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XVI. On the method of deducing the difference of longitude from the azimuths and latitudes of two stations

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Now it is manifest that, the mass of air remaining the same, the quantity $\frac{1+\alpha\theta}{V'}$ has the same value in all circumstances; it is equal to $\frac{1}{M}$, putting M for the bulk of the given mass of air reduced to zero of the thermometer, the pressure being constant. The first of the foregoing formulas is therefore,

$$i = \frac{1}{\beta} \times \frac{V-V'}{M}.$$

The variation of temperature for the same change of volume, is

$$\tau = \frac{1}{\alpha} \times \frac{V-V'}{M}.$$

Now, in Fahrenheit's scale, $\frac{1}{\alpha} = 480°$, and $\frac{1}{\beta} = \frac{3}{8} \times \frac{1}{\alpha}$,

therefore, $i = \frac{3}{8} \times 480 \times \frac{V-V'}{M} = 180° \times \frac{V-V'}{M}$.

In order to find the heat i, we must therefore convert the difference of volume $V-V'$, into degrees at the rate of 180° for M, which is the bulk of the given mass of air at the temperature zero; just as the heat of temperature is found by converting the same difference of volume into degrees at the rate of 480° for the same bulk M. In the second formula, we have $(1 + \alpha\theta)\,\varrho' = D$, D being the density of the mass of air when it is reduced to the bulk M; and thus we readily obtain

$$i = 180°\left(\frac{D}{\varrho} - \frac{D}{\varrho'}\right),$$

which is manifestly equivalent to the other expression. I am not aware that it is possible to warp these formulas from their obvious and unequivocal meaning; and as my sole intention is to explain what I have written, and to rescue it from the fangs of Mr. Meikle's algebra, which perverts whatever it touches, it seems unnecessary to add any thing further. But in quitting this subject, to which I will not return, I cannot help expressing my surprise at finding myself involved in such petty disputation.

Jan. 13, 1829. JAMES IVORY.

XVI. *On the Method of deducing the Difference of Longitude from the Azimuths and Latitudes of two Stations.* By JAMES IVORY, *Esq. M.A. F.R.S. &c.**

I SHALL now extend to all solids of revolution that are little different from a sphere, the same property which is proved of the oblate elliptical spheroid of revolution in the last Number of this Journal.

* Communicated by the Author.

Taking

Taking any station on the surface of the solid, let y denote the ordinate of the meridian perpendicular to the equator, and x the distance of y from the centre: put

$$\varrho = y\frac{\sqrt{dx^2+dy^2}}{dx}; \quad \tau = \frac{x\,dx + y\,dy}{dx};$$

then ϱ is the normal, or the perpendicular to the surface of the solid, limited by the equator, and τ is the distance of the foot of the normal from the centre. The latitude, or λ, is the angle which ϱ makes with the equator; and therefore,

$$x = \tau + \varrho\cos\lambda, \quad y = \varrho\sin\lambda.$$

For any other station we have similarly,

$$x' = \tau' + \varrho'\cos\lambda', \quad y' = \varrho'\sin\lambda':$$

And if ω be the difference of longitude, the three coordinates of the second station referred to the meridian of the first, will be,

$$x'\cos\omega = (\tau' + \varrho'\cos\lambda')\cos\omega$$
$$x'\sin\omega = (\tau' + \varrho'\cos\lambda')\sin\omega$$
$$y' = \varrho'\sin\lambda'.$$

Put m and m' for the azimuths at the first and second stations: then if we make two planes pass, one through ϱ and the second station, and the other through ϱ' and the first station, we shall obtain these equations,

$$\left.\begin{aligned} &Q = \tau\sin\lambda\cos\lambda' - \tau'\sin\lambda'\cos\lambda, \\ &\frac{\sin\omega}{\tan m} + \cos\omega\sin\lambda - \cos\lambda\tan\lambda' = \frac{1}{\cos\lambda'}\cdot\frac{Q}{x'}, \\ &\frac{\sin\omega}{\tan m'} + \cos\omega\sin\lambda' - \cos\lambda'\tan\lambda = -\frac{1}{\cos\lambda}\cdot\frac{Q}{x}. \end{aligned}\right\} (A')$$

But in a spherical triangle, as described at p. 24 of the last Number of this Journal, we have,

$$\frac{\sin\omega}{\tan\mu} + \cos\omega\sin\lambda - \cos\lambda\tan\lambda' = 0,$$

$$\frac{\sin\omega}{\tan\mu'} + \cos\omega\sin\lambda' - \cos\lambda'\tan\lambda = 0;$$

and by subtracting these equations from the former, we get,

$$\text{Sin}\,(\mu - m) \times \frac{\sin\omega}{\sin\mu\sin m} = \frac{1}{\cos\lambda'} \times \frac{Q}{x'},$$

$$\text{Sin}\,(m' - \mu') \times \frac{\sin\omega}{\sin\mu'\sin m'} = \frac{1}{\cos\lambda} \times \frac{Q}{x}.$$

