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X-Ray Emission Region of a White Dwarf with Accretion

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As a model of X-ray stars of the Sco X-1 type the accretion of gas by a white dwarf is studied, and, in particular, the structure of a hot plasma formed by the accretion is examined in detail. It is found that the temperature distribution in the plasma is determined only by the mass of the star, while the density and the thickness of this plasma depend also on the accretion rate. Thus the relative spectrum of thermal X-rays emitted from the hot plasma is determined only by the stellar mass, while the luminosity changes proportionally to the accretion rate. This feature is supported by recent observations of Sco X-1 by Matsuoka et al. The mass of the white dwarf of Sco X-1 is estimated as $0.29M_{\odot}$.

§ 1. Introduction

Recently Hōshi[1] (referred to as H) studied the accretion of gas by a white dwarf as a model of X-ray stars of the Sco $X-1$ type. The accretion is assumed to be steady and spherically symmetric. Near the stellar surface a shock front is formed and a hot plasma between the front and the surface emits thermal bremsstrahlung (in the following the plasma is called the emission region). It is shown that the observed X-rays are explained as the high energy part of the bremsstrahlung and the visible light as the low-energy part modified by absorption and scattering in a cool region before the front. Also the mean temperature $\overline{T}$ of the emission region is shown to vary with the accretion rate A, and the observed correlated variation of the X-ray intensity and spectrum[2] is explained as due to the variation of A. However, he represents the emission region by parameters such as mean temperature $\overline{T}$, mean gas velocity and thickness, not entering into the study of its structure. The meaning of $\overline{T}$ and its relation with an effective temperature T_{eff} obtained from observations are not clear.

The purpose of this paper is to study in detail the structure of the emission region (temperature, density and gas velocity distributions) and to calculate the X-ray spectrum of the thermal bremsstrahlung from this region. If the accretion rate A is not small as in the case of Sco $X-1$ (see § 4), the plasma density is not small, and so the cooling time of the hot plasma is short. Then the thickness x of this region, being the distance traversed by the accreting gas without appreciable cooling, is small compared with the stellar radius R, and the gravity change in this region can be neglected. This fact simplifies greatly our treatment.

In § 2 the equations of motion and energy are solved in the expansion of a small parameter x/R and it is shown that the distributions of temperature, velocity, and density relative to that in the shock front are determined only by the stellar mass M, while the absolute density is proportional to the accretion rate A and the thickness is inversely proportional to it. In § 3 the distribution of X-ray emissivity in the emission region and the spectrum of the superposed X-rays are given. It is shown that the relative spectrum depends only on the stellar mass and the absolute luminosity is proportional to the accretion rate. In § 4 the above features of our model is shown to be supported by recent observations of Sco X−1 by Matsuoka et al.[8] Also the stellar mass of the Sco X−1 white dwarf is estimated as $0.29\,M_{\odot}$.

§ 2. The structure of emission region

We consider a steady gas flow falling radially on the surface of a white dwarf. A shock wave is formed near the stellar surface. Gas density ρ, velocity v (the inward direction is taken as positive), temperature T and pressure p are functions of the distance r from the center of the star. At the shock wave $r=R+x$, these quantities have discontinuities. We use suffixes f and b which refer the front and back sides of the shock wave, respectively. In the case of a strong shock these quantities are given by H as follows:

$$v_f=4v_b=(2GM/R)^{1/2}, \tag{1}$$

$$T_b=(3/8)(GMm_{\rm H}\mu/kR), \tag{2}$$

$$\rho_f=\rho_b/4=(4\pi R^2v_f)^{-1}A, \tag{3}$$

where G is the gravitational constant, k the Boltzmann constant, $m_{\rm H}$ the mass of a hydrogen atom and μ the mean molecular weight of the falling gas. The gas is assumed to be composed of hydrogen and helium with the chemical composition in mass as $X=0.7$ and $Y=0.3$, and to be completely ionized. Then $\mu=0.615$.

We shall study the structure of the emission region in the hydrodynamical approximation. The equations of continuity, motion and energy are given respectively by

$$\rho vr^2=\rho_bv_b(R+x)^2=A/4\pi, \tag{4}$$

$$v(dv/dr)+\rho^{-1}(dp/dr)+GM/r^2=0, \tag{5}$$

$$\rho vT(dS/dr)=\rho\varepsilon_{ff}. \tag{6}$$

Here S is the entropy per unit mass:

$$S=(3kT/2\mu m_{\rm H})\ln(T\rho^{-2/3}). \tag{7}$$

The hot plasma of this region is assumed to be optically transparent and its energy loss is due to only the thermal bremsstrahlung. ε_{ff} is its rate per unit mass. Also the thermal conduction is neglected.

