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YANG-MILLS THEORIES WITH LOCAL SUPERSYMMETRY: LAGRANGIAN, TRANSFORMATION LAWS AND SUPERHIGGS EFFECT

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ABSTRACT

We derive the Lagrangian and transformation laws of the coupled Yang-Mills-matter-supergravity system for unextended n = 1 local supersymmetry. We study the superHiggs effect and the normal Higgs effect of the Yang-Mills gauge group G. In the case of N chiral multiplets "minimally" coupled to supergravity, transforming according to some N-dimensional, generally reducible representation of G, we find a model independent mass formula:

$$\text{Supertrace}\,\mathcal{M}^2 = \sum_{J=0}^{3/2}(-)^{2J}(2J+1)m_J^2 = (N-1)(2m_{3/2}^2-\kappa^2 D^\alpha D^\alpha) + 2\tilde{g}_\alpha D^\alpha \mathrm{Tr}T^\alpha$$

where $m_{3/2}$ is the gravitino mass, κ and $\tilde{g}_\alpha$ the gravitational and gauge couplings, respectively, D^α is the auxiliary component of the gauge multiplet of G and T^α the generators of G in the representation of the matter chiral multiplets.

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1. - INTRODUCTION

Supersymmetric quantum field theories have recently received much attention as possible candidates for the unification of the fundamental forces of Nature. These theories share exceptional properties at the quantum level which are manifest in a softening of quantum divergences due to corrections of infinities between boson and fermion loops.

Non-renormalization theorems[1] in supersymmetric Yang-Mills theories imply that certain ratios of coupling constants are non-renormalized when supersymmetry is unbroken. Their renormalization in the presence of breaking is therefore proportional to the supersymmetry breaking scale.

These theorems have been advocated in connection with the problem of the naturalness of scales of fundamental interactions and in particular with the so-called hierarchy problem of Grand Unified Theories[2].

The absence of quadratic mass renormalization of scalar fields in supersymmetric gauge theories without traceable $U(1)$ factors, offers us a possible way of understanding the small scale of the broken $SU(2) \times U(1)$ gauge theory of electroweak interactions in spite of the fact that a unified theory certainly contains much bigger scales such as the grand unification scale or the Planck mass. It has been realized that this does not necessarily mean that the scale of supersymmetry breaking, related to the vacuum energy of a spontaneously broken supersymmetric theory, should be small[3].

The main difficulty in constructing a realistic model where supersymmetry is spontaneously broken is to get the correct mass spectrum for the scalar partners of quarks and leptons as well as the correct breaking pattern of the gauge symmetry of low-energy physics with $SU(3) \times SU(2) \times U(1)$ broken down to $SU(3) \times U(1)_{em}$. Since it seems that supersymmetry survives radiative corrections and it is not known whether it can be broken dynamically, the only available models are based on a tree-level breaking. These models fall into two categories: the Fayet-Iliopoulos[4] breaking mechanism and the O'Raifeartaigh mechanism[5]. For different reasons, realistic models based on these two mechanisms seem to imply that the true scale of supersymmetry breaking m should be as large as 10^{10} - 10^{12} GeV [3]. This range of scales can be considered as the geometrical mean of μM_{Pl} : $m \simeq \sqrt{\mu M_{Pl}}$ where M_{Pl} is the Planck mass and μ is some small scale $\mu \simeq O(100\text{-}1000\ \text{GeV})$. When supersymmetry is broken at such a large scale m, one may think that gravitational effects are still negligible since the ratio m/M_{Pl} is very small. However, from our investigation it turns out that in general this is not the case.

To understand the effects of supergravity corrections to globally supersymmetric Yang-Mills theories, one must couple chiral multiplets and Yang-Mills multiplets to the supergravity multiplet which contains, as particle fields, the vierbein and the gravitino. This is the task of the present article. We give the Lagrangian and transformation laws of the particle fields under local supersymmetry of an arbitrary supersymmetric Yang-Mills theory coupled to $n = 1$ supergravity.

We cannot demand that the action be polynomial in the scalar fields of chiral multiplets since scalar fields can appear at arbitrary powers due to the presence of the Planck scale.

The requirement of renormalizability in flat space does not help because all non-polynomial interactions which are possible in the supergravity couplings of scalar fields and Yang-Mills fields are suppressed by inverse powers of M_{Pl} so that they disappear if we let $M_{Pl} \to \infty$.

We find that the most general Lagrangian consistent with local supersymmetry depends on two arbitrary functions. The first is an arbitrary real scalar function $\mathcal{G}(z,z^*)$, where z^i are the complex scalar components of the chiral multiplets of the theory. This function must be a scalar under the gauge group G and is the same as defined by Cremmer et al.[6]. The second is an analytic function $f_{\alpha\beta}(z)$ of the z^i's transforming as the symmetric product of the adjoint representation of G.

This result is obtained under the following conditions:

1) The action contains only terms up to second derivatives in the boson fields and first derivatives in the fermion fields.

2) The action contains only supersymmetric invariants which are quadratic in the multiplet of the Yang-Mills field strength.

In this article we do not consider the possibility of a Fayet-Iliopoulos term[7] and mass term[8] for the gauge multiplet which are also possible if the gauge group G contains some Abelian $U(1)$ factors.

The essentials of the results of the present investigation were reported in a previous short paper[9].

The present analysis reveals that gravitational effects in supersymmetric theories can play a crucial unexpected role, especially for the understanding of the supersymmetry breaking mechanism. In fact, when gravitation is turned on, supersymmetry becomes a gauge symmetry and the superHiggs mechanism[10], i.e., the mass generation for the spin 3/2 gravitino field occurs. The superHiggs effect in turn implies some relations between the gravitino mass and the other particle masses.

We demonstrate that the scalars can be raised with respect to their fermionic partners by an amount of $O(\mu)$ provided the gravitino mass is of the same order, i.e., supersymmetry is broken at a scale $m \simeq \sqrt{\mu M_{Pl}}$.

This result invalidates the mass formulae of globally supersymmetric theories[11] and therefore implies that models based on a large supersymmetry breaking must be reconsidered when gravitational effects are present.

The present article is organized as follows: in Section 2 we give the main properties of tensor calculus of local supersymmetry which have been used to derive the final Lagrangian and transformation laws. In Section 3 we give the invariant terms which appear in the Lagrangian before the elimination of the auxiliary field components of the several multiplets involved. The terms which contribute to the action are, at this stage, separately invariant under local supersymmetry transformations.

In Section 4 we eliminate the auxiliary fields and perform Weyl rescalings and chiral rotations which enable us to recast the Lagrangian and transformation laws in the final form, when only physical fields are present. In Section 5 we discuss the super Higgs mechanism and mass formulae of spontaneously broken local supersymmetry.

2. - ELEMENTS OF THE TENSOR CALCULUS

Our action and transformation laws are constructed by using the $N = 1$ tensor calculus for supergravity[12]. We will use superconformal notations as this considerably simplifies several rules. Indeed, as all fields in the $N = 1$ Weyl multiplet are gauge fields, they occur in covariant expressions only through covariant derivatives or curvatures. Therefore, the full superconformal transformation rules and multiplication rules of multiplets are just the linearized expressions with all derivatives replaced by superconformally covariant derivatives.

We recall that the $n = 1$ Weyl multiplet $(e_{a\mu}, \psi_\mu, A_\mu b_\mu)$ contains gauge fields for general co-ordinate transformations, Q supersymmetry, chiral U(1) and dilatations. The gauge fields for Lorentz rotations ω_{1mn}, S supersymmetry ϕ_μ and special conformal transformations $f_{\mu m}$ are solvable through the conventional constraints*).

$$
\begin{aligned}
& R_{\mu\nu}{}^{m}(P) = 0 \\
& \gamma^\mu R_{\mu\nu}(Q) = 0 \\
& \hat{R}_{\mu\nu mn}(M)\, e^{\nu n} + \frac{i}{3}\, \tilde{R}_{\mu m}(A) = 0 \\
& \qquad \left(\tilde{T}_{mn} = \frac{1}{2}\, \varepsilon_{mnpq}\, T^{pq} \right)
\end{aligned}
\tag{2.1}
$$

The hat on the Lorentz connection $R_{\mu\nu mn}(M)$ denotes full covariantization. We note that curvatures are covariant, but since the constraints are not Q invariant (S invariance determines the coefficient in the last constraint), $\delta_Q \omega_{\mu mn}$, $\delta_Q \phi_\mu$ and $\delta_Q f_{\mu m}$ do not have their group-theoretical rule. $\hat{R}_{\mu\nu mn}(M)$ is covariant using the new $\delta_Q \omega_{\mu mn}$ [14].

We use two basic multiplets. The scalar (or chiral) multiplet S with Weyl weight ω (this is the Weyl weight of the lowest field: z) transforms as

$$
\begin{aligned}
\delta z &= \bar{\epsilon}_L \chi_L - i\omega \frac{\alpha}{3} z \\
\delta \chi_L &= \frac{1}{2} \tilde{\not{D}} z\, \epsilon_R + \frac{1}{2} h\, \epsilon_L + \omega z\, \eta_L + i\alpha \left(\frac{1}{2} - \frac{\omega}{3} \right) \chi_L \\
\delta h &= \bar{\epsilon}_R \tilde{\not{D}} \chi_L + 2 \bar{\eta}_L \chi_L (1-\omega) + i\alpha \left(1 - \frac{\omega}{3} \right) h
\end{aligned}
\tag{2.2}
$$

The parameter ε differs from that of Ref. 6) by a factor 1/2. α is the parameter of the U(1) group of the superconformal algebra (R symmetry), normalized such that $\delta A_\mu = \partial_\mu \alpha$; η is the parameter of S symmetry and ε that of Q

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*) In this paper we use the conventions as given in Ref. 13). We also use the following conventions:
a = 1,2 two component spinor indices
m = 1,...,4 Lorentz indices
μ = 1,...,4 world indices
α = 1,...,dim G group indices
i,j = 1,...,dim R representation indices. R is a representation of G. We also set in our formulae the gravitational coupling constant $\kappa = 1$. κ is proportional to M_{Pl}^{-1} where M_{Pl} is the Planck mass.

symmetry. All parameters are local. $\epsilon_{L,R} = (1\pm\gamma_5)/2\ \epsilon$. $\hat{D}_\mu$ is a superconformal covariant derivative. For $\omega = 3$, h has Weyl weight 4 and can be extended to a superconformal density.

$$e^{-1}\mathcal{L}_{S(\omega=3)} = h + \bar{\Psi}_R \cdot \gamma\, \chi_L + \bar{\Psi}_{\mu R}\, \sigma_{\mu\nu}\, \Psi_{\nu R}\, z \tag{2.3}$$

The second multiplet is the vector multiplet V. Its transformation for arbitrary Weyl weight is[15)]

$$\begin{aligned}
\delta C &= \tfrac{i}{2}\bar{\epsilon}_L \zeta_L - \tfrac{i}{2}\bar{\epsilon}_R \zeta_R \\
\delta \zeta_L &= \tfrac{i}{2}\mathcal{H}\epsilon_L - \tfrac{1}{2}\not{B}\epsilon_R - \tfrac{i}{2}\tilde{\not{D}}C\,\epsilon_R - i\omega C\eta_L + \tfrac{i}{2}\alpha\zeta_L \\
\delta \mathcal{H} &= -i\bar{\epsilon}_R\tilde{\not{D}}\zeta_L - i\bar{\epsilon}_R\lambda_R + i(-2+\omega)\bar{\eta}_L\zeta_L + i\alpha\mathcal{H} \\
\delta B_m &= \left(-\tfrac{1}{2}\bar{\epsilon}_L\hat{D}_m\zeta_L - \tfrac{1}{2}\bar{\epsilon}_L\gamma_m\lambda_R + \tfrac{1}{2}(1+\omega)\bar{\eta}_L\gamma_m\zeta_R\right) + h.c. \\
\delta\lambda_R &= \tfrac{1}{2}(\sigma\cdot\hat{F} - iD)\,\epsilon_R + \tfrac{\omega}{2}(i\mathcal{H}\eta_R - \not{B}\eta_L + i\tilde{\not{D}}C\eta_L) + \tfrac{i}{2}\alpha\lambda_R - \omega\not{\Lambda}_K\zeta_L \\
\delta D &= \left(\tfrac{i}{2}\bar{\epsilon}_L\tilde{\not{D}}\lambda_R + i\omega\bar{\eta}_L(\lambda_L + \tfrac{1}{2}\tilde{\not{D}}\zeta_R) - \omega\Lambda_{Km}\hat{D}_m C\right) + h.c.
\end{aligned}$$

$$\hat{F}_{mn} = 2\hat{D}_{[m}B_{n]} + i\,\epsilon_{mnpq}\hat{D}_p\hat{D}_q C \qquad ;\ [mn] = \tfrac{1}{2}(mn - nm) \tag{2.4}$$

Λ_{Km} is the parameter of the special conformal transformations. Our rule of tensor calculus, which corresponds to the superspace rule $\epsilon^{ab}D_aD_bV$, allows us to obtain a scalar multiplet with $\omega = 3$ from a vector multiplet with $\omega = 2$

$$\left(-\mathcal{H}^*,\ -i\lambda_L - i\tilde{\not{D}}\zeta_R,\ D + \hat{\Box}C + i\hat{D}_m B^m\right) \tag{2.5}$$

Since this scalar multiplet has $\omega = 3$, we can insert (2.5) into (2.3) to obtain a density formula for a vector multiplet with $\omega = 2$.

$$e^{-1}\mathcal{L}_{V(\omega=2)} = D + \hat{\Box}C + \left(\tfrac{i}{2}\bar{\Psi}_L\cdot\gamma(\lambda_R + \tilde{\not{D}}\zeta_L) - \tfrac{1}{2}\bar{\Psi}_{\mu_L}\sigma_{\mu\nu}\Psi_{\nu_L}\mathcal{H} + h.c.\right) \tag{2.6}$$

Conversely, one can get a vector multiplet with $\omega = 0$ from a scalar multiplet with $\omega = 0$. The rule is

$$i(S - \bar{S}) = \left(i(z - z^*),\ 2\chi_L,\ -2ih,\ -\hat{D}_m(z + z^*),\ 0,\ 0\right) \tag{2.7}$$

The multiplication rules of scalar and vector multiplets are

$$
\begin{aligned}
S_1 S_2 &= \left(z_1 z_2 \,,\, z_1 \chi_{2L} + z_2 \chi_{1L} \,,\, z_1 h_2 + z_2 h_1 - 2\,\bar{\chi}_{1L}\chi_{2L}\right) \\
V_1 V_2 &= \Big(C_1 C_2 \,,\, C_1 \zeta_2 + C_2 \zeta_1 \,,\, C_1 \mathcal{H}_2 + C_2 \mathcal{H}_1 - \bar{\zeta}_{1L}\zeta_{2L} \,, \\
&\qquad C_1 B_{m2} + \tfrac{i}{2}\bar{\zeta}_{1L}\gamma_m \zeta_{2R} + (1\leftrightarrow 2) \,,\, C_1 \lambda_{2R} + \tfrac{1}{2}\left(-\not{\partial} C_1 \zeta_{2L} + \mathcal{H}_1 \zeta_{2R} - i \not{B}_1 \zeta_{2L}\right) + (1\leftrightarrow 2) \,, \\
&\qquad C_1 D_2 - \bar{\zeta}_{1L}\lambda_{2L} - \bar{\zeta}_{1R}\lambda_{2R} + \tfrac{1}{2}\mathcal{H}_1 \mathcal{H}_2^* - \tfrac{1}{2} B_1 \cdot B_2 \\
&\qquad - \tfrac{1}{2}\hat{D}_m C_1 \hat{D}^m C_2 - \tfrac{1}{2}\bar{\zeta}_{1L}\not{\partial}\zeta_{2R} - \tfrac{1}{2}\bar{\zeta}_{1R}\not{\partial}\zeta_{2L} + (1\leftrightarrow 2)\Big) \\
\tfrac{1}{2}\left(S_1 \bar{S}_2 + \bar{S}_1 S_2\right) &= \Big(\tfrac{1}{2} z_1 z_2^* \,,\, -i z_2^* \chi_{1L} \,,\, -z_2^* h_1 \,,\, \tfrac{i}{2}\left(z_2^* \hat{D}_m z_1 - z_1 \hat{D}_m z_2^*\right) - i\bar{\chi}_{2R}\gamma_m \chi_{1L} \,, \\
&\qquad -i h_1 \chi_{2R} + i \not{\partial} z_2^* \chi_{1L} \,,\, h_1 h_2^* - \hat{D}_m z_1 \hat{D}^m z_2^* - \bar{\chi}_{1L}\overleftrightarrow{\not{\partial}}\chi_{1R}\Big) + (1\leftrightarrow 2)
\end{aligned}
\tag{2.8}
$$

For each multiplication, the resulting multiplet has Weyl weight $\omega_1 + \omega_2$. The last multiplication in (2.8) is possible only if S_1 and S_2 have the same chiral weight. So far we have given the basic properties of superconformal rules. However, most actions we want to construct are not superconformal invariant. In particular, they do not have the right Weyl weight. Therefore we now introduce a compensating multiplet with $\omega = 1$. Multiplication with this compensating multiplet changes the Weyl weight and this can be used to adapt the weight to its right value.

