swedland & Wolstencroft 1974; Swedland, Kemp & Wolstencroft 1974) which is rotating with a period of 142 s or 2·37 min. The mass of the white dwarf is about 1 $M_\odot$ (Robinson 1976; Webbink 1975) and the rotation period is decreasing on a time scale of $P/\dot{P} \sim 10^7$ yr (Herbst, Hesser & Ostriker 1974). If this speed-up of the rotation rate is caused by accretion of angular momentum, then the accretion rate is $\sim 3 \times 10^{16}$ g s^{-1}. This is consistent with the observed upper limits to the rate of change of orbital period (Pringle 1975). For these parameters we see from Fig. 3 that for accretion to take place we require $B \lesssim 10^6$ G and that the dominant emission mechanism is then likely to be bremsstrahlung (although cyclotron emission could dominate by a factor 10 or so). The total luminosity due to accretion is $\sim 10^{34}$ erg s^{-1}. The assumption of $7^{\rm m}$ for the distance modulus (Kraft 1963) yields a distance $\mathscr{D} \sim 250$ pc. If bremsstrahlung emission dominates, we expect an X-ray flux at Earth of $\sim 7 \times 10^{-10}$ erg cm^{-2} with a flat (quasi-bremsstrahlung) spectrum and $kT \sim 60$ keV. This flux could, however, be considerably reduced by absorption and scattering in the material flowing in the white dwarf's Roche lobe: there is evidence that much of the light in the system is due to reflection effects. There remains the additional possibility that much, but not all, of the luminosity can be radiated as cyclotron radiation at ~ 100 μm. This would be modulated with a period of 142 s.

5.3 *Stellar wind*

A variety of stars are observed to emit stellar winds at rates of 10^{-5}–10^{-7} $M_\odot$ yr^{-1} and a compact object orbiting such a star can accrete a fraction of it. The accretion energy so produced is not necessarily the dominant contribution to the luminosity of the system (mainly because the star must be very luminous to produce such a strong wind) but can be readily observable if it is emitted at wavelengths different from the bulk of the luminosity.

A number of authors (Davidson & Ostriker 1973; Illarionov & Sunyaev 1975; Shapiro & Lightman 1976) have considered the accretion process in this case. We briefly repeat their arguments here. Consider a binary system, consisting of stars masses M_1 and M_2, and with semi-major axis a. Assume for simplicity that the orbit is circular. Let M_2, with radius R_2, be a compact accreting object and M_1, with radius R_1, emit a stellar wind with velocity

$$V_\mathrm{w}(R) = (\lambda(R)\, 2GM_1/R_1)^{1/2}$$

where R is the distance from the centre of M_1 and λ is a parameter, probably of order unity for $R \sim a$. If we assume that the wind velocity is larger than both the speed of sound in the wind and the orbital velocities of the two stars (a good approximation provided that the mass ratio is not too extreme and the system is not too close), we may define an accretion radius R_A around M_2 approximately by

$$R_\mathrm{A} = \frac{\alpha G M_2}{V_\mathrm{w}^2} = \frac{\alpha M_2 R_1}{2\lambda(a)\, M_1} \tag{41}$$

where α is another parameter of order unity.

In general $R_\mathrm{A} \ll a$ and thus the fraction of the wind that is accreted by the white dwarf is

$$\frac{F}{F_\mathrm{w}} = \frac{\pi R_\mathrm{A}^2}{4\pi a^2} = \left(\frac{\alpha}{\lambda(a)}\right)^2 \frac{M_2^2}{16 M_1^2} \left(\frac{R_1}{a}\right)^2. \tag{42}$$

The angular momentum per unit mass h of the accreted material is given roughly by (Illarionov & Sunyaev 1975)

$$h = \tfrac{1}{4}\Omega R_\mathrm{A}^2 \tag{43}$$

where Ω is the orbital angular velocity. Therefore the angular momentum of the accreted material begins to dominate the flow—that is, an accretion disc starts to form—at the circularization radius R_c, where $h = (GM_2R_\mathrm{c})^{1/2}$. Thus

$$\frac{R_\mathrm{c}}{a} = \left(\frac{\alpha}{\lambda(a)}\right)^4 \frac{M_2^3(M_1+M_2)}{256 M_1^4} \left(\frac{R_1}{a}\right)^4. \tag{44}$$

If the accreting object does not have a strong magnetic field, then the occurrence or not of radial infall on to the surface depends on the relative sizes of R_c and R_2.