If we define the cooling time as

$$t_c = 3kT/(2\mu m_{\mathrm{H}}\varepsilon_{ff}), \tag{8}$$

Eq. (6) is written as

$$vd \ln(T\rho^{-2/3})/dr = t_c^{-1}. \tag{9}$$

If the bottom of the emission region is taken as a place where the falling gas cools appreciably, the thickness x is estimated as $x \sim vt_c$. If this quantity is calculated on the back side of the shock front, we have

$$x \sim v_b t_{cb} \equiv x_b. \tag{10}$$

The proportional constant is given later (Eq. (24)). For simplicity we assume that the bottom of the emission region coincides with the stellar surface, neglecting the structure of the photosphere.

In order to see the order of magnitude of various quantities involved, we shall give a numerical example. We choose a white dwarf of $M = 0.29\, M_{\odot}$ and $R = 1.25 \times 10^9$ cm. Equations (1) $\sim$ (3) give $v_b = 6.2 \times 10^7$ cm/sec, $T_b = 8.7 \times 10^7$ °K and $\rho_b = 5.2 \times 10^{-2} A$ (in $M_{\odot}$/y) g/cm^3 for the falling gas specified above. In c.g.s. unit,

$$t_c = 4.20 \times 10^{-13} T^{1/2} \rho^{-1} \text{ sec}, \tag{11}$$

where the mean value of the Gaunt factor is taken as the Born approximation value 1.10. Thus $t_{cb} = 3.9 \times 10^{-9}/\rho_b$ sec, and $x_b = 0.24\rho_b^{-1}$ cm $= 4.7/A$ (in $M_{\odot}$/y) cm. The condition $x_b \ll R$ requires

$$A \gg 3.7 \times 10^{-9} M_{\odot} \mathrm{y}^{-1}. \tag{12}$$

As is seen later this condition holds in most of the actual cases.

Introducing non-dimensional quantities

$$y = v/v_b, \qquad z = T/T_b, \qquad u = r/(R+x),$$

we rewrite Eqs. (4) $\sim$ (6) in the following forms:

$$\rho = \rho_b y^{-1} u^{-2}, \tag{13}$$

$$[y - (3zy^{-1})]dy + 3dz + (-6zu^{-1} + 8u^{-2})du = 0, \tag{14}$$

$$\alpha q u^2 y^2 z^{1/2} d \ln(zy^{2/3} u^{4/3}) = du. \tag{15}$$

Here we have used Eqs. (1) $\sim$ (3), and put

$$\alpha = x_b/R, \tag{16}$$

$$q = R/(R+x). \tag{17}$$

The boundary conditions at the shock front are

$$y = z = u = 1, \tag{18}$$

and near the stellar surface where a steady sink of the flow is assumed,

$$y \to 0, \qquad z \to 0 \quad \text{as} \quad u \to q. \tag{19}$$

The unknown parameter q is to be determined by Eqs. (13)$\sim$(15) together with the boundary conditions (18) and (19).

Since $\alpha \ll 1$ as shown later, Eqs. (13)$\sim$(15) are solved in the expansion of α. The zeroth-order quantities are denoted by a suffix 0, and the first-order ones by a suffix 1. The zeroth-order approximation of Eq. (15) gives $u_0=\text{const.}$, so u cannot be taken as the independent variable. We choose y as the independent variable, and assume the following expansions:

$$z=z_0+\alpha z_1, \quad u=u_0+\alpha u_1, \quad q=q_0+\alpha q_1.$$

In this expansion the boundary conditions (18) and (19) become

$$z_0=u_0=1 \quad \text{and} \quad z_1=u_1=0 \quad \text{at} \quad y=1, \tag{20}$$

$$z_0 \to 0, \quad u_0 \to q_0 \quad \text{and} \quad z_1 \to 0, \quad u_1 \to q_1 \quad \text{as} \quad y \to 0, \tag{21}$$

respectively. Then the zeroth-order solutions of Eqs. (14) and (15) are

$$u_0=1, \quad q_0=1 \quad \text{and} \quad z_0=y(4-y)/3. \tag{22}$$

In this approximation the gravity in Eq. (5) is neglected, so that the last in Eq. (22) is nothing but the conservation of the momentum flux ρv^2+p. It is also noted that this relation between T and v is independent of the accretion rate A, because T_b and v_b is determined only by the stellar mass.