The choice of this compensating multiplet determines the set of auxiliary fields for $n = 1$ Poincaré supergravity. The compensating multiplet we choose is a scalar multiplet which is related to the minimal set of auxiliary fields of $n = 1$ Poincaré supergravity[16].

The compensating scalar multiplet has components

$$
\mathcal{E} = \left(a \,,\, \xi \,,\, \tfrac{1}{3} u\right) \tag{2.9}
$$

with $\omega = 1$. u is the complex scalar auxiliary field of Poincaré supergravity as defined in Ref. 6). Poincaré supergravity is obtained by fixing the gauges of dilatation, U(1), S supersymmetry and special conformal transformations:

$$
a = 1 \quad , \quad \xi = 0 \quad , \quad b_\mu = 0 \tag{2.10}
$$

These conditions are invariant only under the new (Poincaré) supersymmetry

$$
\begin{aligned}
\delta^P(\epsilon) &= \delta_Q(\epsilon) + \delta_S\left(\eta_L = -\tfrac{1}{6} u\, \epsilon_L - \tfrac{i}{6}\not{A}\epsilon_R\right) \\
&\quad + \delta_K\left(\Lambda_{K\mu} = \tfrac{1}{4}\bar{\epsilon}\phi_\mu - \tfrac{1}{4}\bar{\eta}\psi_\mu\right)
\end{aligned}
\tag{2.11}
$$

Equation (2.11) gives the Poincaré local supersymmetry transformations in terms of local superconformal transformations. We can now construct a density formula for the transformation given by (2.11) of a scalar (chiral) multiplet S with $\omega = 0$. We first construct $S\mathcal{E}^3$, which has $\omega = 3$ and then use Eq. (2.3). In the Poincaré gauge defined by (2.10) we have

$$\mathcal{E}^n = \left(1,\ 0,\ \frac{n}{3}u\right) \tag{2.12}$$

so that [using (2.8)]

$$S\mathcal{E}^n = \left(z,\ \chi,\ h + \frac{n}{3}uz\right) \tag{2.13}$$

The Poincaré density formula is therefore given by[12)]

$$e^{-1}\mathcal{L}_{S(\omega=0)} = h + uz + \bar{\Psi}_R \cdot \gamma\, \chi_L + \bar{\Psi}_{\mu R}\, \sigma^{\mu\nu} \psi_{\nu R}\, z \tag{2.14}$$

The same trick can be done for a vector multiplet. $\mathcal{E}\bar{\mathcal{E}}$ is a vector multiplet with $\omega = 2$. In the Poincaré gauge, using (2.8), we have

$$\mathcal{E}\bar{\mathcal{E}} = \left(1, 0, -\frac{2}{3}u, -\frac{2}{3}A_m, 0, \frac{2}{9}(uu^* - A_m A^m)\right) \tag{2.15}$$

If we now take a vector multiplet with Weyl weight $\omega = 0$, then $V\mathcal{E}\bar{\mathcal{E}}$ has Weyl weight $\omega = 2$ and we can use the density formula (2.6) for $V\mathcal{E}\bar{\mathcal{E}}$ to get a Poincaré density formula for the vector multiplet V [12)]

$$\begin{aligned} e^{-1}\mathcal{L}_{V(\omega=0)} = {} & D - \frac{i}{2}\bar{\Psi}\cdot\gamma\gamma_5\lambda - \frac{1}{3}(u^*\mathcal{H} + u\mathcal{H}^*) \\ & + \frac{2}{3}B_m A^m - \frac{i}{3}\bar{\zeta}\gamma_5\gamma\cdot R^P + \frac{i}{4}\varepsilon^{mnrs}\bar{\Psi}_m\gamma_n\psi_r\left(B_s - \frac{1}{2}\bar{\Psi}_s\zeta\right) \\ & - \frac{2}{3}C\, e^{-1}\mathcal{L}_{SG} \end{aligned} \tag{2.16}$$

with

$$R_\mu{}^P = e^{-1}\varepsilon_{\mu\nu}{}^{\rho\sigma}\gamma_5\gamma^\nu\Big[\partial_\rho + \tfrac{1}{2}\omega_{\rho mn}(e,\psi)\sigma^{mn} + \tfrac{i}{2}\gamma_5 A_\rho + \tfrac{1}{6}\gamma_\rho(u - i\not{A}\gamma_5)\Big]\psi_\sigma \tag{2.17}$$

$$\mathcal{L}_{SG} = -\tfrac{1}{2}e\,R(e,\omega(e,\psi)) - \tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho(\omega(e,\psi))\psi_\sigma - \tfrac{e}{3}uu^* + \tfrac{e}{3}A_m A^m \tag{2.18}$$

A comment is in order. The Poincaré density formula, as well as the Poincare transformation rules, are ω independent. If S has weight ω, then $S' = S\mathcal{E}^{-\omega}$ always has zero weight. In the Poincaré gauge its components can be read off from (2.13) ($h' = h - \omega/3\, uz$). This multiplet defines the ω independent Poincaré transformations[12]. For a vector multiplet this is done with $V' = V(\mathcal{E}\bar{\mathcal{E}})^{-\omega/2}$

$$\begin{aligned}
&\delta^P z = \bar{\epsilon}_L\chi_L \;;\; \delta^P\chi_L = \tfrac{1}{2}\tilde{\not{D}}^P z\,\epsilon_R + \tfrac{1}{2}h'\epsilon_L \\
&\delta^P h' = \bar{\epsilon}_R\tilde{\not{D}}^P\chi_L - \tfrac{1}{3}\bar{\epsilon}_L u\chi_L - \tfrac{i}{6}\bar{\epsilon}_R\not{A}\chi_L
\end{aligned} \tag{2.19}$$

$$\begin{aligned}
&\delta^P C = \tfrac{i}{2}\bar{\epsilon}_L\zeta_L + h.c. \;;\; \delta^P\zeta_L = \tfrac{i}{2}\mathcal{H}'\epsilon_L - \tfrac{1}{2}\not{B}'\epsilon_R - \tfrac{i}{2}\tilde{\not{D}}^P C\,\epsilon_R \\
&\delta^P\mathcal{H}' = -i\bar{\epsilon}_R\tilde{\not{D}}^P\zeta_L - i\bar{\epsilon}_R\lambda'_R + \tfrac{i}{3}\bar{\zeta}_L\epsilon_L u + \tfrac{1}{6}\bar{\zeta}_L\not{A}\epsilon_R \\
&\delta^P B'_m = -\tfrac{1}{2}\bar{\epsilon}_L(\hat{D}^P_m - \tfrac{i}{2}A_m)\zeta_L - \tfrac{1}{2}\bar{\epsilon}_L\gamma_m\lambda'_R + \tfrac{1}{12}\bar{\zeta}_R\gamma_m(u\epsilon_L + i\not{A}\epsilon_R) + h.c. \\
&\delta^P\lambda'_R = \tfrac{1}{2}(\sigma\cdot\hat{F}^P - iD')\epsilon_R \;;\; \delta^P D' = \tfrac{i}{2}\bar{\epsilon}_L(\not{D}^P - \tfrac{i}{2}\not{A})\lambda'_R + h.c. \\
&\hat{F}^P_{mn} = 2D_{[m}B'_{n]} + \bar{\psi}_m\hat{D}^P_n\zeta + \bar{\psi}_{[m}\gamma_{n]}\lambda' + \bar{\zeta}e^\mu_{[m}D_{n]}\psi_\mu \\
&\mathcal{H}' = \mathcal{H} + \tfrac{\omega}{3}Cu \;;\; B'_m = B_m + \tfrac{\omega}{3}CA_m \;;\; \lambda'_R = \lambda_R + \tfrac{\omega}{6}(u\zeta_R - i\not{A}\zeta_L) \\
&D' = D + \tfrac{\omega}{6}(\mathcal{H}u^* + \mathcal{H}^*u - 2B_mA^m + \tfrac{i}{2}\bar{\zeta}\gamma_5\gamma\cdot R^P) + \tfrac{\omega^2}{18}C(uu^* - A_mA^m)
\end{aligned} \tag{2.20}$$

This completes the tensor calculus of the $N = 1$ scalar (chiral) and vector multiplets.

We now introduce an extra invariance group. To make this group local, i.e., a Yang-Mills group, the parameter must belong to a supermultiplet. Therefore, one needs to introduce a scalar multiplet parameter corresponding to each generator of the group[17] and write

$$\Lambda = \tilde{g}\Lambda^\alpha T^{\alpha\,j}_i \tag{2.21}$$

$\tilde{g}$ is the gauge coupling constant, T^α are representation matrices and Λ^α are scalar multiplets of weight $\omega = 0$. If the group G is not simple $\tilde{g}$ stands for $\tilde{g}_\alpha$ where $\tilde{g}_\alpha$ refers to the simple and $U(1)$ factors of G. The components of Λ^α will be denoted by

$$\Lambda^{\alpha} = (y^{\alpha}, \rho^{\alpha}, v^{\alpha}) \tag{2.22}$$

For the connection we introduce a Lie algebra valued vector multiplet

$$V = \tilde{g}\, V^{\alpha} T_i^{\alpha j} \tag{2.23}$$

which, under a finite Yang-Mills transformation, transforms as

$$e^{2V} \to e^{-i\bar{\Lambda}}\, e^{2V}\, e^{i\Lambda} \tag{2.24}$$

In the infinitesimal, (2.24) reads

$$\delta V = \frac{i}{2}(\Lambda - \bar{\Lambda}) + \frac{i}{2}[V, \Lambda + \bar{\Lambda}] + \frac{i}{6}[V,[V, \Lambda - \bar{\Lambda}]] + \mathcal{O}(V^3) \tag{2.25}$$

If we want to discuss a theory invariant under a Yang-Mills group, we can take gauge choices for all transformations in Λ but a real scalar. This can be done by using the Wess-Zumino gauge[18] in which

$$C^{\alpha} = \zeta^{\alpha} = \mathcal{H}^{\alpha} = 0 \tag{2.26}$$

for the components of V^{α}.

Equations (2.26) are not invariant under supersymmetry and gauge transformations. In fact, under supersymmetry transformations and (2.25), we get

$$\begin{aligned} \delta C^{\alpha} &= \frac{i}{2}(y - y^{*}) \\ \delta \zeta_L^{\alpha} &= \rho_L^{\alpha} - \frac{1}{2} \not{\partial} \epsilon_R \\ \delta \mathcal{H}^{\alpha} &= -i v^{\alpha} - \frac{i}{2} \bar{\epsilon}_R \gamma^{\mu} \not{\partial}^{\alpha} \psi_{\mu R} - i \bar{\epsilon}_R \lambda_R^{\alpha} \end{aligned} \tag{2.27}$$

Therefore the Wess-Zumino gauge is only invariant for $y = y^*$ (a real parameter which is the actual parameter of the Yang-Mills group) and under a new supersymmetry

$$\delta^{YM}(\epsilon) = \delta_Q(\epsilon) + \delta\left(\rho_L^\alpha = \tfrac{1}{2}\not{D}^\alpha \epsilon_R\right) + \delta\left(v^\alpha = -\tfrac{1}{2}\bar{\epsilon}_R \gamma^\mu \not{D}^\alpha \psi_{\mu R} - \bar{\epsilon}_R \lambda_R^\alpha\right) \tag{2.28}$$

If we apply this rule to δB_μ, $\delta\lambda$ and δD we get

$$\delta B_\mu = -\tfrac{1}{2}\bar{\epsilon}_L \gamma_\mu \lambda_R - \tfrac{1}{2}\bar{\epsilon}_R \gamma_\mu \lambda_L - D_\mu y$$

$$\delta \lambda_R = \tfrac{1}{2}(\sigma \cdot \hat{F} - i D)\epsilon_R + i[\lambda_R, y] \tag{2.29}$$

$$\delta D = \tfrac{i}{2}\bar{\epsilon}_L \hat{\not{D}} \lambda_R - \tfrac{i}{2}\bar{\epsilon}_R \hat{\not{D}} \lambda_L + i[D, y]$$

where $\hat{F}_{\mu\nu}$ and $\hat{D}_\mu$ are now covariant with respect to local supersymmetry and the Yang-Mills group.

For scalar multiplets S_i which transform according to some (generally complex) representation of the group G

$$S \to e^{-i\Lambda} S \tag{2.30}$$

or in the infinitesimal

$$\delta S = -i\Lambda S \tag{2.31}$$

we find in components

$$\delta S = -i\left(yz,\ y\chi_L + \rho_L z,\ yh + vz - \bar{\rho}_L \chi_L\right) \tag{2.32}$$

Therefore, from (2.28) we find that in the transformation law given by (2.2) all derivatives are replaced by Yang-Mills covariant derivatives and in the transformation law of δh there is an extra term[19] due to the last term in Eq. (2.28):

$$\delta^{extra} h = i\bar{\epsilon}_R \lambda_R z = i\tilde{g}\,\bar{\epsilon}_R \lambda_R^\alpha T_i^{\alpha\, j} z_j \tag{2.33}$$

Another useful formula which allows us to compute $S_1 \times S_2$ as given by the first equation in (2.8) using chiral multiplication instead of vector multiplication is

$$S_1\, T(S_2) + S_2\, T(S_1) \tag{2.34}$$

where $T(S)$ is the kinetic multiplet as introduced in Ref. 6). $T(S)$ is the curved generalization of the superfield $\bar{D}\bar{D}\bar{S}$ of flat supersymmetry.