We see at once that the specific angular momentum of matter accreted from a wind is much less (by a factor $\sim (R_\mathrm{A}/a)^2$ than that of matter transferred by Roche lobe overflow and that therefore a disc is less likely to form when a stellar wind provides the main mass transfer. We stress that the value of R_c estimated above is very uncertain. Shapiro & Lightman (1976) have made a more detailed estimate, but nevertheless the main uncertainty lies in the function $\lambda(R)$. Theoretical estimates suggest (Lucy & Solomon 1970) that in a steady wind driven by radiation pressure $\lambda(R) = 1-(R_1/R)$, although this could be seriously wrong if the accretion luminosity gave rise to significant ionization in the acceleration zone of the wind (Hatchett, Buff & McCray 1975). (We consider the effect of radiation on the accretion radius in the next section.) On the other hand, observational data (Kuan & Kuhi 1975) suggest that, for the star P Cygni, $\lambda = (R_1/R)$, in which case $F/F_{\rm W}$ and $R_{\rm c}/a$ are ***independent of the binary separation.*** If the mass-loss rate in the wind is variable, further complications occur since the velocity and density structure of the wind can then vary independently.

We conclude therefore that radial and disc accretion can arise from stellar wind transfer and that it is not a trivial matter to decide which in fact occurs. Moreover, since $R_{\rm c}$ is so sensitive to the wind parameters the flow can switch to and fro between radial (hard X-rays emitted) and disc (soft or no X-rays) on the same time scale as the variation of the wind flow (*cf.* Shapiro & Lightman 1975). This may be of some importance in the understanding of the transient X-ray sources.

One further point is worth making at this stage. We mentioned in Section 4 that if $R_{\rm M} > R_\Omega$ the accretion flow is obstructed from latching on to the magnetic field lines by a centrifugal barrier. If, however, $R_{\rm c} < R_\Omega < R_{\rm M}$ some accretion can nevertheless take place. In particular, the matter that falls on to the magnetosphere within a distance R_Ω of the rotation axis (that is $\sim R_\Omega^2/4R_{\rm M}^2$ of the accretion flux) can be accreted. The rest is expelled by centrifugal forces. It is even possible that the expelled material could form a centrally-driven accretion disc (*cf.* Lynden-Bell & Pringle 1974) and so extract a considerable amount of angular momentum from the accreting object. In this case the rotation rate of the star could be decreasing, even though accretion is taking place (*cf.* Fabian 1975).

5.4 *Interactions between the outgoing radiation and infalling matter*

The accretion radius, $R_{\rm A}$, is modified if the radiative heating of the ambient gas is sufficient to produce thermal velocities comparable with the wind velocity, $V_{\rm W}$, near $R_{\rm A}$. It may be that no static self-consistent solution exists and that the accretion flow becomes unstable (Buff & McCray 1974). The gas temperature can be estimated from the parameter $\xi = L/nr^2$, where L is the accretion luminosity, n the number density of the gas and r the distance from the accreting object (Tarter, Tucker & Salpeter 1969; Buff & McCray 1974). At the accretion radius,

$$\xi(R_{\rm A}) \approx 2{\cdot}2 \left(\frac{V_{\rm W}}{30\ {\rm km\ s^{-1}}}\right)\left(\frac{M}{M_\odot}\right) R_9^{-1}, \tag{45}$$

while the thermal velocity in the wind is given by

$$\frac{V_{\rm s}}{30\ {\rm km\ s^{-1}}} \simeq 1{\cdot}6 \left(\frac{T}{10^5\ {\rm K}}\right)^{1/2}.$$

Fig. 1 of Buff & McCray (1974) shows that

$$\frac{T}{10^5\ {\rm K}} \simeq \left(\frac{\xi}{30}\right)^2 \qquad (10 < \xi < 10^4) \tag{46}$$

for a thermal spectrum ($kT \sim 10$ keV) with no low energy cut-off ($\epsilon_{\rm min} = 0$), so

$$\frac{V_{\rm s}}{V_{\rm W}} \simeq 0{\cdot}12 \left(\frac{M}{M_\odot}\right) R_9^{-1} \tag{47}$$

and stable accretion can occur. Note that, for radial infall within $R_{\rm A}$, the flow speed $\propto r^{-1/2}$ (free-fall) and the sound speed $V_{\rm s} \propto \xi \propto r^{-1/2}$ and the flow is self-consistent for all $r < R_{\rm A}$. If $\xi > 10^4$ or if $\epsilon_{\rm min} > 0$, (46) is an overestimate for T and stable accretion occurs *a fortiori*. When the wind speed is low, we see from (45) that $\xi < 10$. We find approximately that, when $V_W \lesssim 15\ \mathrm{km\ s^{-1}}$ and when $\epsilon_{\min}$ is small, $V_S \gtrsim V_W$ and the above can break down, with the accretion flow becoming non-static.