In order to obtain physical quantities as a function of the distance r, we must go to the first approximation. The relation between the relative distance and relative velocity is given by

$$\begin{aligned}(r-R-x)/R &\cong \alpha u_1 \\ &= (8/9\sqrt{3})\alpha[15\sin^{-1}(1-(y/2)) - (5\pi/2) - (39\sqrt{3}/4) \\ &\quad + \{15+(5/2)y+2y^2\}\{y(1-(y/4))\}^{1/2}],\end{aligned} \tag{23}$$

where the boundary conditions (20) are used. When the boundary conditions (21) are applied to Eq. (23), we obtain

$$x=(2/3\sqrt{3})(13\sqrt{3}-20\pi)x_b=0.605x_b, \tag{24}$$

where $\alpha q_1=-x/(R+x)\cong -x/R$ is used. In the case of $M=0.29\,M_\odot$, we have $x=2.82/A(\text{in } M_\odot/\text{y})\,\text{cm}$. For Sco X$-$1, A is $4.1\times10^{-6}\ M_\odot/\text{y}$, and so $x\sim6.9\times10^5\,\text{cm}$ (see § 4). This leads a small value of $\alpha\sim9.2\times10^{-4}$.

The first-order approximation of the relative temperature as a function of the relative velocity is

$$\begin{aligned}z_1 &= (64/9\sqrt{3})y[(\pi/6)+(13\sqrt{3}/16)-\sin^{-1}(1-(y/2)) \\ &\quad -\{1+y-(y^2/2)+(y^3/8)\}\{y(1-(y/4))\}^{1/2}].\end{aligned} \tag{25}$$

In Fig. 1 we show z_0, z_1 and $z_0+\alpha z_1$ as functions of the relative height $(r-R)/x$. For the value of α, we choose 0.185, which corresponds to $A=4.3\times10^{-8}M_\odot/\text{y}$.

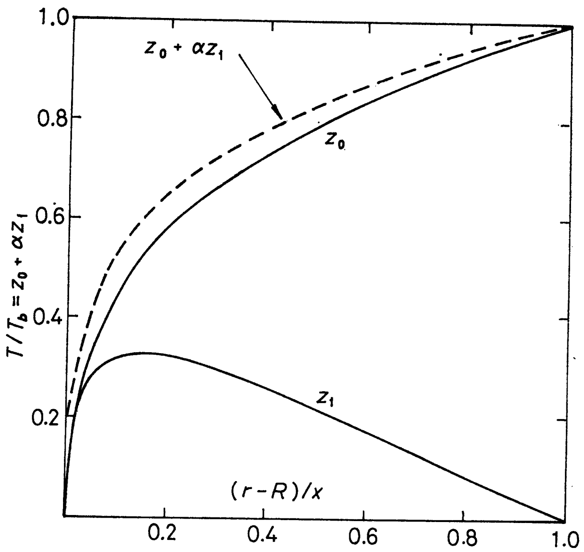

Fig. 1. Temperature distribution in the emission region. In the ordinate are plotted $T/T_b=z$, its zeroth-order approximation z_0 and its correction z_1, where $z=z_0+\alpha z_1$, α being an expansion parameter, and T_b is the temperature on the back side of the shock front. The abscissa is the height from the stellar surface $r-R$ divided by the thickness x of the region.

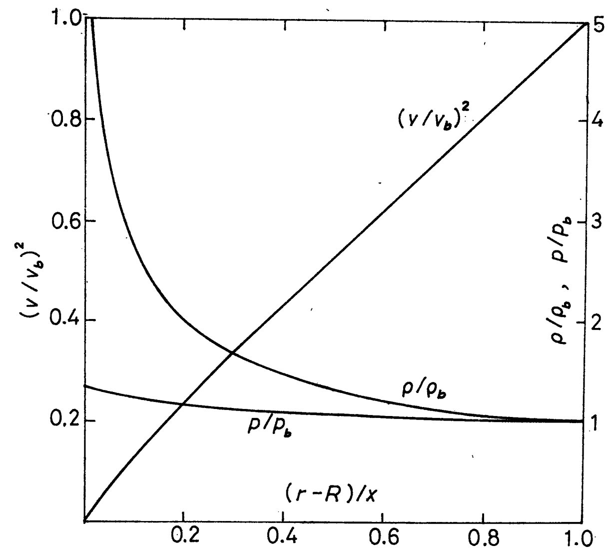

Fig. 2. Square of velocity, density, and pressure distributions in the emission region. All quantities are normalized to 1 at the shock front as in Fig. 1.