The extension of the components of $T(S)$ to a Yang-Mills covariant expression, in the Wess-Zumino gauge, is given by

$$\begin{aligned}
T(S_i) &= \left(h^{*i} + \tfrac{1}{3} u^* z^{*i},\ \tilde{\Psi}_L^{\,i},\ \mathcal{H}^i \right) \\
\tilde{\Psi}_L^{\,i} &= \hat{\not{D}}^{\rho} \chi_R^{\,i} + \tfrac{i}{6} \not{A} \chi_R^{\,i} + \tfrac{1}{6} \gamma \cdot R_R^{\rho} z^{*i} - i \tilde{g}\, z^{*j} T_j^{\alpha\, i} \lambda_L^{\alpha} \\
\mathcal{H}^i &= \hat{\Box}^c z^{*i} - \tfrac{2}{3} u \left(h^{*i} + \tfrac{1}{3} u^* z^{*i} \right) + \tilde{g}\, z^{*j} T_j^{\alpha\, i} D^{\alpha} \\
&\quad - 2 i \tilde{g}\, \bar{\chi}_R^{\,j} T_j^{\alpha\, i} \lambda_R^{\alpha}
\end{aligned} \tag{2.35}$$

where $\hat{\Box}^c$ is the conformal d'Alembertian[6)] with derivatives replaced by Yang-Mills covariant derivatives. The density formula for the gauge covariantized expression of (2.34) can then be computed using the chiral density formula and it gives

$$\begin{aligned}
& e^{-1} \mathcal{L} \left(c^i{}_j\, S_{1i}\, T(S_{2j}) + (c^i{}_j)^*\, S_{2j}\, T(S_{1i}) \right) = \\
& c^i{}_j \Big\{ \tfrac{1}{6} \left(R + e^{-1} \varepsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\sigma \right) z_{1i}\, z_2^{*j} \\
& \quad - \hat{D}_\mu z_{1i}\, \hat{D}^\mu z_2^{*j} - \bar{\chi}_{1Li} \not{D} \chi_{2R}^{\,j} - \bar{\chi}_{2R}^{\,j} \not{D} \chi_{1Li} \\
& \quad + \left(h_{1i} + \tfrac{1}{3} u z_{1i} \right) \left(h_2^{*j} + \tfrac{1}{3} u^* z_2^{*j} \right)
\end{aligned}$$

$$
\begin{aligned}
&- \frac{1}{9} A_m^2 z_{1i} z_2^{*j} + \frac{i}{3} A^m \left(- \bar{\chi}_{1Li} \gamma_m \chi_{2R}^j + z_2^{*j} \hat{D}_m z_{1i} - z_{1i} \hat{D}_m z_2^{*j} \right) \\
&+ \frac{e^{-1}}{8} \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_\nu \psi_\rho \left(z_{1i} \hat{D}_\sigma z_2^{*j} - z_2^{*j} \hat{D}_\sigma z_{1i} \right) \\
&+ \frac{1}{3} \bar{\chi}_{1Li} \sigma^{\nu\mu} D_\mu \psi_\nu z_2^{*j} + \frac{1}{3} \bar{\chi}_{2R}^j \sigma^{\nu\mu} D_\mu \psi_\nu z_{1i} \\
&+ \bar{\chi}_{1Li} \sigma^{\nu\mu} \psi_\nu \hat{D}_\mu z_2^{*j} + \bar{\chi}_{2R}^j \sigma^{\nu\mu} \psi_\nu \hat{D}_\mu z_{1i} \\
&- D_\mu \bar{\chi}_{1Li} \sigma^{\nu\mu} \psi_\nu z_2^{*j} - D_\mu \bar{\chi}_{2R}^j \sigma^{\nu\mu} \psi_\nu z_{1i} \\
&+ 2 i \tilde{g} T_i^{\alpha k} \bar{\lambda}_L^\alpha \chi_{1Lk} z_2^{*j} - 2 i \tilde{g} \bar{\chi}_{2R}^k T_k^{\alpha j} \lambda_R^\alpha z_{1i} \\
&+ \tilde{g} D^\alpha T_i^{\alpha k} z_{1k} z_2^{*j} \\
&- \frac{i}{2} \tilde{g} \bar{\psi} \cdot \gamma \gamma_5 \lambda^\alpha T_i^{\alpha k} z_{1k} z_2^{*j} \Big\} \\
&+ h.c.
\end{aligned}
\tag{2.36}
$$

where $C^i{}_j z_{1i} z^{*j}{}_2$ is invariant under G, i.e.,

$$
c^i{}_j T_i^{\alpha k} z_{1k} z_2^{*j} - c^i{}_j T_k^{\alpha j} z_{1i} z_2^{*k} = 0 \tag{2.37}
$$

The other combination

$$Re\left[\mathcal{L}\left(c^i{}_j\, S_{1i}\, T(S_{2j}) - (c^i{}_j)^*\, S_{2j}\, T(S_{1i})\right)\right] \tag{2.38}$$

is easily seen to be a total divergence. Equation (2.36) extends the rule used in Ref. 6) in order to obtain the generalized kinetic terms of scalar fields.

3. - CONSTRUCTION OF THE ACTION

In this section we construct the action of the coupled Yang-Mills-matter-supergravity system. We start by considering the flat action invariant under global supersymmetry. Its superspace expression is given by[20)]

$$\int d^4x\, d^4\theta\;\; \Phi(S,\bar{S}) + \int d^4x\;\; Re \int d^2\theta\;\; g(S) \tag{3.1}$$

where $\phi(S,\bar{S})$ is an arbitrary real function of the chiral multiplets S_i and their complex conjugate $\bar{S}^i$ and $g(S)$ is an arbitrary function of S^i often called "superpotential".

The function $\phi(S,\bar{S})$ corresponds to a non-polynomial modification of the action for the kinetic term. The latter is normally given by $S\bar{S}$. The second term in (3.1) is a generalized Wess-Zumino interaction. If the chiral multiplets S^i transform according to some representation of a compact Lie group G, the action is assumed to be G invariant. This requires that the superfields $\phi(S,\bar{S})$ and $g(S)$ be G invariant. In terms of the first components of the vector multiplet $\phi(S,\bar{S})$ and of the chiral multiplet $g(S)$ this implies

$$\begin{gathered} z^{*j}\, T_j^{\alpha\, i}\, \phi'_i = \phi'^j\, T_j^{\alpha\, i}\, z_i \\ g'^j\, T_j^{\alpha\, i}\, z_i = 0 \\ \left(\phi'^i = \frac{\partial\phi}{\partial z_i}\;;\; \phi'_j = \frac{\partial\phi}{\partial z^{*j}}\;;\; g'^j = \frac{\partial g}{\partial z_j}\right) \end{gathered} \tag{3.2}$$

The flat actions corresponding to (3.1) have been given, in component form, in Refs. 21) and 11), respectively.

In order to extend the action given by (3.1) to local supersymmetry and local Yang-Mills invariance, the following modifications are needed[9]:

$$\int d^4x\, d^4\theta\, E\left[\Phi\left(S, \bar{S}\, e^{2V}\right) + Re\left(\frac{1}{R}\, g(S)\right)\right] \tag{3.3}$$

where V is the gauge multiplet of G, E is the superspace determinant and R is the chiral scalar curvature superfield[20]. The component form of Eq. (3.3) was obtained in Ref. 6) in the particular case $\tilde{g} = 0$ and for a single chiral multiplet S.

The remaining part of the Yang-Mills supergravity coupling corresponds to the supergravity modification of the flat Yang-Mills action:

$$\int d^4x\; Re \int d^2\theta\, \left(W_a^{\alpha}\, \varepsilon^{ab}\, W_b^{\alpha} \right) \tag{3.4}$$

where W_a^{α} is a gauge-covariant chiral superfield which contains the vector field strength[17]. Since W_a^{α} is gauge covariant, it is called the super-covariant field strength multiplet. The modification of (3.4) to non-polynomial interactions with scalar fields is[9]

$$\int d^4x\; Re \int d^2\theta\, \left(f_{\alpha\beta}(S)\; W_a^{\alpha}\, \varepsilon^{ab}\, W_b^{\beta} \right) \tag{3.5}$$

where $f_{\alpha\beta}(S)$ is a chiral superfield transforming as the symmetric product of the adjoint representation of G. Since (3.5) is a chiral action, generalization to curved space reads

$$\int d^4x\, d^4\theta\, E\, Re\left(\frac{1}{R}\, f_{\alpha\beta}(S)\; W_a^{\alpha}\, \varepsilon^{ab}\, W_b^{\beta} \right) \tag{3.6}$$

A non-trivial function $f_{\alpha\beta}(z) \neq \delta_{\alpha\beta}$ is known to be present in all n extended supergravities with $n \geq 4$.

In order to write the component form of (3.3) and (3.6) we have to work out the components of the vector multiplet

$$\Phi\,(S,\,\bar{S}\,e^{2V}) \tag{3.7}$$

and of the chiral multiplets with conformal weight $\omega = 0$ and $\omega = 3$, respectively

$$g(S) \tag{3.8}$$

$$f_{\alpha\beta}(S)\;W_a^{\alpha}\,\epsilon^{ab}\,W_b^{\beta} = f\,W^2 \tag{3.9}$$

and apply the density formulae of the previous sections. In the Wess-Zumino gauge $V^{\alpha}V^{\beta}V^{\gamma} = 0$ and (3.7) simplifies as follows

$$\begin{aligned}\Phi\,(S,\bar{S}\,e^{2V}) = \Phi\,(S,\bar{S}) &+ 2\tilde{g}\,\Phi_i'\,\bar{S}^j\,T_j^{\alpha\,i}\,V^{\alpha} \\ &+ 2\tilde{g}^2\,\Phi_{ij}''\,\bar{S}^k\,T_k^{\alpha\,i}\,\bar{S}^l\,T_l^{\beta\,j}\,V^{\alpha}V^{\beta}\end{aligned} \tag{3.10}$$

The components $C(\phi)$, $\zeta_L(\phi)$, $\mathcal{H}(\phi)$, $B_m(\phi)$ $\lambda_R(\phi)$ and $D(\phi)$ of the local vector multiplet in (3.10) are given by

$$\begin{aligned}
C(\phi) &= \phi(z,z^*) \\
\zeta_L(\phi) &= -2i\,\phi'^i\,\chi_{Li} \\
\mathcal{H}(\phi) &= -2\phi'^i h_i + 2\,\phi''^{ij}\,\bar{\chi}_{Li}\,\chi_{Lj} \\
B_m(\phi) &= i\,\phi'^i\,\hat{D}_m z_i - i\,\phi_i'\,\hat{D}_m z^{*i} - 2i\,\phi_i''^j\,\bar{\chi}_R^i\,\gamma_m\,\chi_{Lj} \\
\lambda_R(\phi) &= -2i\,\phi_i''^j h_j\,\chi_R^i + 2i\,\phi'''^{ij}_{\;\;k}\,\chi_R^k\,\bar{\chi}_{Li}\,\chi_{Lj} \\
&\quad + 2i\,\phi_i''^j\,\tilde{\not{D}}\,z^{*i}\,\chi_{Lj} + 2\tilde{g}\,\lambda_R^{\alpha}\,\tilde{D}^{\alpha}
\end{aligned} \tag{3.11}$$

$$
\begin{aligned}
D(\phi) = {} & 2\phi''^{\,i}_{\,j}\, h_i h^{*j} - 2\phi'''^{\,ij}_{\,k}\, \bar{\chi}_{Li}\chi_{Lj} h^{*k} - 2\phi'''^{\,k}_{ij}\, \bar{\chi}^i_R \chi^j_R h_k \\
& + 2\phi''''^{\,ij}_{\,k\ell}\, \bar{\chi}_{Li}\chi_{Lj}\, \bar{\chi}^k_R \chi^\ell_R - 2\phi''^{\,j}_{\,i}\, \hat{D}_\mu z^{*i} \hat{D}^\mu z_j \\
& - 2\phi''^{\,j}_{\,i}\, \bar{\chi}_{Lj} \hat{\not{D}} \chi^i_R - 2\left(\phi'''^{\,mi}_{\,jk}\, \hat{D}_\mu z^{*k} + \phi'''^{\,ik}_{\,j}\, \hat{D}_\mu z_k\right) \bar{\chi}_{Li}\gamma^\mu \chi^j_R \\
& + 2\tilde{g}\, D^\alpha \tilde{D}^\alpha + 4i\tilde{g}\, \bar{\lambda}^\alpha_L \tilde{D}'^{\alpha i} \chi_{Li} - 4i\tilde{g}\, \bar{\lambda}^\alpha_R D'^\alpha_i \chi^i_R
\end{aligned}
$$

$$
\left(\tilde{D}^\alpha = \phi'_i T^{\alpha\, i}_j z^{*j} = \phi'^i T^{\alpha\, j}_i z_j\right) \tag{3.11}
$$

(The derivatives are supersymmetric and Yang-Mills covariant.)
The components $z(g)$, $\chi_L(g)$, $h(g)$ and $z(fW^2)$, $\chi_L(fW^2)$ and $h(fW^2)$ of the two local chiral multiplets defined by (3.8) and (3.9) are given by

$$
\begin{aligned}
z(g) &= g(z) \quad ; \quad \chi_L(g) = g'^i \chi_{Li} \\
h(g) &= g'^i h_i - g''^{ij} \bar{\chi}_{Li}\chi_{Lj}
\end{aligned} \tag{3.12}
$$

$$
\begin{aligned}
z(fW^2) &= -\frac{1}{2} f_{\alpha\beta}\, \bar{\lambda}^\alpha_L \lambda^\beta_L \\
\chi_L(fW^2) &= \frac{1}{2} f_{\alpha\beta}\left(\sigma\cdot\hat{F}^{-\alpha} - iD^\alpha\right)\lambda^\beta_L - \frac{1}{2} f'^i_{\alpha\beta}\, \chi_{Li}\, \bar{\lambda}^\alpha_L \lambda^\beta_L \\
h(fW^2) &= f_{\alpha\beta}\left(-\bar{\lambda}^\alpha_L \hat{\not{D}} \lambda^\beta_R - \frac{1}{2}\hat{F}^{-\alpha}_{\mu\nu}\hat{F}^{-\beta}_{\mu\nu} + \frac{1}{2} D^\alpha D^\beta\right) \\
& \quad + f'^i_{\alpha\beta}\, \bar{\chi}_{Li}\left(-\sigma\cdot\hat{F}^{-\alpha} + iD^\alpha\right)\lambda^\beta_L - \frac{1}{2} f'^i_{\alpha\beta}\, h_i\, \bar{\lambda}^\alpha_L \lambda^\beta_L \\
& \quad + \frac{1}{2} f''^{\,ij}_{\alpha\beta}\, \bar{\chi}_{Li}\chi_{Lj}\, \bar{\lambda}^\alpha_L \lambda^\beta_L
\end{aligned} \tag{3.13}
$$

respectively.