Now, $\frac{\sin\omega}{\sin\mu} = \frac{\sin\beta'}{\cos\lambda'}$, and $\frac{\sin\omega}{\sin\mu'} = \frac{\sin\beta'}{\cos\lambda}$; therefore,

$$\text{Sin}\,(\mu - m) = \frac{\sin m}{x'} \times \frac{Q}{\sin\beta'},$$

$$\text{Sin}\,(m' - \mu') = \frac{\sin m'}{x'} \times \frac{Q}{\sin\beta'}.$$

 Let

Let ϕ and ϕ' denote the depressions of the chord γ below the horizons of the first and second stations; then, according to what is shown at p. 242 of this Journal for October 1828, we shall have these two equations:

$$x' \sin \omega = \gamma \cos \phi \sin m$$
$$x \sin \omega = \gamma \cos \phi' \sin m';$$

wherefore, $\frac{\sin m}{x'} \times \cos \phi = \frac{\sin m'}{x} \times \cos \phi'$.

By combining this equation with the former one, we readily obtain,

$$\text{Sin } (\mu - m) \cos \phi = \sin (m' - \mu') \cos \phi'.$$

On the suppositions made, $\mu - m$ and $m' - \mu'$ are small arcs; also $\cos \phi$ and $\cos \phi'$ are always nearly in a ratio of equality; wherefore we may conclude without sensible error, that

$$\text{Sin } (\mu - m) = \sin (m' - \mu'),$$
$$m + m' = \mu + \mu'$$
$$\text{Tan}.\frac{\omega}{2} = \frac{\cos \frac{\lambda - \lambda'}{2}}{\sin \frac{\lambda + \lambda'}{2}} \times \tan \frac{m + m'}{2}.$$

This demonstration comprehends the elliptical spheroid as a particular case.

When the two latitudes are equal, $\cos \phi = \cos \phi'$; and we learn from the equations (A') that m and m' are respectively equal to one and another to μ and μ'. But the formula for the difference of longitude is true, independently of the situation of the stations.

In an elliptical spheroid when the latitudes are equal, the excentricity disappears from the equations (A'), and it is therefore indeterminate. And when the latitudes are very nearly, although not exactly, equal, there is so near an approach to the condition which makes the excentricity indeterminate, that no dependence can be practically placed on any result respecting the figure of the earth obtained by means of the equations, or by means of the angles they contain. In order to find the excentricity, we must have recourse to the measured distance between the stations, as I have pointed in the last Number of this Journal.

If we put s for the geodetical line between the stations on the spheroid, and represent by σ a line traced on the surface of the sphere, in such a manner that every two points of s and σ that are upon the same meridian, have the same latitude; then the sum of the three angles of the triangle on the surface of the spheroid, that is, the sum of ω and the inclinations of s to the meridians at its extreme points, will exceed 180° by a quantity proportional to the surface of the trilateral figure on the

the sphere contained by σ and the two meridians. This proposition is rigorously true, and is only a particular case of a more general theorem demonstrated by Professor Gauss in the Memoirs of the Royal Society of Göttingen: but it must be observed that the trilateral figure on the surface of the sphere is not a spherical triangle, because the line σ is not contained in the plane of any great circle. And as the line σ depends upon the geodetical line s, which, supposing the latitudes and the difference of longitude to remain unchanged, varies with the excentricity, it follows necessarily that the sum of the azimuths at the extremities of s, is not independent of the excentricity, but varies from one spheroid to another. The sum of the azimuths mentioned is not, in any spheroid, exactly equal to the sum of the angles at the base of a triangle on the surface of the sphere formed by the two meridians, and a great circle which cuts them at the latitudes of the stations.

Jan. 13, 1829. J. Ivory.

XVII. *On the Nature of Light and Shadow, demonstrating that a Black Shadow can be rarefied, without Refraction, into all the Colours of the Rainbow.* By Joseph Reade, M.D.

To the Editors of the Philosophical Magazine and Annals.

Gentlemen,

I BEG leave to return you my thanks for the correctness with which you have published my experiments on light*, and I hope the following novel experiment may be favourably received by your scientific readers.

Experiment 1.—Having placed a table at about ten feet from a well lighted window, I placed on it a candle in a high candlestick. I now held a quire of white paper parallel to the table, and at right angles with the lighted candle: on holding this paper rather close to the blaze, two shadows were produced by means of a piece of coiled paper held immediately near the quire; the one next to the candle was a bright orange, the other a bright blue. On turning the quire of paper towards the window, so as to cut off the light of the candle, this orange shadow changed to a perfect black; and on turning the quire of paper towards the candle, and excluding the light of the sun, the blue shadow likewise changed to a perfect black. Here I changed orange and blue colours into black, and *vice versâ*, without any possibility of refraction. This experiment may be made by holding the paper behind the candle.

Experiment 2.—The former experiment was made with the paper between the candle and the window: I now held the paper close behind the candle, and perceived two shadows, the one

* See Phil. Mag. vol. lxiii. p. 27, &c.