The figure shows that the temperature T drops relatively slowly as the gas flows over the emission region; for example, T drops to the half of T_b when the gas flows over 90 % of the emission region. Also this figure shows that the dominant feature of the temperature distribution is determined only by the stellar mass and is not influenced by the accretion rate as long as the latter is not too small, i.e., Eq. (12) holds. This is partly because α is small, but also because the numerical value of z_1 given by Eq. (25) happens to be small. In Fig. 2, square of relative velocity $y^2=(v/v_b)^2$, relative density ρ/ρ_b and relative pressure

p/p_b are given as functions of $(r-R)/x$. It is seen the increase of density as the gas approaches to the surface is slow over most of the emission region, but is very sharp near the surface. This leads to the similar behavior of the emissivity of the falling gas. It is to be noted that in the vicinity of the surface many effects which are not considered here become important (see § 4).

§ 3. X-ray luminosity and spectrum

In order to calculate the total X-ray luminosity from the emission region, it is necessary to consider the radiative transfer there. Here we assume, for simplicity, that the half of the emission which goes outwards is observed as X-rays and the other half which goes inwards to the surface is absorbed at the photosphere and re-emitted as radiation of much longer wavelengths. Then the energy spectrum $L(\varepsilon)d\varepsilon$ is given by

$$L(\varepsilon)=2\pi\int_R^{R+x}\rho\varepsilon_{ff}(r,\varepsilon)r^2dr\,.$$

The emissivity spectrum $\varepsilon_{ff}(r,\varepsilon)$ can be written with the value of ε_{ff} at the shock front ε_{ffb} as

$$\varepsilon_{ff}(r,\varepsilon)=\varepsilon_{ffb}(kT_b)^{-1}(yu^2)^{-1}z^{-1/2}\exp(-\varepsilon/kT)\,.$$

If we denote by $L_0(\varepsilon)$ the expression $L(\varepsilon)$ where the relations between y, z and r are taken from Eqs. (22) and (23), we have

$$L_0(\varepsilon)=(GM/2R)A\int_0^1 d(r/x)P_0(r,\varepsilon)\,.$$

Here the relative energy spectrum $P_0(r,\varepsilon)$ is given by

$$P_0(r,\varepsilon)=(9\sqrt{3}/16)\,(x/kT_bx_b)y^{-5/2}(4-y)^{-1/2}\exp(-\beta/z_0)\,, \tag{26}$$

where $\beta=\varepsilon/kT_b$, and is normalized as

$$\int_0^\infty d\varepsilon\int_0^1 d(r/x)P_0(r,\varepsilon)=1\,.$$

Then we have

$$L_0=\int_0^\infty L_0(\varepsilon)d\varepsilon=(1/2)\,(GM/R)A \tag{27}$$

as shown in H. This relation (27) means that, if the thickness of the emission region is small compared with the stellar radius, the X-ray luminosity is given by the gravitational energy gained by the gas falling on to the shock front, and the energy gain in the emission region can be neglected.

Based on Eq. (26) we can examine which part of the region contributes most to the emission at a given X-ray energy. In Fig. 3 we show Eq. (26) for cases $\beta=2$, 1 and 0.25. As $kT_b=7.5$ keV for a white dwarf of mass $0.29M_\odot$,