To get local supersymmetric densities which correspond to the three multiplets given by (3.12), (3.13) and (3.11) we insert their components in the density formula for chiral and vector multiplets, as defined in Section 2 and finally obtain

$$
\begin{aligned}
e^{-1}\mathcal{L}(\phi) = & -\frac{1}{6}\phi\, e^{-1}\mathcal{L}_{SG} + \phi''^{i}_{j}\Big(-\frac{1}{2}D_\mu z_i D^\mu z^{*j} - \bar{\chi}_{Li}\not{D}\chi_R^j + \frac{1}{2}h_i h^{*j}\Big) \\
& - \phi'''^{ij}_{k}\,\bar{\chi}_{Li}\chi_{Lj}h^{*k} + \phi'''^{ij}_{k}\,\bar{\chi}_{Li}\not{D}z_j\chi_R^k + \frac{1}{2}\phi'''^{ij}_{kl}\,\bar{\chi}_{Li}\chi_{Lj}\bar{\chi}_R^k\chi_R^l \\
& + \frac{1}{3}u^*(\phi'^i h_i - \phi''^{ij}\bar{\chi}_{Li}\chi_{Lj}) + \frac{i}{3}A^\mu\Big(\frac{1}{2}\phi''^i_j\,\bar{\chi}_R^j\gamma_\mu\chi_{Li} + \phi'^i(D_\mu z_i - \bar{\psi}_{\mu L}\chi_{Li})\Big) \\
& + \phi''^i_j\,\bar{\psi}_{\mu L}\not{D}z^{*j}\gamma_\mu\chi_{Li} - \frac{4}{3}\phi'^i\bar{\chi}_{Li}\sigma^{\mu\nu}D_\mu\psi_{\nu L} \qquad (3.14) \\
& - \frac{1}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\,(\phi'^i D_\sigma z_i + \frac{1}{2}\phi''^j_i\,\bar{\chi}_R^i\gamma_\sigma\chi_{Lj}) \\
& - \frac{1}{2}\phi''^i_j\,\bar{\psi}_{\mu L}\chi_{Li}\bar{\psi}^\mu_R\chi_R^j \\
& + \frac{1}{6}\phi'^i\bar{\chi}_{Li}\Big(\psi_{\mu L}\bar{\psi}\cdot\gamma\psi^\mu + \frac{1}{2}\sigma^{\mu\nu}\psi_L^\rho\bar{\psi}_\nu\gamma_\rho\psi_\mu + \sigma^{\mu\nu}\psi_{\nu L}\bar{\psi}_\mu\gamma\cdot\psi\Big) \\
& + \frac{1}{2}\tilde{g}\phi'^i T_i^{\alpha j} z_j(D^\alpha + i\bar{\psi}_L\cdot\gamma\lambda_R^\alpha) - 2i\tilde{g}\phi''^j_i T_j^{\alpha k} z_k\,\bar{\lambda}_R^\alpha\chi_R^i \\
& + h.c.
\end{aligned}
$$

$$
\begin{aligned}
e^{-1}\mathcal{L}(g) = & -\frac{1}{2}g''^{ij}\bar{\chi}_{Li}\chi_{Lj} + \frac{1}{2}g'^i h_i + \frac{1}{2}gu + \frac{1}{2}\bar{\psi}_R\cdot\gamma\chi_{Li}g'^i \qquad (3.15) \\
& + \frac{1}{2}g\,\bar{\psi}_{\mu R}\sigma^{\mu\nu}\psi_{\nu R}
\end{aligned}
$$

$$
e^{-1}\mathcal{L}(f_{\alpha\beta}) = \frac{1}{2}f_{\alpha\beta}\Big(-\frac{1}{4}F^\alpha_{\mu\nu}F^\beta_{\mu\nu} - \frac{1}{2}\bar{\lambda}^\alpha\tilde{\not{D}}\lambda^\beta + \frac{1}{2}D^\alpha D^\beta \qquad (3.16)
$$

$$+\frac{1}{4}F^{\alpha}_{\mu\nu}\bar{\psi}_{\rho}\sigma^{\mu\nu}\gamma^{\rho}\lambda^{\beta}+\frac{1}{4}F^{\alpha}_{\mu\nu}\tilde{F}^{\beta}_{\mu\nu}-\frac{1}{2}D_{\mu}(\bar{\lambda}^{\alpha}_{L}\gamma^{\mu}\lambda^{\beta}_{R})\Big)$$

$$+\frac{1}{2}f'^{i}_{\alpha\beta}\Big(\bar{\chi}_{Li}(-\sigma\cdot\hat{F}^{-\alpha}+iD^{\alpha})\lambda^{\beta}_{L}-\frac{1}{2}h_{i}\bar{\lambda}^{\alpha}_{L}\lambda^{\beta}_{L} \qquad (3.16)\ \text{cont.}$$

$$-\frac{1}{2}\bar{\psi}_{R}\cdot\gamma\chi_{Li}\bar{\lambda}^{\alpha}_{L}\lambda^{\beta}_{L}\Big)+\frac{1}{4}f''^{ij}_{\alpha\beta}\bar{\chi}_{Li}\chi_{Lj}\bar{\lambda}^{\alpha}_{L}\lambda^{\beta}_{L}+h.c.$$

$\tilde{D}_{\mu}\lambda^{\alpha}$ is a covariant derivative with respect to supergravity and gauge transformations, but without the $\psi_{\mu}D^{\alpha}$ term. The other derivatives are Yang-Mills and general co-ordinate covariant, but have no ψ_{μ} torsion. The three terms given above give the sum of (3.3) and (3.4).

$$\mathcal{L}=\mathcal{L}(\phi)+\mathcal{L}(g)+\mathcal{L}(f_{\alpha\beta}) \qquad (3.17)$$

The three terms in (3.17) are separately invariant under local supersymmetry transformations and Yang-Mills transformations. The component form of the local supersymmetry transformations on the vector, chiral and gravitational multiplets are defined as follows

$$\delta B^{\alpha}_{\mu}=-\frac{1}{2}\bar{\epsilon}_{L}\gamma_{\mu}\lambda^{\alpha}_{R}-\frac{1}{2}\bar{\epsilon}_{R}\gamma_{\mu}\lambda^{\alpha}_{L}$$

$$\delta\lambda^{\alpha}_{R}=\frac{1}{2}\sigma^{\mu\nu}\hat{F}^{\alpha}_{\mu\nu}\epsilon_{R}-\frac{i}{2}D^{\alpha}\epsilon_{R} \qquad (3.18)$$

$$\delta D^{\alpha}=\frac{i}{2}\bar{\epsilon}_{L}(\tilde{\not{D}}^{\rho}-\frac{i}{2}\not{A})\lambda_{R}-\frac{i}{2}\bar{\epsilon}_{R}(\tilde{\not{D}}^{\rho}+\frac{i}{2}\not{A})\lambda_{L}$$

$$\delta z_{i}=\bar{\epsilon}_{L}\chi_{i}$$

$$\delta\chi_{i}=\frac{1}{2}\tilde{\not{D}}z_{i}\epsilon_{R}+\frac{1}{2}h_{i}\epsilon_{L} \qquad (3.19)$$

$$\delta h_{i}=\bar{\epsilon}_{R}(\tilde{\not{D}}^{\rho}-\frac{i}{2}\not{A})\chi_{Li}-\frac{1}{3}\bar{\chi}_{Li}(u\epsilon_{L}+i\not{A}\epsilon_{R})+i\tilde{g}\bar{\epsilon}_{R}\lambda^{\alpha}_{R}T^{\alpha j}_{i}z_{j}$$

$$\delta e_\mu{}^m = \frac{1}{2}\bar{\epsilon}_L \gamma^m \psi_{\mu R} + \frac{1}{2}\bar{\epsilon}_R \gamma^m \psi_{\mu L}$$

$$\delta \psi_{\mu L} = \left(\partial_\mu + \frac{1}{2}\omega_{\mu mn}(e,\psi)\sigma^{mn} + \frac{i}{2}A_\mu\right)\epsilon_L + \frac{1}{6}\gamma_\mu\left(u^*\epsilon_R - i\not{A}\epsilon_L\right) \tag{3.20}$$

$$\delta u = \frac{1}{2}\bar{\epsilon}_R \gamma\cdot R^P_L \quad ; \quad \delta A_\mu = \frac{3i}{4}\bar{\epsilon}_L\left(R^P_\mu - \frac{1}{3}\gamma_\mu \gamma\cdot R^P\right)_L + h.c.$$

In order to get the final form of the action, as derived in Ref. 6) for a single chiral multiplet, we have to eliminate the auxiliary fields h_i, D^α, u and A_m. Moreover, an appropriate Weyl rescaling on the vierbein, as well as field redefinitions on the fermion fields, have to be done in order to recast the supergravity part contained in the first term of (3.17) in canonical form and to obtain the final action in a simple form in terms of the particle fields $(B^\alpha_m, \lambda^\alpha_R)$, (z_i, χ_{Li}) and $(e_{a\mu}, \psi_\mu)$.

We end this section by showing, using superspace considerations, that the action given by the sum of (3.3) and (3.6) only depends on a particular combination of the functions $\Phi(S,\bar{S})$ and $g(S)$, namely on the superfield

$$\Phi'(S,\bar{S}) = \Phi(S,\bar{S}) / |g(S)|^{2/3} \tag{3.21}$$

To prove this we make a superWeyl rescaling on the supervierbein

$$E \to E' \exp(\Sigma + \bar{\Sigma}) \tag{3.22}$$

where Σ is a chiral parameter.

Under this rescaling the second term in (3.3) becomes

$$\int d^4x\, d^4\theta\ \mathrm{Re}\,\frac{E'}{R'} \exp 3\Sigma\ \ g(S) \tag{3.23}$$

If we now choose Σ such that

$$\exp 3\Sigma \; g(S) = 1 \quad \rightarrow \quad \exp \Sigma = g(S)^{-1/3} \tag{3.24}$$

the two terms in (3.3) become

$$\int d^4x \; d^4\theta \; E' \Phi'(S, \bar{S} e^{2V}) + \int d^4x \; Re \; d^4\theta \; \frac{E'}{R'} \tag{3.25}$$

The last term in (3.25) is a supersymmetric cosmological term. Under the redefinition (3.22) the action given by (3.5) is left invariant

$$\int d^4x \; d^4\theta \; Re \left(\frac{E'}{R'} f_{\alpha\beta}(S) \; W_a^{\alpha} \; \varepsilon^{ab} \; W_b^{\beta} \right) \tag{3.26}$$

Equations (3.25) and (3.26) indeed show that the final action only depends on two arbitrary functions of the fields z^i. A G invariant real function

$$\Phi'(z, z^*) = \Phi(z, z^*) \, / \, |g(z)|^{2/3} \tag{3.27}$$

and an analytic function transforming as the symmetric product of the adjoint representation of G

$$f_{\alpha\beta}(z) = f_{\beta\alpha}(z) \tag{3.28}$$

The function $\Phi'(z,z^*)$ is related, up to a multiplicative constant factor, to the function $\mathcal{G}(z,z^*)$, to be defined in the next section by the relation

$$\Phi'(z, z^*) = e^{\frac{1}{3}\mathcal{G}(z,z^*)} \tag{3.29}$$

4. - FINAL FORM OF THE ACTION AND TRANSFORMATION LAWS

The action which couples the matter-Yang-Mills supergravity multiplets is given by the sum of (3.14), (3.15) and (3.16). These terms contain the auxiliary fields h_i, D^{α},u and A_m which can be eliminated through their own field equations.

The part of the Lagrangian which contains the auxiliary fields is

$$
\begin{aligned}
e^{-1}\mathcal{L}_{AUX} = {} & \frac{1}{18}\phi(uu^* - A_m A^m) + \frac{1}{2}\phi''^{i}_{j} h_i h^{*j} - \phi'''^{ij}_{k} h^{*k}\bar{\chi}_{Li}\chi_{Lj} \\
& + \frac{1}{2} g'^{i} h_i - \frac{1}{4} f'^{i}_{\alpha\beta} h_i \bar{\lambda}^{\alpha}_{L}\lambda^{\beta}_{L} + \frac{1}{3}u^*\left(\phi'^{i}h_i - \phi''^{ij}\bar{\chi}_{Li}\chi_{Lj} + \frac{3}{2}g^*\right) \\
& + \frac{i}{3}A^{\mu}\left(\frac{1}{2}\phi''^{i}_{j}\bar{\chi}^{j}_{R}\gamma_{\mu}\chi_{Li} + \phi'^{i}(D_{\mu}z_i - \bar{\psi}_{\mu L}\chi_i) + \frac{3}{4}\bar{\lambda}^{\alpha}_{L}\gamma_{\mu}\lambda^{\beta}_{R} f_{\alpha\beta}\right) \\
& + \frac{1}{2}\tilde{g}\,\phi'^{i} T^{\alpha j}_{i} z_j D^{\alpha} + \frac{1}{4} f_{\alpha\beta} D^{\alpha}D^{\beta} + \frac{i}{2} f'^{i}_{\alpha\beta}\bar{\chi}_{Li}\lambda^{\alpha}_{L} D^{\beta} + h.c.
\end{aligned}
\tag{4.1}
$$

If we define the new expressions

$$
\mathcal{J} = 3\log\frac{-\phi}{3} \qquad ; \quad \tilde{u} = u + \mathcal{J}'^{i} h_i \tag{4.2}
$$

Eq. (4.1) then reads

$$
\begin{aligned}
e^{-1}\mathcal{L}_{AUX} = {} & \frac{1}{18}\phi(\tilde{u}\tilde{u}^* - A_m A^m) + \frac{1}{6}\phi\,\mathcal{J}''^{i}_{j} h_i h^{*j} + \frac{1}{2} g\left[\tilde{u} - h_i\left(\mathcal{J}'^{i} - \frac{g'^{i}}{g}\right)\right] \\
& - \frac{1}{9}\tilde{u}^*\left(\mathcal{J}''^{ij} + \frac{1}{3}\mathcal{J}'^{i}\mathcal{J}'^{j}\right)\phi\,\bar{\chi}_{Li}\chi_{Lj} - \frac{1}{3}\phi\, h^{*k}\bar{\chi}_{Li}\chi_{Lj}\left(\mathcal{J}'''^{ij}_{k} + \frac{2}{3}\mathcal{J}''^{i}_{k}\mathcal{J}'^{j}\right) \\
& - \frac{1}{4} f'^{i}_{\alpha\beta} h_i \bar{\lambda}^{\alpha}_{L}\lambda^{\beta}_{L} + \frac{i}{3}A^{\mu}\left(\frac{1}{2}\phi''^{i}_{j}\bar{\chi}^{j}_{R}\gamma_{\mu}\chi_{Li} + \phi'^{i}(D_{\mu}z_i - \bar{\psi}_{\mu L}\chi_i) + \frac{3}{4}\bar{\lambda}^{\alpha}_{L}\gamma_{\mu}\lambda^{\beta}_{R} f_{\alpha\beta}\right) \\
& + \frac{1}{2}\tilde{g}\,\phi'^{i} T^{\alpha j}_{i} z_j D^{\alpha} + \frac{1}{4} f_{\alpha\beta} D^{\alpha}D^{\beta} + \frac{i}{2} f'^{i}_{\alpha\beta}\bar{\chi}_{Li}\lambda^{\alpha}_{L} D^{\beta} + h.c.
\end{aligned}
\tag{4.3}
$$

The field equations are

$$
\begin{aligned}
\tilde{u} &= \frac{g}{\phi}\left[-\frac{1}{2}g^* + \frac{1}{9}\left(\mathcal{J}''^{ij} + \frac{1}{3}\mathcal{J}'^{i}\mathcal{J}'^{j}\right)\phi\,\bar{\chi}_{Li}\chi_{Lj}\right] \\
\frac{\phi}{3} h_i \mathcal{J}''^{i}_{k} &= -\frac{g^*}{2}\left(\frac{g^{*\prime}_{k}}{g^*} - \mathcal{J}'_{k}\right) + \frac{1}{3}\phi\left(\mathcal{J}'''^{ij}_{k} + \frac{2}{3}\mathcal{J}''^{i}_{k}\mathcal{J}'^{j}\right)\bar{\chi}_{Li}\chi_{Lj} \\
&\qquad + \frac{1}{4} f^{*\prime}_{\alpha\beta k}\bar{\lambda}^{\alpha}_{R}\lambda^{\beta}_{R} \\
\frac{2}{3}\phi A_{\mu} &= \frac{i}{2}\phi''^{i}_{j}\bar{\chi}^{j}_{R}\gamma_{\mu}\chi_{Li} + i\phi'^{i}(D_{\mu}z_i - \bar{\psi}_{\mu L}\chi_i) + \frac{3i}{4}\bar{\lambda}^{\alpha}_{L}\gamma_{\mu}\lambda^{\beta}_{R} f_{\alpha\beta} + h.c. \\
Re\, f_{\alpha\beta} D^{\beta} &= \tilde{g}\,\phi'^{i} T^{\alpha j}_{i} z_j + \frac{i}{2} f'^{i}_{\alpha\beta}\bar{\chi}_{Li}\lambda^{\beta}_{L} - \frac{i}{2} f^{*\prime}_{\alpha\beta i}\bar{\chi}^{i}_{R}\lambda^{\beta}_{R}
\end{aligned}
\tag{4.4}
$$