If the accretion flow is radial, for $r < R_A$,

$$\xi(r) \simeq 4 \times 10^2 \left(\frac{r}{10^9\ \mathrm{cm}}\right)^{-1/2} \left(\frac{M}{M_\odot}\right)^{3/2} R_9^{-1}. \tag{48}$$

If appreciable funnelling takes place this is an overestimate of the effective ξ at some radii, since (a) n increases more steeply with decreasing radius and (b) the full accretion luminosity does not reach all points of the flow. Hatchett, Buff & McCray (1976) have shown that the elements C, N and O are completely ionized if $\xi \gtrsim 200$, and that to ionize the heavier elements (S and Fe) requires $\xi \gtrsim 10^3$. Thus the elements producing absorption in the keV range retain their K shell electrons during infall.

Note that the dynamical effects of the outgoing radiation on the gas are negligible, even in the spherically symmetric case, provided the effective cross-section σ per electron is less than $(L_{\mathrm{edd}}/L)\,\sigma_T$, L_{edd} being the ' Eddington limit ' $(4\pi G M m_p c)/\sigma_T$. The precise value of σ depends on the state of ionization of the gas, particularly the H and He (*cf.* Hatchett *et al.* 1976). We have shown that hard X-rays can escape only when $L \lesssim 10^{-2} L_{\mathrm{edd}}$, and under these conditions we expect that the neglect of these dynamical effects is indeed self-consistent.

If $D \lesssim d$ the emergent X-rays must pass through a column density N of absorbing matter,

$$N \simeq 2 \times 10^{25} F_{20} f^{-1/2} (M/M_\odot)^{-1/2} R_9^{1/2}\ \mathrm{cm^{-2}} \tag{49}$$

where the f-dependence allows for the escape of X-rays from the sides of the accretion column. Of course, if $\phi \gtrsim 2$, X-rays escape from the sides of the emitting region and do not have to pass through unshocked, un-ionized material. If we crudely approximate the absorption cross-section $\sigma_p(E)$ of Brown & Gould (1969) by $\sigma_p(E) = 4 \times 10^{-22} E^{-3}\ \mathrm{cm^2}$, and define a cut-off energy E_a by $\sigma_p(E_a) N = 1$, we find

$$E_a \simeq 19\, F_{20}^{1/3} f^{-1/6} (M/M_\odot)^{-1/6} R_9^{-1/6}\ \mathrm{keV}. \tag{50}$$

This holds provided that $E_a \gtrsim 3$ keV, where absorption by C, N and O is negligible. If equation (50) gives formally $E_a < 3$ keV, the low-energy cut-off depends more critically on the ionization state of the lighter elements and the secondary radiation emitted by the infalling gas should be considered. In general, however, the dimensionless parameter γ defined by Tarter & Salpeter (1969) is less than unity and we expect (50) to provide a reasonable estimate of E_a, although fluorescence lines and other features may become evident.

5.5 *Application to Mira and similar systems*

Mira B is thought to be a white dwarf which is accreting material from Mira itself (Warner 1972). The accretion radius is of comparable size to Mira, allowing a comparatively large fraction (~ 1 per cent) of the mass in the wind to be accreted, despite the relatively large separation ($\sim 10^{15}$ cm). Following the discussion in Section 5.3, a disc forms at an approximate radius of

$$R_c \simeq 4 \times 10^{10} \left(\frac{P_b}{261\ \mathrm{yr}}\right)^{-2} \left(\frac{V_W}{10\ \mathrm{km\ s^{-1}}}\right)^{-4} \mathrm{cm}. \tag{51}$$

Here P_b is the binary orbital period. The steep dependence of R_c on V_W means that spherically symmetric accretion may occur at least some of the time. As mentioned in Section 5.3, density gradients in the highly variable wind from Mira can act to substantially reduce the rate of accretion of angular momentum per unit mass. Ionization instabilities can occur as outlined in Section 5.4. We therefore envisage the possibility that Mira B is a sporadic X-ray source, and it may be responsible for the transient sources Cet X-1 and Cet X-2 (Fabian, Pringle & Webbink 1975a; P. J. N. Davison, private communication). There are at present no indications of a magnetic field on the dwarf, but of course if there is a strong enough one to disrupt the disk, radial infall can occur all the time. SY Fornacis is a system similar to Mira (Feast 1975) and we note that the Mira-type variable RS Centauri has been proposed as a candidate for A1118-61 (Fabian *et al.* 1975b).