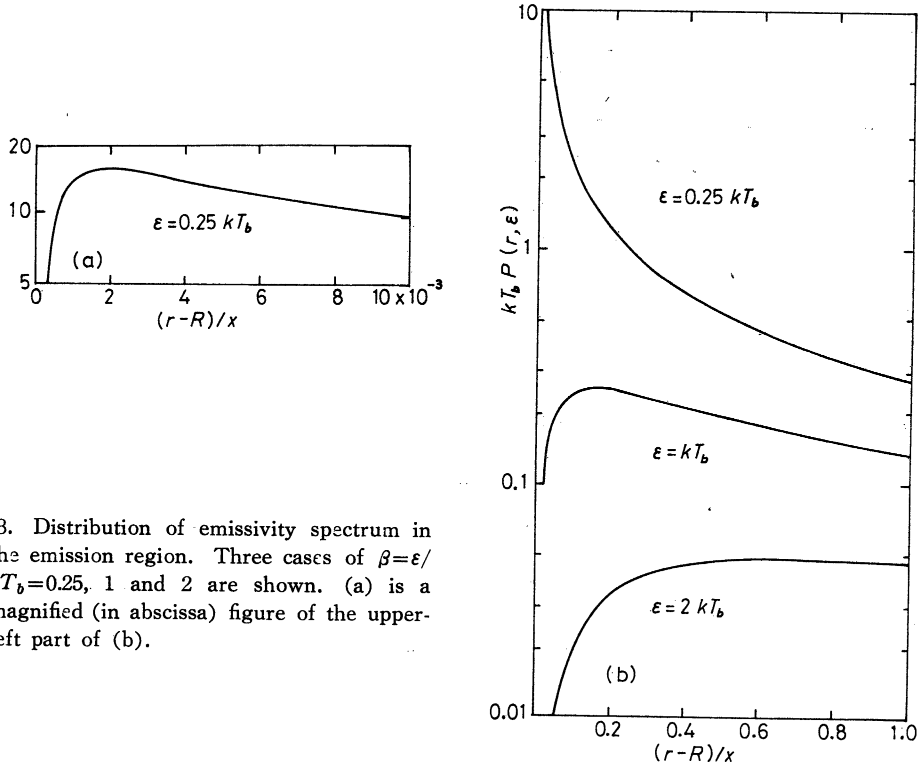

Fig. 3. Distribution of emissivity spectrum in the emission region. Three cases of $\beta=\varepsilon/kT_b=0.25$, 1 and 2 are shown. (a) is a magnified (in abscissa) figure of the upper-left part of (b).

we see from the figure that at $\varepsilon\sim2$ keV the contribution from the region close to the surface is very large, while at $\varepsilon=15$ keV the upper emission region makes a dominant contribution.

The energy spectrum integrated over the emission region is

$$P_0(\varepsilon)=\int_0^1 d(r/x)P_0(r,\varepsilon). \tag{28}$$

This integral cannot be expressed analytically, but at the limiting values of ε asymptotic expressions are given as follows: For $\beta=\varepsilon/kT_b\gg1$,

$$P_0(\varepsilon)\sim(18/\varepsilon)[1-(3/2\beta)+(25/4\beta^2)+\cdots]\exp(-\beta)$$

and for $\beta\ll1$,

$$P_0(\varepsilon)\sim(16kT_b)^{-1}[-15\ln\beta+6\ln(4/3)-15C+\cdots],$$

where C is Euler's constant.

Thus, with increase of energy ε the spectrum goes down more rapidly than that of the thermal bremsstrahlung, and with decrease of ε it grows logarithmically. For an arbitrary values of ε, the results of numerical integration are shown in Fig. 4 where $P_0(\varepsilon)kT_b$ is plotted against ε/kT_b.

Equation (27) shows that, if the stellar mass M is specified, the luminosity is proportional to the accretion rate A, and Eqs. (26) and (28) show that the spectral shape is determined only by M and is independent of A. This results holds under the assumption that the thickness of the emission region is small compared with the stellar radius, or the accretion rate is large, i.e., Eq. (12) holds. However, it holds more generally, because the correction due to the next-order approximation is small as we shall show in the following.

If we expand the spectrum in $\alpha = x_b/R$ as

$$L(\varepsilon) = (GM/2R)A[P_0(\varepsilon) + \alpha P_1(\varepsilon) + \cdots],$$

we have

$$P_1(\varepsilon) = (kT_b)^{-1}\int_0^1 dz_0[z_0(4-3z_0)]^{-1/2}\exp(-\beta/z_0)$$
$$\times[11z_0/4 - 3 + (3-z_0)(4-3z_0)^{1/2}/2 + (9/32)(z_1(z_0)/y(z_0))z_0^{1/2}(4-3z_0)^{-1}],$$

where $y(z_0)$ and $z_1(z_0)$ are given by Eqs. (22) and (25). Numerical calculation shows that except for the high-energy region $\varepsilon > kT_b$, $P_1(\varepsilon)$ is smaller than 0.3 times $P_0(\varepsilon)$. When the effective temperature $T_{\rm eff}$ is introduced as that of the thermal bremsstrahlung which is fitted to a certain part of a given spectrum, the increase of $T_{\rm eff}$ can be realized by the decrease of accretion rate, but its amount is numerically small as long as the stellar mass is smaller than $1.3M_\odot$ (see Fig. 6) and $A \gg 10^{-9} M_\odot$/y (see Eq. (12)). Thus we can neglect the contribution of P_1 to the spectrum and say that the spectral shape is determined only by the stellar mass.