Insertion of (4.4) back into (4.1) gives

$$
\begin{aligned}
e^{-1}\mathcal{L}_{AUX} = & -\frac{9}{\phi}\left|\frac{g^*}{2} - \frac{1}{9}\left(\mathcal{J}''^{ij} + \frac{1}{3}\mathcal{J}'^{i}\mathcal{J}'^{j}\right)\phi\,\bar{\chi}_{Li}\chi_{Lj}\right|^2 \\
& - \frac{3}{\phi}\left(\mathcal{J}''^{-1}\right)^{k}_{\ell}\left[\frac{g^*}{2}\left(\frac{g'^{k*}}{g^*} - \mathcal{J}'_k\right) - \frac{\phi}{3}\left(\mathcal{J}''^{ij}_{k} + \frac{2}{3}\mathcal{J}''^{i}_{k}\mathcal{J}'^{j}\right)\bar{\chi}_{Li}\chi_{Lj} - \frac{1}{4}f^{*\prime}_{\alpha\beta k}\bar{\lambda}^{\alpha}_{R}\lambda^{\beta}_{R}\right] \\
& \qquad \left[\frac{g}{2}\left(\frac{g'^{\ell}}{g} - \mathcal{J}'^{\ell}\right) - \frac{\phi}{3}\left(\mathcal{J}''^{m\ell}_{mn} + \frac{2}{3}\mathcal{J}''^{\ell}_{m}\mathcal{J}'_n\right)\bar{\chi}^{m}_{R}\chi^{n}_{R} - \frac{1}{4}f'^{\ell}_{\gamma\delta}\bar{\lambda}^{\gamma}_{L}\lambda^{\delta}_{L}\right] \\
& - \frac{1}{4\phi}\left[\phi''^{i}_{j}\bar{\chi}^{j}_{R}\gamma_\mu\chi_{Li} + \phi'^{i}\left(D_\mu z_i - \bar{\Psi}_{\mu L}\chi_{Li}\right) - \phi'_i\left(D_\mu z^{*i} - \bar{\Psi}_{\mu R}\chi^{i}_{R}\right) + \frac{3}{2}\mathrm{Re}\,f_{\alpha\beta}\bar{\lambda}^{\alpha}_{L}\gamma_\mu\lambda^{\beta}_{R}\right]^2 \\
& - \frac{1}{2}\left(\mathrm{Re}\,f\right)^{-1}_{\alpha\beta}\left(\tilde{g}\,\phi'^{i}T^{\alpha j}_{i}z_j + \frac{i}{2}f'^{i}_{\alpha\gamma}\bar{\chi}_{Li}\lambda^{\gamma}_{L} - \frac{i}{2}f^{*\prime}_{\alpha\gamma i}\bar{\chi}^{i}_{R}\lambda^{\gamma}_{R}\right) \\
& \qquad \left(\tilde{g}\,\phi'^{k}T^{\beta\ell}_{k}z_\ell + \frac{i}{2}f'^{k}_{\beta\delta}\bar{\chi}_{Lk}\lambda^{\delta}_{L} - \frac{i}{2}f^{*}_{\beta\delta k}\bar{\chi}^{k}_{R}\lambda^{\delta}_{R}\right)
\end{aligned}
\tag{4.5}
$$

The complete Lagrangian corresponding to (3.17) is therefore given by

$$
e^{-1}\mathcal{L} = e^{-1}\mathcal{L}_{AUX} + e^{-1}\hat{\mathcal{L}} \tag{4.6}
$$

where

$$
\begin{aligned}
e^{-1}\hat{\mathcal{L}} = & \frac{\phi}{12}\Big[R(\omega(e)) + \bar{\Psi}_\mu\gamma_5\gamma_\nu D_\rho\Psi_\sigma\,\varepsilon^{\mu\nu\rho\sigma}e^{-1} + \frac{1}{16}\left(\bar{\Psi}_\mu\gamma_\lambda\Psi_\rho + 2\bar{\Psi}_\lambda\gamma_\mu\Psi_\rho\right)\bar{\Psi}^{\mu}\gamma^{\lambda}\Psi^{\rho} \\
& \qquad - \frac{1}{4}\left(\bar{\Psi}_\mu\gamma\cdot\Psi\right)^2 + e^{-1}\partial_\mu\left(e\,\bar{\Psi}\cdot\gamma\Psi^{\mu}\right)\Big] \\
& + \phi''^{i}_{j}\left(-\frac{1}{2}D_\mu z_i D^{\mu}z^{*j} - \bar{\chi}_{Li}\not{D}\chi^{j}_{R}\right) + \phi''^{ij}_{k}\bar{\chi}_{Li}\not{D}z_j\chi^{k}_{R} + \phi''^{i}_{j}\bar{\Psi}_{\mu L}\not{D}z^{*j}\gamma^{\mu}\chi_{Li} \\
& - \frac{4}{3}\phi'^{i}\bar{\chi}_{Li}\sigma^{\mu\nu}D_\mu\Psi_{\nu L} - \frac{1}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu\gamma_\nu\Psi_\rho\left(\phi'^{i}D_\sigma z_i + \frac{1}{2}\phi''^{j}_{i}\bar{\chi}^{i}_{R}\gamma_\sigma\chi_{Lj}\right) \\
& - \frac{1}{2}\phi''^{i}_{j}\bar{\Psi}_{\mu L}\chi_{Li}\bar{\Psi}^{\mu}_{R}\chi^{j}_{R} + \frac{1}{2}\phi''^{ij}_{k\ell}\bar{\chi}_{Li}\chi_{Lj}\bar{\chi}^{k}_{R}\chi^{\ell}_{R} \\
& + \frac{1}{6}\phi'^{i}\bar{\chi}_{Li}\left(\Psi_{\mu L}\bar{\Psi}\cdot\gamma\Psi^{\mu} + \frac{1}{2}\sigma^{\mu\nu}\Psi^{\rho}_{L}\bar{\Psi}_\nu\gamma_\rho\Psi_\mu + \sigma^{\mu\nu}\Psi_{\nu L}\bar{\Psi}_\mu\gamma\cdot\Psi\right) \\
& + \frac{i}{2}\tilde{g}\,\phi'^{i}T^{\alpha j}_{i}z_j\bar{\Psi}_L\cdot\gamma\lambda^{\alpha}_{R} - 2i\tilde{g}\,\phi''^{i}_{k}T^{\alpha j}_{i}z_j\bar{\lambda}^{\alpha}_{R}\chi^{k}_{R}
\end{aligned}
\tag{4.7}
$$

$$
\begin{aligned}
&-\tfrac{1}{2} g''^{ij} \bar{\chi}_{Li} \chi_{Lj} + \tfrac{1}{2} g'^{i} \bar{\Psi}_R \cdot \gamma \chi_{Li} + \tfrac{1}{2} g \, \bar{\Psi}_{\mu R} \sigma^{\mu\nu} \psi_{\nu R} \\
&+ \tfrac{1}{2} f_{\alpha\beta} \Big(-\tfrac{1}{4} F^{\alpha}_{\mu\nu} F^{\beta}_{\mu\nu} - \tfrac{1}{2} \bar{\lambda}^{\alpha} \not{\partial} \lambda^{\beta} + \tfrac{1}{2} \bar{\lambda}^{\alpha} \gamma^{\mu} \sigma^{\rho\sigma} \psi_{\mu} F^{\beta}_{\rho\sigma} \\
&\quad + \tfrac{1}{4} \bar{\lambda}^{\alpha} \gamma^{\mu} \sigma^{\rho\sigma} \psi_{\mu} \bar{\Psi}_{\rho} \gamma_{\sigma} \lambda - \tfrac{1}{16} \bar{\lambda}^{\alpha} \gamma^{\mu} \sigma^{mn} \lambda^{\beta} \left(2 \bar{\Psi}_{\mu} \gamma_{m} \psi_{n} + \bar{\Psi}_{m} \gamma_{\mu} \psi_{n} \right) \\
&\quad + \tfrac{1}{4} F^{\alpha}_{\mu\nu} \tilde{F}^{\beta}_{\mu\nu} - \tfrac{1}{2} D_{\mu} \left(\bar{\lambda}^{\alpha}_{L} \gamma^{\mu} \lambda^{\beta}_{R} \right) \Big) \qquad \text{(4.7) cont.} \\
&+ \tfrac{1}{2} f'^{i}_{\alpha\beta} \Big(-\bar{\chi}_{Li} \sigma_{\mu\nu} \lambda^{\beta}_{L} \left(F^{-\alpha}_{\mu\nu} - \tfrac{1}{2} \bar{\Psi}_{\rho L} \sigma_{\mu\nu} \gamma^{\rho} \lambda^{\alpha}_{R} + \tfrac{1}{2} \bar{\Psi}_{R} \cdot \gamma \sigma_{\mu\nu} \lambda^{\alpha}_{L} \right) \\
&\qquad - \tfrac{1}{2} \bar{\Psi}_{R} \cdot \gamma \chi_{Li} \bar{\lambda}^{\alpha}_{L} \lambda^{\beta}_{L} \Big) \\
&+ \tfrac{1}{4} f''^{ij}_{\alpha\beta} \bar{\chi}_{Li} \chi_{Lj} \bar{\lambda}^{\alpha}_{L} \lambda^{\beta}_{L} \qquad + \text{h.c.}
\end{aligned}
$$

The Einstein term present in (4.7) as well as the Rarita-Schwinger term is not in canonical form. To achieve this we perform the following Weyl rescaling on the vierbein as well as on all fermion fields

$$
\begin{aligned}
e_{m\mu} &\to e_{m\mu} \, e^{\sigma} \qquad ; \quad e \to e^{4\sigma} e \\
\lambda &\to e^{-3\sigma/2} \lambda \\
\chi &\to e^{-\sigma/2} \chi \\
\psi_{\mu} &\to e^{\sigma/2} \psi_{\mu}
\end{aligned}
\qquad (4.8)
$$

with

$$
e^{2\sigma} = -\frac{3}{\Phi} \qquad ; \qquad \sigma = -\frac{1}{6} J
$$

Under the Weyl rescaling of the vierbein we get[6)]

$$
\begin{aligned}
&\tfrac{1}{6} e \Phi R \to -\tfrac{1}{2} e R - \tfrac{3}{4} e \left(\partial_{\mu} \log \Phi \right)^2 + \text{4-div.} \qquad (4.9) \\
&\omega_{\mu mn} \to \omega_{\mu mn} - 2 \, e^{\nu}_{[m} e_{n]\mu} \partial_{\nu} \sigma
\end{aligned}
$$

The total Lagrangian becomes the sum of a pure bosonic part and of a fermionic part:

$$e^{-1}\mathcal{L} = e^{-1}\mathcal{L}_B + e^{-1}\mathcal{L}_F \tag{4.10}$$

where

$$\begin{aligned} e^{-1}\mathcal{L}_B = {} & \frac{1}{4} g g^* e^{-\mathcal{J}} \left(3 + \mathcal{J}''^{-1}{}^k_\ell \left(\frac{g_k^{*\prime}}{g^*} - \mathcal{J}'_k\right)\left(\frac{g'^\ell}{g} - \mathcal{J}'^\ell\right)\right) \\ & - \frac{1}{2}\tilde{g}^2 \, \mathrm{Re}\, f^{-1}_{\alpha\beta} \left(\mathcal{J}'^i T_i^{\alpha j} z_j\right)\left(\mathcal{J}'^k T_k^{\beta \ell} z_\ell\right) - \frac{1}{2} R \\ & + \mathcal{J}''^i_j D_\mu z_i D^\mu z^{*j} - \frac{1}{4}\mathrm{Re}\, f_{\alpha\beta} F^\alpha_{\mu\nu} F^\beta_{\mu\nu} + \frac{i}{4}\mathrm{Im}\, f_{\alpha\beta} F^\alpha_{\mu\nu} \tilde{F}^\beta_{\mu\nu} \end{aligned} \tag{4.11}$$

$$\begin{aligned} e^{-1}\mathcal{L}_F = {} & \frac{1}{2} e^{3\sigma}\left(\mathcal{J}''^{ij} + \frac{1}{3}\mathcal{J}'^i \mathcal{J}'^j\right) g\, \bar{\chi}_{Li}\chi_{Lj} \\ & + \frac{g}{2}\left(\frac{g'^\ell}{g} - \mathcal{J}'^\ell\right) e^{3\sigma} \mathcal{J}''^{-1}{}^k_\ell \left(\mathcal{J}'''^{ij}_k + \frac{2}{3}\mathcal{J}''^i_k \mathcal{J}'^j\right)\bar{\chi}_{Li}\chi_{Lj} \\ & + \frac{g}{2}\left(\frac{g'^\ell}{g} - \mathcal{J}'^\ell\right)\frac{3}{4\phi} e^{\sigma} \mathcal{J}''^{-1}{}^k_\ell f^{*\prime}_{\alpha\beta k} \bar{\lambda}^\alpha_R \lambda^\beta_R \\ & - \frac{\phi}{18} e^{2\sigma}\left(\mathcal{J}''^{ij} + \frac{1}{3}\mathcal{J}'^i\mathcal{J}'^j\right)\bar{\chi}_{Li}\chi_{Lj}\left(\mathcal{J}''_{k\ell} + \frac{1}{3}\mathcal{J}'_k\mathcal{J}'_\ell\right)\bar{\chi}^k_R\chi^\ell_R \\ & - \frac{\phi}{6} e^{2\sigma}\mathcal{J}''^{-1}{}^k_\ell\left(\mathcal{J}'''^{ij}_k + \frac{2}{3}\mathcal{J}''^i_k\mathcal{J}'^j\right)\bar{\chi}_{Li}\chi_{Lj}\left(\mathcal{J}'''^\ell_{mn} + \frac{2}{3}\mathcal{J}''^\ell_m\mathcal{J}'_n\right)\bar{\chi}^m_R\chi^n_R \\ & - \frac{1}{4} f'^\ell_{\alpha\beta}\bar{\lambda}^\alpha_L\lambda^\beta_L \mathcal{J}''^{-1}{}^k_\ell\left(\mathcal{J}'''^{ij}_k + \frac{2}{3}\mathcal{J}''^i_k\mathcal{J}'^j\right)\bar{\chi}_{Li}\chi_{Lj} \\ & - \frac{3}{32}\frac{1}{\phi} e^{-2\sigma}\mathcal{J}''^{-1}{}^k_\ell f'^\ell_{\alpha\beta}\bar{\lambda}^\alpha_L\lambda^\beta_L f^{*\prime}_{\gamma\delta k}\bar{\lambda}^\gamma_R\lambda^\delta_R \\ & - \frac{1}{2\phi} e^{2\sigma}\phi''^i_j\bar{\chi}^j_R\gamma^\mu\chi_{Li}\phi'^k D_\mu z_k \\ & - \frac{1}{2\phi} e^{2\sigma}\phi'^i D_\mu z_i\left(\phi'_j\bar{\psi}_{\mu R}\chi^j_R - \phi'^j\bar{\psi}_{\mu L}\chi_{Lj}\right) \\ & - \frac{3}{4\phi}\mathrm{Re}\, f_{\alpha\beta}\phi'^i D_\mu z_i \bar{\lambda}^\alpha_L\gamma^\mu\lambda^\beta_R \end{aligned} \tag{4.12}$$