The accretion rate on to Mira B also depends strongly on the wind velocity, V_W, and consequently it is difficult to make any precise estimate of the X-ray luminosity, L_X. If we take a value, $L_X \sim 10^{33}$ erg s^{-1}, similar to the optical luminosity estimated by Warner (1972), we derive an X-ray flux at the Earth of $\sim 3 \times 10^{-9}$ erg cm^{-2} s^{-1} for a distance of ~ 50 pc. A relatively hard bremsstrahlung spectrum coupled with strong absorption in the infalling material and wind could render such a source undetectable by, for example, the *Uhuru* satellite. The value of L_X used above could at times be a serious underestimate. Some of the optical light might originate in an accretion disc, with the contribution from the white dwarf itself emerging in the ultraviolet. Moreover, Mira B is only clearly seen when Mira is near minimum.

6. SUMMARY

We have extended the calculations of Hoshi (1973) and Aizu (1973) to produce a self-consistent model for X-radiation from accreting, possibly magnetized, white dwarfs. An accretion rate of $\gtrsim 10^{15}$ g s^{-1} and a quasi-radial accretion flow is required to produce a stand-off shock which heats the material to X-ray temperatures ($\sim 6 \times 10^8$ K for a 1 $M_\odot$ white dwarf (see Fig. 1)). The gas radiates X-rays by optically thin bremsstrahlung. Radial flow to the stellar surface is likely when the accretion takes place from a stellar wind and/or when the dwarf has a strong enough magnetic field to channel the flow on to the magnetic polar caps. In the latter case cyclotron emission can dominate (see Figs 2 and 3). If the X-ray luminosity exceeds $\sim 10^{36}$ erg s^{-1}, electron scattering in the infalling material is sufficient to degrade the keV X-rays and in any case the low-energy absorption cut-off is likely to be substantial, particularly for the more luminous sources.

Thus the characteristics of the sources we describe here are as follows. The X-ray luminosities lie roughly in the range 10^{32}–10^{36} erg s^{-1} (and cannot therefore account for the bright variable sources seen by *Uhuru*). The X-ray spectrum is quasi-bremsstrahlung with $kT \sim 30$–100 keV and with a substantial low-energy cut-off (note that little or none of the X-ray emission need fall in the range detectable by *Uhuru* and similar instruments). The optical (or ultraviolet) luminosity should be at least comparable to that emitted as X-rays. It is possible that a comparable, or greater, luminosity is radiated by the cyclotron mechanism

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APPENDIX

When $F_{20} \lesssim 3 \times 10^{-2} f(M/M_\odot)^{3/2} R_9^{-1/2}$, the height of the shock above the stellar surface $D \gtrsim R$. In this case the temperature behind the shock $\propto (R+D)^{-1}$ is lower than that used in previous calculations and at first sight it seems that the above analysis is not applicable. Most of the energy, however, is still liberated close to the star, within the region $R \leqslant r \lesssim 2R$ where r is the radial coordinate. We obtain below an approximate solution for the shocked gas which, as we shall see, is a good approximation in the region $R \leqslant r \lesssim 2R$ when $D \gg R$.

Since the flow velocities behind the shock are subsonic we may assume the approximate validity of the hydrostatic equation

$$\frac{dp}{dr} = -\frac{GM\rho}{r^2}. \tag{A1}$$

For simplicity we assume spherical symmetry. Well below the shock (where most of the energy is released) we may assume roughly that the energy liberated is radiated *locally* by bremsstrahlung at a rate $\epsilon\rho^2 T^{1/2}$, where ϵ is a constant (apart from the temperature dependence of the Gaunt factor). Thus we may write

$$\frac{FGM}{r^2} = 4\pi r^2 . \epsilon\rho^2 T^{1/2}. \tag{A2}$$

Solving these, together with the gas equation

$$p = \frac{\mathscr{R}\rho T}{\mu} \tag{A3}$$

we obtain

$$p^{4/3} - p_s^{4/3} = \frac{4}{11}(GM)^{5/3}\left(\frac{\mathscr{R}}{\mu}\right)^{1/3}\left(\frac{F}{4\pi\epsilon}\right)^{2/3}\left[\frac{1}{r^{11/3}} - \frac{1}{(R+D)^{11/3}}\right]$$

where p_s is the pressure just after the shock at $r = R+D$. For $R \lesssim r \ll R+D$, we find

$$T \approx \frac{4}{11}\left(\frac{\mu}{\mathscr{R}}\right)\frac{GM}{r} \tag{A4}$$

and $p \propto r^{-11/4}$, $\rho \propto r^{-7/4}$ where the constants of proportionality *are independent of the position of the shock.*

This is not a surprising result. The amount of energy to be liberated at $r \sim R$ and the temperature at $r \sim R$ have been fixed. These, together with the assumption that bremsstrahlung is the dominant cooling mechanism, imply a density for the region. Thus the temperature and density of the radiating region for $D \gg R$ do not differ appreciably from the solutions with $D \lesssim R$ and the above analysis is approximately valid for this case too.