In H the mean temperature $\overline{T}$ is introduced and the differential

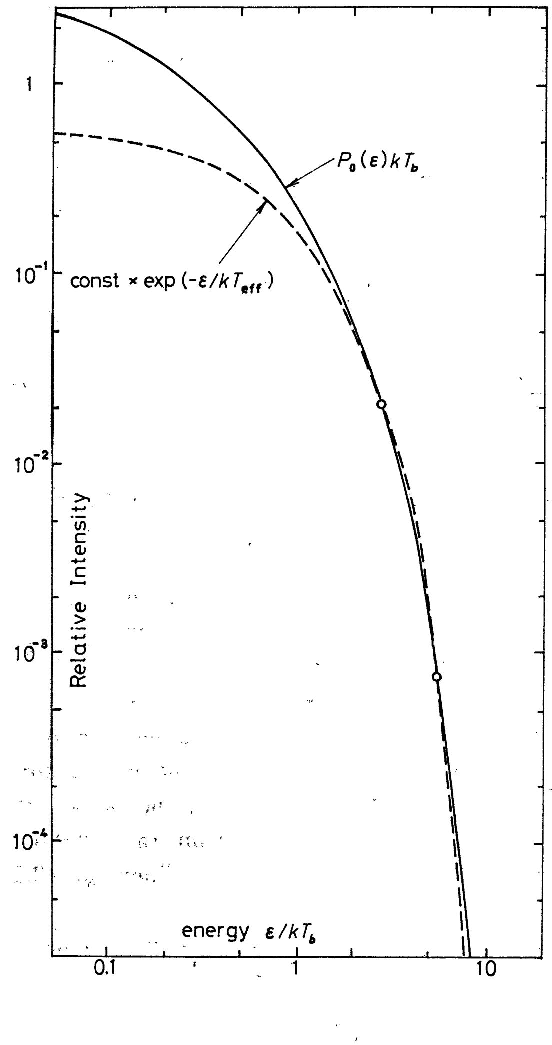

Fig. 4. Energy spectrum. The solid curve is $P_0(\varepsilon)kT_b$ as a function of ε/kT_b. The dashed curve is the spectrum of pure bremsstrahlung which has $kT_{\rm eff}=6$ keV and coincides with the above spectrum at 20 keV and 40 keV, marked by points ○.

equations of motion and energy are replaced by the algebraic ones. As $\overline{T}$ depends on A, it is stated that the variation of A leads to the observed correlation of variations of luminosity and spectrum as summarized in Kitamura et al.[2] The above calculation (also z_1 in Fig. 1) shows that this is qualitatively correct but quantitatively too small. The difference between H and us will be due to that of some numerical factors involved.

§ 4. Comparison with observations

Since the energy spectrum of our model is essentially an exponential one, our model applies only to the Sco X−1 type. Also our model is too simple to discuss the time variation. Here we discuss only the average luminosity and temperature of Sco X−1. Now recent observations of Sco X−1 by Matsuoka et al.[3] show that the temperature of the hot plasma determined in the energy range $20\sim40$ keV does not change appreciably and is 6 ± 1 keV, while the intensity changes by more than a factor of two. In the lower and higher energy ranges the observed spectra show complicated behaviors which indicate the effect of absorption and scattering in the regions behind and/or before the shock front and also the existence of the non-thermal component. So the comparison is limited to the energy range $20\sim40$ keV. Here the features of our model that the temperature distribution of the plasma in the emission region is determined only by the stellar mass and does not change even if the accretion rate changes, while the luminosity changes proportionally to the accretion rate, are in good agreement with the above observations.

In Fig. 4 we also show the spectrum of pure bremsstrahlung which has the effective temperature $kT_{\rm eff}=6$ keV and coincides with our spectrum at energies 20 keV and 40 keV. This shows that our spectrum can fit with observations equally well as that of the pure thermal bremsstrahlung.