$$
\begin{aligned}
&-\frac{1}{8\phi}\left(\phi''^{i}_{j}\,\bar{\chi}^{j}_{R}\gamma_{m}\chi_{Li}\right)^{2}e^{2\sigma}+\frac{1}{2\phi}\,\phi''^{i}_{j}\,\bar{\chi}^{j}_{R}\gamma^{\mu}\chi_{Li}\,\phi'^{k}\bar{\psi}_{\mu L}\,\chi_{Lk}\,e^{2\sigma}\\
&-\frac{3}{8\phi}\,\phi''^{i}_{j}\,\bar{\chi}^{j}_{R}\gamma^{m}\chi_{Li}\,\mathrm{Re}\,f_{\alpha\beta}\,\bar{\lambda}^{\alpha}_{L}\gamma_{m}\lambda^{\beta}_{R}+\frac{3}{4\phi}\,\phi'^{i}\bar{\psi}_{\mu L}\,\chi_{Li}\,\mathrm{Re}\,f_{\alpha\beta}\,\bar{\lambda}^{\alpha}_{L}\gamma^{\mu}\lambda^{\beta}_{R}\\
&+\frac{1}{4\phi}\,e^{2\sigma}\phi'^{ij}\bar{\psi}_{\mu L}\,\chi_{Lj}\left(\phi'_{i}\,\bar{\psi}_{\mu R}\,\chi^{i}_{R}-\phi'^{i}\bar{\psi}_{\mu L}\,\chi_{Li}\right)\\
&-\frac{9}{32\phi}\,\mathrm{Re}\,f_{\alpha\beta}\,\mathrm{Re}\,f_{\gamma\delta}\,\bar{\lambda}^{\alpha}_{L}\gamma^{m}\lambda^{\beta}_{R}\,\bar{\lambda}^{\gamma}_{L}\gamma_{m}\lambda^{\delta}_{R}\;e^{-2\sigma}\\
&-\frac{i}{2}\,\mathrm{Re}\,f^{-1}_{\alpha\beta}\,\tilde{g}\,\phi'^{i}\,T^{\alpha j}_{i}\,z_{j}\,f'^{k}_{\beta\gamma}\,\bar{\chi}_{Lk}\,\lambda^{\gamma}_{L}\,e^{2\sigma}\\
&+\frac{1}{8}\,\mathrm{Re}\,f^{-1}_{\alpha\beta}\,f'^{i}_{\alpha\gamma}\,\bar{\chi}_{Li}\,\lambda^{\gamma}_{L}\left(f'^{j}_{\beta\delta}\,\bar{\chi}_{Lj}\,\lambda^{\delta}_{L}-f'_{\beta\delta j}\,\bar{\chi}^{j}_{R}\,\lambda^{\delta}_{R}\right)\\
&-\frac{1}{4}\,\bar{\psi}_{\mu}\gamma_{5}\gamma_{\nu}D_{\rho}\psi_{\sigma}\,\varepsilon^{\mu\nu\rho\sigma}e^{-1}-\frac{1}{64}\left(\bar{\psi}_{\mu}\gamma_{\lambda}\psi_{\rho}+2\,\bar{\psi}_{\lambda}\gamma_{\mu}\psi_{\rho}\right)\bar{\psi}^{\mu}\gamma^{\lambda}\psi^{\rho}\\
&+\frac{1}{16}\left(\bar{\psi}_{\mu}\gamma\cdot\psi\right)^{2}+3\,\frac{\phi''^{i}_{j}}{\phi}\,\bar{\chi}_{Li}\,\partial\!\!\!/\,\chi^{j}_{R}-3\,\frac{\phi'''^{ij}_{k}}{\phi}\,\bar{\chi}_{Li}\,\partial\!\!\!/\,z_{j}\,\chi^{k}_{R}
\end{aligned}
\tag{4.12 cont.}
$$

$$
\begin{aligned}
&-3\,\frac{\phi''^{i}_{j}}{\phi}\,\bar{\psi}_{\mu L}\,\partial\!\!\!/\,z^{*j}\gamma_{\mu}\chi_{Li}+4\,\frac{\phi'^{i}}{\phi}\,\bar{\chi}_{Li}\,\sigma^{\mu\nu}D_{\mu}\psi_{\nu}\\
&-3\,\frac{\phi'^{i}}{\phi}\,\bar{\chi}_{Li}\,\gamma^{\nu}\gamma^{\mu}\psi_{L\nu}\,D_{\mu}\sigma\\
&+\frac{3}{8}\,e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}\gamma_{\nu}\psi_{\rho}\left(\frac{\phi'^{i}}{\phi}\,D_{\sigma}z_{i}+\frac{1}{2}\,\frac{\phi''^{j}_{i}}{\phi}\,\bar{\chi}^{i}_{R}\gamma_{\sigma}\chi_{Lj}\right)\\
&+\frac{3}{2}\,\frac{\phi''^{i}_{j}}{\phi}\,\bar{\psi}_{\mu L}\chi_{Li}\,\bar{\psi}^{\mu}_{R}\chi^{j}_{R}-\frac{3}{2}\,\frac{\phi'''^{ij}_{k\ell}}{\phi}\,\bar{\chi}_{Li}\chi_{Lj}\,\bar{\chi}^{k}_{R}\chi^{\ell}_{R}\\
&-\frac{1}{2}\,\phi'^{i}\bar{\chi}_{Li}\left(\psi_{\mu L}\,\bar{\psi}\cdot\gamma\psi^{\mu}+\frac{1}{2}\,\sigma_{\mu\nu}\psi_{\rho L}\,\bar{\psi}^{\nu}\gamma^{\rho}\psi^{\mu}+\sigma^{\mu\nu}\psi_{\nu L}\,\bar{\psi}_{\mu}\gamma\cdot\psi\right)\\
&-\frac{3i}{2}\,\tilde{g}\,\frac{\phi'^{i}}{\phi}\,T^{\alpha j}_{i}\,z_{j}\,\bar{\psi}_{L}\cdot\gamma\,\lambda^{\alpha}_{R}+6i\tilde{g}\,\frac{\phi''^{j}_{i}}{\phi}\,T^{\alpha k}_{j}\,z_{k}\,\bar{\lambda}^{\alpha}_{R}\,\chi^{i}_{R}\\
&-\frac{1}{2}\,g''^{ij}e^{3\sigma}\bar{\chi}_{Li}\chi_{Lj}+\frac{1}{2}\,g'^{i}e^{3\sigma}\bar{\psi}_{R}\cdot\gamma\chi_{Li}+\frac{1}{2}\,g\,e^{3\sigma}\bar{\psi}_{\mu R}\,\sigma^{\mu\nu}\psi_{\nu R}
\end{aligned}
$$

$$+ \frac{1}{2} \mathrm{Re}\, f_{\alpha\beta} \Big(-\frac{1}{2} \bar{\lambda}^{\alpha} \not{D} \lambda^{\beta} + \frac{1}{2} \bar{\lambda}^{\alpha} \gamma^{\mu} \sigma^{\rho\sigma} \psi_{\mu} \left(F^{\beta}_{\rho\sigma} + \frac{1}{2} \bar{\psi}_{\rho} \gamma_{\sigma} \lambda \right)$$

$$- \frac{1}{32} \bar{\lambda}^{\alpha} \gamma_5 \gamma_{\nu} \lambda^{\beta} e^{-1} \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_{\mu} \gamma_{\rho} \psi_{\sigma} \Big) \, .$$

$$- \frac{i}{4} \mathrm{Im}\, f_{\alpha\beta} \; D^{\mu} \left(\bar{\lambda}^{\alpha}_{L} \gamma^{\mu} \lambda^{\beta}_{R} \right) \tag{4.12 cont.}$$

$$+ \frac{1}{2} f'^{i}_{\alpha\beta} \Big(-\bar{\chi}_{Li} \sigma^{\mu\nu} \lambda^{\beta}_{L} \left(F^{-\alpha}_{\mu\nu} - \frac{1}{2} \bar{\psi}_{\rho L} \sigma_{\mu\nu} \gamma^{\rho} \lambda^{\alpha}_{R} + \frac{1}{2} \bar{\psi}_{R} \cdot \gamma \sigma_{\mu\nu} \lambda^{\alpha}_{L} \right)$$

$$- \frac{1}{2} \bar{\psi}_{R} \cdot \gamma \chi_{Li} \bar{\lambda}^{\alpha}_{L} \lambda^{\beta}_{L} \Big)$$

$$+ \frac{1}{4} f''^{ij}_{\alpha\beta} \bar{\chi}_{Li} \chi_{Lj} \bar{\lambda}^{\alpha}_{L} \lambda^{\beta}_{L} \qquad + h.c.$$

The kinetic fermionic part can be further diagonalized by the following shift

$$\psi_{\mu L} \to \psi_{\mu L} - \gamma_{\mu} \frac{\phi'_i}{\phi} \chi^{i}_{R} \tag{4.13}$$

If we now perform the following chiral rotations on the fermion fields

$$\psi_{\mu L} \to \left(\frac{g}{g^*} \right)^{1/4} \psi_{\mu L} \; ; \; \chi_{Li} \to \left(\frac{g}{g^*} \right)^{-1/4} \chi_{Li} \; ; \; \lambda^{\alpha}_{L} \to \left(\frac{g}{g^*} \right)^{1/4} \lambda^{\alpha}_{L} \tag{4.14}$$

and define the function $\mathcal{G}(z,z^*)$:

$$\mathcal{G} = J - \log \frac{|g|^2}{4} \tag{4.15}$$

so that

$$\mathcal{G}'^{i} = J'^{i} - \frac{g'^{i}}{g} \qquad ; \quad \mathcal{G}''^{i}_{j} = J''^{i}_{j}$$

The final Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_F \tag{4.16}$$

where

$$
\begin{aligned}
e^{-1}\mathcal{L}_B = {} & e^{-\mathcal{G}}\left(3 + \mathcal{G}'_k \mathcal{G}''^{-1\,k}{}_{\ell}\, \mathcal{G}'^{\ell}\right) \\
& - \tfrac{1}{2}\tilde{g}^2 \operatorname{Re} f^{-1}_{\alpha\beta}\left(\mathcal{G}'^{i} T_i^{\alpha j} z_j\right)\left(\mathcal{G}'^{k} T_k^{\beta \ell} z_\ell\right) \\
& - \tfrac{1}{2} R + \mathcal{G}''^{i}{}_{j}\, D_\mu z_i D^\mu z^{*j} - \tfrac{1}{4}\operatorname{Re} f_{\alpha\beta} F^\alpha_{\mu\nu} F^\beta_{\mu\nu} + \tfrac{i}{4}\operatorname{Im} f_{\alpha\beta} F^\alpha_{\mu\nu} \tilde{F}^\beta_{\mu\nu}
\end{aligned}
\tag{4.17}
$$

and

$$
\mathcal{L}_F = \mathcal{L}_{F,K} + \mathcal{L}_{F,M} \tag{4.18}
$$

$L_{F,K}$ are those fermionic terms which contain at least one (covariant) derivative on the fields and $L_{F,M}$ the fermionic part without derivative terms.

$$
\begin{aligned}
e^{-1}\mathcal{L}_{F,K} = {} & \tfrac{1}{2}\operatorname{Re} f_{\alpha\beta}\Big(-\tfrac{1}{2}\bar{\lambda}^\alpha \not{D}\lambda^\beta + \tfrac{1}{2}\bar{\lambda}^\alpha \gamma^\mu \sigma^{\rho\sigma}\psi_\mu F^\beta_{\rho\sigma} \\
& - \tfrac{1}{2}\bar{\lambda}^\alpha_L \gamma_\mu \lambda^\beta_R \mathcal{G}'^{i} D^\mu z_i\Big) - \tfrac{i}{8}\operatorname{Im} f_{\alpha\beta}\, e^{-1} D_\mu\left(e\,\bar{\lambda}^\alpha \gamma_5 \gamma^\mu \lambda^\beta\right) \\
& - \tfrac{1}{2} f'^{i}_{\alpha\beta}\, \bar{\chi}_{Li}\, \sigma\cdot F^\alpha \lambda^\beta_L - \tfrac{1}{4}\bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \varepsilon^{\mu\nu\rho\sigma} e^{-1} \\
& + \tfrac{1}{8} e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_\nu \psi_\rho\, \mathcal{G}'^{i} D_\sigma z_i - \mathcal{G}''^{j}_{i}\, \bar{\psi}_{\mu L} \not{D} z^{*i} \gamma^\mu \chi_{Lj} \\
& - \bar{\chi}_{Li} \not{D} z_j \chi^k_R \left(\mathcal{G}'''^{ij}_{k} + \tfrac{1}{2}\mathcal{G}''^{i}_{k}\mathcal{G}'^{j}\right) + \mathcal{G}''^{i}_{j}\, \bar{\chi}_{Li} \not{D} \chi^j_R \quad + h.c.
\end{aligned}
\tag{4.19}
$$

$$
\begin{aligned}
e^{-1}\mathcal{L}_{F,M} = {} & e^{-\mathcal{G}/2}\bar{\psi}_{\mu R}\sigma^{\mu\nu}\psi_{\nu R} + \tfrac{1}{4} e^{-\mathcal{G}/2}\mathcal{G}'^{\ell}\mathcal{G}''^{-1\,k}{}_{\ell} f^{*\prime}_{\alpha\beta k}\, \bar{\lambda}^\alpha_R \lambda^\beta_R \\
& + e^{-\mathcal{G}/2}\left[\mathcal{G}'''^{ij} - \mathcal{G}'^{i}\mathcal{G}'^{j} - \mathcal{G}'^{\ell}\mathcal{G}''^{-1\,k}{}_{\ell}\mathcal{G}'''^{ij}_{k}\right]\bar{\chi}_{Li}\chi_{Lj} \\
& - \tfrac{i}{2}\tilde{g}\,\mathcal{G}'^{i} T_i^{\alpha j} z_j\, \bar{\psi}_L\cdot\gamma\lambda^\alpha_R - e^{-\mathcal{G}/2}\mathcal{G}'^{i}\bar{\psi}_R\cdot\gamma\chi_{Li} \\
& + \tfrac{i}{2}\left(\operatorname{Re} f\right)^{-1}_{\alpha\beta} f'^{k}_{\beta\gamma}\,\tilde{g}\,\mathcal{G}'^{i} T_i^{\alpha j} z_j\, \bar{\chi}_{Lk}\lambda^\gamma_L + 2i\tilde{g}\,\mathcal{G}''^{j}_{i} T_j^{\alpha k} z_k\, \bar{\lambda}^\alpha_R \chi^i_R
\end{aligned}
\tag{4.20}
$$

$$
\begin{aligned}
&+ \frac{1}{32}\, \mathcal{G}^{\prime\prime -1}{}^{k}_{\ell}\, f^{\prime\ell}_{\alpha\beta}\, f^{*\prime}_{\gamma\delta k}\, \bar{\lambda}^{\alpha}_{L} \lambda^{\beta}_{L}\, \bar{\lambda}^{\gamma}_{R} \lambda^{\delta}_{R} + \frac{3}{32} \left(\mathrm{Re}\, f_{\alpha\beta}\, \bar{\lambda}^{\alpha}_{L} \gamma_{m} \lambda^{\beta}_{R} \right)^{2} \\
&+ \frac{1}{8}\, \mathrm{Re}\, f_{\alpha\beta}\, \bar{\lambda}^{\alpha} \gamma^{\mu} \sigma^{\rho\sigma} \psi_{\mu}\, \bar{\psi}_{\rho} \gamma_{\sigma} \lambda^{\beta} \\
&+ \frac{1}{2}\, f^{\prime i}_{\alpha\beta} \left(\bar{\chi}_{Li} \sigma^{\mu\nu} \lambda^{\alpha}_{L} \bar{\psi}_{\nu L} \gamma_{\mu} \lambda^{\beta}_{R} + \frac{1}{4}\, \bar{\psi}_{R} \cdot \gamma \chi_{Li}\, \bar{\lambda}^{\alpha}_{L} \lambda^{\beta}_{L} \right) \\
&+ \frac{1}{8}\, \mathcal{G}^{\prime\prime j}_{i}\, \bar{\chi}^{i}_{R} \gamma_{d} \chi_{Lj} \left(\varepsilon^{abcd} \bar{\psi}_{a} \gamma_{b} \psi_{c} - \bar{\psi}^{a} \gamma_{5} \gamma^{d} \psi_{a} \right) \qquad \text{(4.20) cont.} \\
&+ \frac{1}{16}\, \bar{\chi}_{Li} \gamma^{\mu} \chi^{j}_{R}\, \bar{\lambda}^{\delta}_{R} \gamma_{\mu} \lambda^{\gamma}_{L} \left(-2\, \mathcal{G}^{\prime\prime i}_{j}\, \mathrm{Re}\, f_{\gamma\delta} + \mathrm{Re}\, f^{-1}_{\alpha\beta}\, f^{\prime i}_{\alpha\gamma}\, f^{*\prime}_{\beta\delta j} \right) \\
&+ \frac{1}{16}\, \bar{\chi}_{Li} \chi_{Lj}\, \bar{\lambda}^{\gamma}_{L} \lambda^{\delta}_{L} \left(-4\, \mathcal{G}^{\prime\prime\prime ij}_{k}\, \mathcal{G}^{\prime\prime -1}{}^{k}_{\ell}\, f^{\prime\ell}_{\gamma\delta} + 4\, f^{\prime\prime ij}_{\gamma\delta} - \mathrm{Re}\, f^{-1}_{\alpha\beta}\, f^{\prime i}_{\alpha\gamma}\, f^{\prime j}_{\beta\delta} \right) \\
&+ \left(-\frac{1}{2}\, \mathcal{G}^{\prime\prime\prime ij}_{k\ell} + \frac{1}{2}\, \mathcal{G}^{\prime\prime\prime ij}_{m}\, \mathcal{G}^{\prime\prime -1}{}^{m}_{n}\, \mathcal{G}^{\prime\prime\prime n}_{k\ell} - \frac{1}{4}\, \mathcal{G}^{\prime\prime i}_{k}\, \mathcal{G}^{\prime\prime j}_{\ell} \right) \bar{\chi}_{Li} \chi_{Lj}\, \bar{\chi}^{k}_{R} \chi^{\ell}_{R} \\
&\qquad + h.c.
\end{aligned}
$$

R in (4.17) and the fermion derivatives in (4.19) include again the ψ_μ torsion terms in the Lorentz connection $\omega_{\mu mn}(e,\psi)$. This completes the derivation of the final Lagrangian as a function of $\mathcal{G}$ and $f_{\alpha\beta}$.

One realizes that the chiral part of (4.16), which is obtained by setting the gauge coupling $\tilde{g} = 0$, has the same structure as in the case of a single chiral multiplet[6)] with the addition of trivial index contractions[*)].

We are now interested in deriving the transformation laws of the physical fields $e_{a\mu}$, ψ_μ, z_i, χ_{Li}, λ^α, B^α_m under which the Lagrangian given by the sum of (4.17) and (4.18) is invariant. The transformation laws of the vector, chiral and gravitational multiplets are given by Eqs. (3.18), (3.19) and (3.20), respectively. The elimination of the auxiliary fields only affects the transformation laws of the fermion fields λ^α_R, $\psi_{\mu L}$, χ_{Li}. Inserting Eq. (4.4) in to the transformation laws, we get

---

*) The potential term contained in (4.17) has also been given by Barbieri et al. [third reference in Ref. 7)] and the chiral Lagrangian without gauge interactions and without transformation laws has been given in Ref. 22).

$$\delta\lambda_R^\alpha = \frac{1}{2}\sigma^{\mu\nu}\hat{F}_{\mu\nu}^{\alpha}\epsilon_R - \frac{i}{2}\epsilon_R \, Re f_{\alpha\beta}^{-1}\left(\tilde{g}\,\phi'^i T_i^{\beta j} z_j + \frac{i}{2} f'^i_{\beta\gamma}\bar{\chi}_{Li}\lambda_L^\gamma - \frac{i}{2} f^{*\prime}_{\beta\gamma i}\bar{\chi}_R^i \lambda_R^\gamma\right)$$

$$\delta\chi_{Li} = \frac{1}{2}\tilde{\not{D}} z_i \epsilon_R + \frac{3}{2\phi}\epsilon_L \mathcal{J}''^{-1\,k}_{\;\;i}\Big[-\frac{g^*}{2}\Big(\frac{g^{*\prime}_k}{g^*} - \mathcal{J}'_k\Big) +$$

$$+ \frac{1}{3}\phi\left(\mathcal{J}'''^{\,jl}_{\;\;k} + \frac{2}{3}\mathcal{J}''^{\,j}_{\;\;k}\mathcal{J}'^{\,l}\right)\bar{\chi}_{Lj}\chi_{Ll} + \frac{1}{4} f^{*\prime}_{\alpha\beta k}\,\bar{\lambda}_R^\alpha\lambda_R^\beta\Big]$$

$$\delta\psi_{\mu L} = \left(\partial_\mu + \frac{1}{2}\omega_{\mu mn}(e,\psi)\,\sigma^{mn}\right)\epsilon_L \tag{4.21}$$

$$-\frac{1}{2\phi}(\delta_{\mu\nu} - \sigma_{\mu\nu})\epsilon_L\Big[\phi''^i_{\;j}\,\bar{\chi}_R^j\gamma^\nu\chi_{Li} + \frac{3}{2}\bar{\lambda}_L^\alpha\gamma^\nu\lambda_R^\beta f_{\alpha\beta}$$

$$+ \phi'^i\left(D^\nu z_i - \bar{\psi}_L^\nu\chi_{Li}\right) - \phi'_i\left(D^\nu z^{*i} - \bar{\psi}_R^\nu\chi_R^i\right)\Big]$$

$$+\frac{3}{2\phi}\gamma_\mu\epsilon_R\Big[-\frac{1}{2}g + \frac{1}{9}\left(\mathcal{J}''_{ij} + \frac{1}{3}\mathcal{J}'_i\mathcal{J}'_j\right)\phi\,\bar{\chi}_R^i\chi_R^j\Big]$$

$$-\frac{1}{2\phi}\gamma_\mu\epsilon_R\,\mathcal{J}'_l\,\mathcal{J}''^{-1\,l}_{\;\;k}\Big[-\frac{g}{2}\Big(\frac{g'^k}{g} - \mathcal{J}'^k\Big) + \frac{\phi}{3}\Big(\mathcal{J}''^{\,k}_{ij} + \frac{2}{3}\mathcal{J}''^{\,k}_{\;i}\mathcal{J}'_j\Big)\bar{\chi}_R^i\chi_R^j + \frac{1}{4}f'^k_{\alpha\beta}\,\bar{\lambda}_L^\alpha\lambda_L^\beta\Big]$$

After the Weyl rescaling defined by Eq. (4.8), performing an additional rescaling on the parameter $\varepsilon \to e^{\sigma/2}\varepsilon$, the transformation laws become

$$\delta\lambda_R^\alpha = \frac{1}{2}\sigma^{\mu\nu}\hat{F}_{\mu\nu}^{\alpha}\epsilon_R - \frac{i}{2}\epsilon_R \, Re f_{\alpha\beta}^{-1}\left(-\tilde{g}\,\mathcal{J}'^i T_i^{\beta j} z_j + \frac{i}{2} f'^i_{\beta\gamma}\bar{\chi}_{Li}\lambda_L^\gamma - \frac{i}{2} f^{*\prime}_{\beta\gamma i}\bar{\chi}_R^i \lambda_R^\gamma\right)$$

$$-\frac{1}{4}\lambda_R^\alpha\left(\mathcal{J}'^i\bar{\epsilon}_L\chi_{Li} + \mathcal{J}'_i\bar{\epsilon}_R\chi_R^i\right)$$

$$\delta\chi_{Li} = \frac{1}{2}\tilde{\not{D}} z_i \epsilon_R + \frac{1}{2}\epsilon_L \mathcal{J}''^{-1\,k}_{\;\;i}\Big[\frac{g^*}{2}\Big(\frac{g^{*\prime}_k}{g^*} - \mathcal{J}'_k\Big)e^{-\mathcal{J}/2}$$

$$+\left(\mathcal{J}'''^{\,jl}_{\;\;k} + \frac{2}{3}\mathcal{J}''^{\,j}_{\;\;k}\mathcal{J}'^{\,l}\right)\bar{\chi}_{Lj}\chi_{Ll} - \frac{1}{4}f^{*\prime}_{\alpha\beta k}\,\bar{\lambda}_R^\alpha\lambda_R^\beta\Big] \tag{4.22}$$

$$-\frac{1}{12}\chi_{Li}\left(\mathcal{J}'^j\bar{\epsilon}_L\chi_{Lj} + \mathcal{J}'_j\bar{\epsilon}_R\chi_R^j\right)$$

$$\delta\psi_{\mu L} = \left(\partial_\mu + \frac{1}{2}\omega_{\mu mn}(e,\psi)\,\sigma^{mn}\right)\epsilon_L - \frac{1}{12}\gamma_\mu\left(\mathcal{J}'^i\not{D} z_i + \mathcal{J}'_i\not{D} z^{*i}\right)\epsilon_L$$

$$-\frac{1}{2}(\delta_{\mu\nu} - \sigma_{\mu\nu})\epsilon_L\Big[\left(\frac{1}{3}\mathcal{J}''^{\,i}_{\;j} + \frac{1}{9}\mathcal{J}'^i\mathcal{J}'_j\right)\bar{\chi}_R^j\gamma^\nu\chi_{Li} - \frac{1}{2}\bar{\lambda}_L^\alpha\gamma^\nu\lambda_R^\beta f_{\alpha\beta}$$

$$+\frac{1}{3}\mathcal{J}'^i\left(D^\nu z_i - \bar{\psi}_L^\nu\chi_{Li}\right) - \frac{1}{3}\mathcal{J}'_i\left(D^\nu z^{*i} - \bar{\psi}_R^\nu\chi_R^i\right)\Big]$$

$$+\frac{1}{2}\gamma_\mu \epsilon_R \left[\frac{1}{2} g\, e^{-\mathcal{J}/2} + \frac{1}{3}\left(\mathcal{J}''_{ij} + \frac{1}{3}\mathcal{J}'_i \mathcal{J}'_j\right)\bar{\chi}_R^{\,i}\chi_R^{\,j}\right]$$

$$-\frac{1}{6}\gamma_\mu \epsilon_R \mathcal{J}'_l \mathcal{J}''^{-1\,l}{}_k \Big[-\frac{g}{2} e^{-\mathcal{J}/2}\left(\mathcal{J}'^k - \frac{g'^k}{g}\right) + \left(\mathcal{J}'''^k_{ij} + \frac{2}{3}\mathcal{J}'^k_i \mathcal{J}'_j\right)\bar{\chi}_R^{\,i}\chi_R^{\,j}$$

$$- \frac{1}{4} f'^k_{\alpha\beta}\, \bar{\lambda}_L^{\alpha}\lambda_L^{\beta}\Big] \qquad (4.22)\ \text{cont.}$$

$$+\frac{1}{12}\psi_{\mu L}\left(\mathcal{J}'^i \bar{\epsilon}_L \chi_{Li} + \mathcal{J}'_j \bar{\epsilon}_R \chi_R^{\,j}\right)$$

$$\delta e_\mu^{\ m} = \frac{1}{2}\bar{\epsilon}_L \gamma^m \psi_{\mu R} + \frac{1}{6} e_\mu^{\ m} \mathcal{J}'^i \bar{\epsilon}_L \chi_{Li} \quad + h.c.$$

The chiral rotations defined by Eq. (4.14) give the following modifications to the transformation laws of the fermion fields after a chiral rotation on the parameters $\varepsilon_L \to (g/g^*)^{1/4}\varepsilon_L$

$$\delta\lambda_R^{\alpha} = \frac{1}{2}\sigma^{\mu\nu}\hat{F}^{\alpha}_{\mu\nu}\epsilon_R - \frac{i}{2}\epsilon_R \,\mathrm{Re} f^{-1}_{\alpha\beta}\left(-\tilde{g}\mathcal{G}'^i T_i^{\beta j} z_j + \frac{i}{2} f'^i_{\beta\gamma}\bar{\chi}_{Li}\lambda_L^{\gamma} - \frac{i}{2} f'^{*i}_{\beta\gamma}\bar{\chi}_R^{\,i}\lambda_R^{\gamma}\right)$$

$$-\frac{1}{4}\lambda_R^{\alpha}\mathcal{G}'^i \bar{\epsilon}_L \chi_{Li} + \frac{1}{4}\lambda_R^{\alpha}\mathcal{G}'_i \bar{\epsilon}_R \chi_R^{\,i} - \frac{1}{2}\lambda_R^{\alpha}\mathcal{J}'_i \bar{\epsilon}_R \chi_R^{\,i}$$

$$\delta\chi_{Li} = \frac{1}{2}\tilde{\not{D}} z_i \epsilon_R + \frac{1}{2}\epsilon_L \mathcal{G}''^{-1\,k}{}_i\left[-\mathcal{G}'_k e^{-\mathcal{G}/2} + \mathcal{G}'''^{j\ell}_k \bar{\chi}_{Lj}\chi_{L\ell} - \frac{1}{4} f'^*_{\alpha\beta k}\bar{\lambda}_R^{\alpha}\lambda_R^{\beta}\right]$$

$$+\frac{1}{4}\chi_{Li}\left(\mathcal{G}'_j \bar{\epsilon}_R \chi_R^{\,j} - \mathcal{G}'^j \bar{\epsilon}_L \chi_{Lj}\right) - \frac{1}{3}\chi_{Li}\bar{\epsilon}_R \chi_R^{\,j}\mathcal{J}'_j + \frac{1}{6}\sigma_{\mu\nu}\chi_{Li}\bar{\chi}_{Lj}\sigma^{\mu\nu}\epsilon_L \mathcal{J}'^j$$

$$\delta\psi_{\mu L} = \left(\partial_\mu + \frac{1}{2}\omega_{\mu mn}(e,\psi)\sigma^{mn}\right)\epsilon_L - \frac{1}{6}\gamma_\mu \mathcal{J}'_i \not{D} z^{*i}\epsilon_L - \frac{1}{4}\left(\mathcal{G}'^i D_\mu z_i - \mathcal{G}'_i D_\mu z^{*i}\right)\epsilon_L$$

$$-\frac{1}{2}\left(\delta_{\mu\nu} - \sigma_{\mu\nu}\right)\epsilon_L\Big[\left(\frac{1}{3}\mathcal{G}''^i_j + \frac{1}{9}\mathcal{J}'^i \mathcal{J}'_j\right)\bar{\chi}_R^{\,j}\gamma^\nu\chi_{Li} - \frac{1}{2}\bar{\lambda}_L^{\alpha}\gamma^\nu\lambda_R^{\beta} f_{\alpha\beta}$$

$$-\frac{1}{3}\mathcal{J}'^i \bar{\psi}_{\nu L}\chi_{Li} + \frac{1}{3}\mathcal{J}'_i \bar{\psi}_{\nu R}\chi_R^{\,i}\Big] \qquad (4.23)$$

$$+\frac{1}{2}\gamma_\mu \epsilon_R\Big[e^{-\mathcal{G}/2} + \left(\frac{1}{3}\mathcal{J}''_{ij} - \frac{1}{9}\mathcal{J}'_i \mathcal{J}'_j - \frac{1}{3}\mathcal{J}'_\ell \mathcal{G}''^{-1\,\ell}{}_k \mathcal{G}'''^k_{ij}\right)\bar{\chi}_R^{\,i}\chi_R^{\,j}$$

$$+\frac{1}{3} e^{-\mathcal{G}/2}\mathcal{J}'_\ell \mathcal{G}''^{-1\,\ell}{}_k \mathcal{G}'^k + \frac{1}{12}\mathcal{J}'_\ell \mathcal{G}''^{-1\,\ell}{}_k f'^k_{\alpha\beta}\bar{\lambda}_L^{\alpha}\lambda_L^{\beta}\Big]$$

$$+\frac{1}{6}\psi_{\mu L}\left(-\mathcal{J}'^i \bar{\epsilon}_L \chi_{Li} + 2\mathcal{J}'_i \bar{\epsilon}_R \chi_R^{\,i}\right) + \frac{1}{4}\psi_{\mu L}\left(\mathcal{G}'^i \bar{\epsilon}_L \chi_{Li} - \mathcal{G}'_i \bar{\epsilon}_R \chi_R^{\,i}\right)$$