Now we shall estimate the mass of the white dwarf and the mean accretion rate for Sco X−1. Our spectrum $P_0(\varepsilon)$ (Eqs. (26) and (28)) includes a parameter T_b. By fitting our spectrum to that of the thermal bremsstrahlung of a single temperature $T_{\rm eff}$ at energies $\varepsilon=20$ keV and 40 keV, we obtain a relation between T_b and $T_{\rm eff}$, which is shown in Fig. 5. From this we obtain $T_b\cong8.7\times10^7$°K for $kT_{\rm eff}=6$ keV. The relation between T_b and the stellar mass (Eq. (2)) is shown graphically in Fig. 6 (Chandrasekhar's Table[4] and revised values of atomic constants being used). From this figure we find the above value of T_b corresponds to $M\sim0.29M_\odot$. If the distance to Sco X−1 is 300 pc, the X-ray luminosity $L\sim4\times10^{36}$ erg/sec. Then Eq. (27) gives the accretion rate $A\doteq4.1\times10^{-6}M_\odot$/y. This value satisfies the condition of Eq. (12). Then $\rho_b=2.1\times10^{-7}$ g/cm^3 and $t_{cb}=1.9\times10^{-2}$ s.

As for the optical spectrum the situation is the same as in Hōshi's paper.[1] Thus our model seems to be essentially correct for the thermal X-rays from

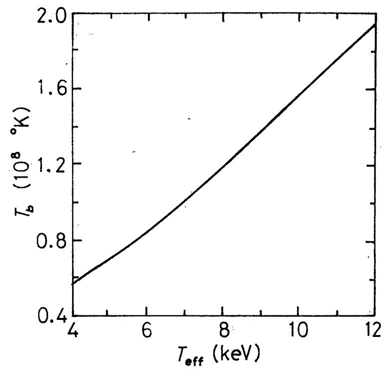

Fig. 5. Relation between the temperature T_b at the shock front and the effective temperature T_{eff} of thermal bremsstrahlung which give the same slope in the energy range 20∼40 keV.

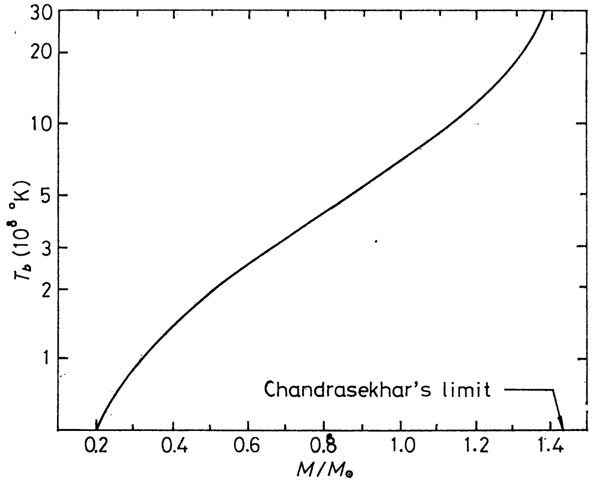

Fig. 6. Temperature T_b is plotted against the mass M of a white dwarf.

the Sco X−1 type stars, but is too simple to explain various observational features and needs at least the following improvements:

1. The accretion is assumed to be spherically symmetric, but this cannot be expected in cases of binaries with rotation and/or magnetic fields.[5]

2. Transfer problem of X-rays in the emission region must be studied. In particular, near the stellar surface absorption and re-emission, radiation pressure, thermal conduction, and cooling processes other than the thermal bremsstrahlung must be considered. These will influence the low-energy part of the spectrum. Also for the soft X-rays absorption and scattering by the falling gas before the shock front will be important.

3. Non-steady treatment of gas is important also naer the surface.

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I am indebted to Professor S. Hayakawa for pointing out that our result of constant plasma temperature is supported by the latest observations.[3]

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Note added in proof: As is pointed out by Hayakawa and Matsuoka the free-bound absorption of X-rays by the accreting gas before the shock front is appreciable and our model cannot apply to the actual case in the present form. One way to reduce this absorption is, as pointed out by Hayakawa, to take into account the existence of the atmosphere of the star and to increase its effective radius as well as the mass. A study of this effect by the author shows that the reduction of the absorption is marginal for the X-ray emission, even if the nuclear shell burning near the bottom of the non-degenerate surface layer is appreciable. Details of this study will be published elsewhere.