Finally, the shift defined by Eq. (4.13) brings the transformation laws under local supersymmetry in to their final form

$$\delta e_\mu^m = \frac{1}{2}\,\bar{\epsilon}_L\,\gamma^m\,\psi_{\mu R} \quad + h.c.$$

$$\delta\psi_{\mu L} = \left(\partial_\mu + \frac{1}{2}\omega_{\mu mn}(e,\psi)\,\sigma^{mn}\right)\epsilon_L + \frac{1}{2}\sigma_{\mu\nu}\,\epsilon_L\,\mathcal{G}''^{j}_{i}\,\bar{\chi}^i_R\,\gamma^\nu\chi_{Lj}$$

$$+\frac{1}{2}\gamma_\mu\,\epsilon_R\,e^{-\mathcal{G}/2} + \frac{1}{4}\psi_{\mu L}\left(\mathcal{G}'^{i}\,\bar{\epsilon}_L\,\chi_{Li} - \mathcal{G}'_i\,\bar{\epsilon}_R\,\chi^i_R\right)$$

$$-\frac{1}{4}\,\epsilon_L\left(\mathcal{G}'^{i}\,D_\mu z_i - \mathcal{G}'_i\,D_\mu z^{*i}\right) + \frac{1}{4}\left(g_{\mu\nu} - \sigma_{\mu\nu}\right)\epsilon_L\,\bar{\lambda}^\alpha_L\,\gamma^\nu\lambda^\beta_R\,f_{\alpha\beta}$$

$$\delta B_\mu^\alpha = -\frac{1}{2}\,\bar{\epsilon}_L\,\gamma_\mu\,\lambda^\alpha_R \quad + h.c.$$

$$\delta\lambda^\alpha_R = \frac{1}{2}\sigma^{\mu\nu}\hat{F}^\alpha_{\mu\nu}\,\epsilon_R - \frac{i}{2}\epsilon_R\,\mathrm{Re}f^{-1}_{\alpha\beta}\left(-\tilde{g}\,\mathcal{G}'^{i}\,T_i^{\beta j}z_j + \frac{i}{2}f'^{i}_{\beta\gamma}\,\bar{\chi}_{Li}\,\lambda^\gamma_L - \frac{i}{2}f^{*\prime}_{\beta\gamma i}\,\bar{\chi}^i_R\,\lambda^\gamma_R\right)$$

$$-\frac{1}{4}\,\lambda^\alpha_R\left(\mathcal{G}'^{i}\,\bar{\epsilon}_L\,\chi_{Li} - \mathcal{G}'_i\,\bar{\epsilon}_R\,\chi^i_R\right) \tag{4.24}$$

$$\delta z_i = \bar{\epsilon}_L\,\chi_{Li}$$

$$\delta\chi_{Li} = \frac{1}{2}\hat{\not{D}}z_i\,\epsilon_R - \frac{1}{2}\epsilon_L\,e^{-\mathcal{G}/2}\mathcal{G}''^{-1\,j}_{\;\;i}\,\mathcal{G}'_j - \frac{1}{8}\epsilon_L\,\bar{\lambda}^\alpha_R\,\lambda^\beta_R\,\mathcal{G}''^{-1\,k}_{\;\;i}\,f_{\alpha\beta k}$$

$$+\frac{1}{2}\epsilon_L\,\mathcal{G}''^{-1\,k}_{\;\;i}\,\mathcal{G}'''^{j\ell}_{k}\,\bar{\chi}_{Lj}\,\chi_{L\ell} + \frac{1}{4}\chi_{Li}\left(\mathcal{G}'_j\,\bar{\epsilon}_R\,\chi^j_R - \mathcal{G}'^{j}\,\bar{\epsilon}_L\,\chi_{Lj}\right)$$

In the transformation laws given by Eq. (4.24), as derived by Eq. (4.23), we have dropped a local Lorentz transformation with field-dependent parameter given by

$$\Omega^{mn} = -\Omega^{nm} = \frac{1}{3}\,\bar{\epsilon}_L\,\sigma^{mn}\chi_{Li}\,\mathcal{T}'^{i} + h.c. \tag{4.25}$$

We note here that the final Lagrangian and transformation laws could be further modified by making an additional transformation on the left-handed fermions of the chiral multiplet χ_{Li}. This is in order to recast the spin 1/2 kinetic terms in canonical form. The transformation is

$$\chi_{Lj} \to \left(-2 \quad \mathcal{G}''\right)^{-1/2}{}^{i}_{j}\, \chi_{Li} \tag{4.26}$$

This transformation was actually performed in Ref. 6). However, since we will discuss the mass matrix only in the case in which $-2\mathcal{G}^{i}{}_{j} = \delta^{i}{}_{j}$, we do not perform the additional transformation given by Eq. (4.26). This is left as an exercise for the reader.

5. - SUPERHIGGS MECHANISM AND QUADRATIC MASS RELATIONS

In this final section we discuss the superHiggs mechanism which, in the case of a single chiral multiplet, was discussed in Ref. 6). The superHiggs mechanism for an arbitrary number of chiral multiplets was discussed in full in Ref. 9) under the condition of "minimal" coupling defined as

$$\mathcal{G}''^{i}_{j} = -\frac{1}{2}\,\delta^{i}_{j} \qquad ; \quad f_{\alpha\beta} = \delta_{\alpha\beta} \tag{5.1}$$

and the additional requirement that at the extremum the cosmological constant vanishes and, furthermore,

$$D^{\alpha} \equiv -\tilde{g}\, \mathcal{G}'^{i}\, T_{i}^{\alpha j}\, z_{j} = 0 \tag{5.2}$$

In order to discuss the superHiggs effect we have to consider the part of the Lagrangian which contains the scalar potential V in $\mathcal{L}_B$ and the quadratic part of the fermion terms in $\mathcal{L}_F$. They are

$$\begin{aligned} V(z,z^*) = &-e^{-\mathcal{G}}\left(3 + \mathcal{G}'^{i}\, \mathcal{G}''^{-1}{}^{j}_{i}\, \mathcal{G}'_{j}\right) \\ &+ \frac{1}{2}\tilde{g}^2\, \mathrm{Re}\, f^{-1}_{\alpha\beta}\left(\mathcal{G}'^{i}\, T_{i}^{\alpha j}\, z_{j}\right)\left(\mathcal{G}'^{k}\, T_{k}^{\beta \ell}\, z_{\ell}\right) \end{aligned} \tag{5.3}$$

$$
\begin{aligned}
\mathcal{L}_F^{(2)} = {} & e^{-\mathcal{G}/2}\, \bar{\psi}_{\mu R}\, \sigma^{\mu\nu} \psi_{\nu R} \\
& - \bar{\psi}_R \cdot \gamma \left(e^{-\mathcal{G}/2} \mathcal{G}^{\prime i} \chi_{Li} - \frac{i}{2} \tilde{g}\, \mathcal{G}^{\prime i} T_i^{\alpha j} z_j\, \lambda_L^{\alpha} \right) \\
& + e^{-\mathcal{G}/2} \left[\mathcal{G}^{\prime\prime ij} - \mathcal{G}^{\prime i} \mathcal{G}^{\prime j} - \mathcal{G}^{\prime l} \mathcal{G}^{\prime\prime -1 k}{}_{l}\, \mathcal{G}^{\prime\prime\prime ij}{}_{k} \right] \bar{\chi}_{Li}\, \chi_{Lj} \\
& - 2 i \tilde{g}\, \mathcal{G}_i^{\prime\prime k} z^{*j} T_j^{\alpha i}\, \bar{\lambda}_L^{\alpha} \chi_{Lk} \qquad (5.4) \\
& + \frac{1}{2} f_{\alpha\beta}^{\prime k} \left(\frac{1}{2} e^{-\mathcal{G}/2} \mathcal{G}_l^{\prime}\, \mathcal{G}^{\prime\prime -1 l}{}_{k}\, \bar{\lambda}_L^{\alpha} + i \tilde{g} (\mathrm{Re} f)^{-1}_{\alpha\gamma} \mathcal{G}^{\prime i} T_i^{\gamma j} z_j\, \bar{\chi}_{Lk} \right) \lambda_L^{\beta} + \\
& \qquad + h.c.
\end{aligned}
$$

The Goldstone fermion is uniquely defined by the combination of the fermion fields which couple to $\psi_{\mu R}$. It is given by

$$
- \bar{\psi}_R \cdot \gamma\, \eta_L \qquad (5.5)
$$

with

$$
\eta_L = e^{-\mathcal{G}/2} \mathcal{G}^{\prime i} \chi_{Li} - \frac{i}{2} \tilde{g}\, \mathcal{G}^{\prime i} T_i^{\alpha j} z_j\, \lambda_L^{\alpha} \qquad (5.6)
$$

where the scalar-dependent factors which multiply χ_{Li} and λ_L^{α} are meant to be computed at the extremum of the scalar potential

$$
V_o^{\prime i} = \frac{\partial V}{\partial z_i} = 0 \qquad (5.7)
$$

Since we are interested in the superHiggs effect in Minkowski space, we also demand the cancellation of the cosmological constant at the extremum given in Eq. (5.7).

$$
V = 0 \qquad (5.8)
$$

at the extremum.

We now discuss in detail the mass formulae which one obtains in the case of "minimal" coupling of the Yang-Mills system to supergravity. This is defined by Eq. (5.1) in which case all the kinetic terms are canonical and the Lagrangian depends only on the superpotential g(z). In fact, in this case,

$$\mathcal{G}(z,z^*) = -\tfrac{1}{2} z_i z^{*i} - \log \frac{|g|^2}{4}$$

$$f_{\alpha\beta}(z) = \delta_{\alpha\beta} \tag{5.9}$$

Under this assumption (5.3) and (5.4) simplify into

$$V(z,z^*) = e^{-\mathcal{G}}\left[2\mathcal{G}^{ii}\mathcal{G}'_i - 3\right] + \tfrac{1}{2} D^\alpha D^\alpha \tag{5.10}$$

where

$$D^\alpha = -\tilde{g}\,\mathcal{G}^{ii} T_i^{\alpha j} z_j = \tfrac{1}{2}\tilde{g}\, z^{*i} T_i^{\alpha j} z_j \tag{5.11}$$

$$\mathcal{L}_F^{(2)} = e^{-\mathcal{G}/2}\left[\bar{\Psi}_{\mu R}\sigma^{\mu\nu}\Psi_{\nu R} - \bar{\Psi}_R\cdot\gamma\,\mathcal{G}^{ii}\chi_{Li} + \bar{\chi}_{Li}(\mathcal{G}^{iij} - \mathcal{G}^{ii}\mathcal{G}^{ij})\chi_{Lj}\right]$$

$$-\tfrac{i}{2}\bar{\Psi}_R\cdot\gamma\,\lambda_L^\alpha D^\alpha + 2i\,\bar{\lambda}_L^\alpha \chi_{Li} D'^{\alpha i} \tag{5.12}$$

where

$$D'^{\alpha i} = \frac{\partial D^\alpha}{\partial z_i} \tag{5.13}$$

We now have to separate the Goldstone mode from the other $N + \alpha - 1$ fermionic fields. N is the number of chiral multiplets and α is the dimension of G. This is achieved by rewriting (5.12) as follows

$$\mathcal{L}_F^{(2)} = e^{-\mathcal{G}/2}\left[\bar{\Psi}_{\mu R}\sigma^{\mu\nu}\Psi_{\nu R} - \bar{\Psi}_R\cdot\gamma\,\tilde{\eta}_L - \tfrac{2}{3}\bar{\tilde{\eta}}_L\tilde{\eta}\right]$$

$$+\bar{\chi}_{Li}\mathcal{M}^{ij}\chi_{Lj} + 2\bar{\chi}_{Li}\mathcal{M}^{i\alpha}\lambda_L^\alpha + \bar{\lambda}_L^\alpha\mathcal{M}^{\alpha\beta}\lambda_L^\beta + h.c. \tag{5.14}$$

where

$$\tilde{\eta}_L = \mathcal{G}^{ii}\chi_{Li} + \tfrac{i}{2} e^{\mathcal{G}/2} D^\alpha \lambda_L^\alpha \tag{5.15}$$

The mass matrix in the second line of (5.14) is given by

$$\mathcal{M}^{ij} = e^{-\mathcal{G}/2}\left[\mathcal{G}''^{ij} - \frac{1}{3}\mathcal{G}'^{i}\mathcal{G}'^{j}\right]$$

$$\mathcal{M}^{i\alpha} = \frac{i}{3}\mathcal{G}'^{i} D^{\alpha} + i\, D'^{\alpha i} \tag{5.16}$$

$$\mathcal{M}^{\alpha\beta} = -\frac{1}{6} e^{\mathcal{G}/2} D^{\alpha} D^{\beta}$$

It is now straightforward to check that η_L, as given by (5.6), is a massless eigenstate of the fermion mass matrix given by (5.16).

We now compute the trace of the scalar squared mass matrix

$$\mathrm{Tr}\, \mathcal{M}_0^2 = 4\, \mathrm{Tr}\, V''^{k}_{j} = 4 \frac{\partial V}{\partial z_j\, \partial z^{*j}} =$$

$$= -(N+4)\, D^{\alpha} D^{\alpha} + 2\, \mathcal{G}'^{i} \mathcal{G}'_{i}\, D^{\alpha} D_{\alpha} + 2\, e^{-\mathcal{G}}\left(4\, \mathcal{G}''_{ij}\, \mathcal{G}''^{ij} + N\right)$$

$$+ 4\, D'^{\alpha i} D'^{\alpha}{}_{i} + 4\, D^{\alpha} D''^{\alpha}{}_{i}^{\,i} \tag{5.17}$$

The trace of the squared spin 1 mass matrix is given by[11)]

$$\mathrm{Tr}\, \mathcal{M}_1^2 = 3 \cdot 4\, D'^{\alpha}{}_{i}\, D'^{\alpha i} = 12\, D'^{\alpha}{}_{i}\, D'^{\alpha i} \tag{5.18}$$

In the case of "minimal" coupling to supergravity, Eq. (5.18), as well as D^{α} given by (5.11), coincide with their "flat" values. For the trace of the spin 1/2 squared mass matrix we get

$$\mathrm{Tr}\, \mathcal{M}_{1/2}^2 = 8\left(\mathcal{M}^{ij}\mathcal{M}_{ij} + 2\, \mathcal{M}^{\alpha i}\mathcal{M}_{\alpha i} + \mathcal{M}^{\alpha\beta}\mathcal{M}_{\alpha\beta}\right) =$$

$$= 8\, e^{-\mathcal{G}} \left(\mathcal{G}''^{ij} - \tfrac{1}{3} \mathcal{G}'^{i} \mathcal{G}'^{j} \right) \left(\mathcal{G}''_{ij} - \tfrac{1}{3} \mathcal{G}'_{i} \mathcal{G}'_{j} \right)$$
$$+ 16 \left(D'^{\alpha i} + \tfrac{1}{3} \mathcal{G}'^{i} D^{\alpha} \right) \left(D'^{\alpha}{}_{i} + \tfrac{1}{3} \mathcal{G}'_{i} D^{\alpha} \right) \tag{5.19}$$
$$+ \frac{2}{9} e^{\mathcal{G}} (D^{\alpha})^2 (D^{\beta})^2$$

If we now use the following relations

$$V = 0 \quad \Rightarrow \quad (D^{\alpha})^2 = 2\, e^{-\mathcal{G}} \left(3 - 2 \mathcal{G}'^{i} \mathcal{G}'_{i} \right) \tag{5.20}$$

$$V'_i \mathcal{G}'^{i} = \tfrac{1}{2} (D^{\alpha})^2 \mathcal{G}'^{i} \mathcal{G}'_{i} - \tfrac{1}{2} (D^{\alpha})^2 - e^{-\mathcal{G}} \mathcal{G}'^{i} \mathcal{G}'_{i}$$
$$+ 2\, e^{-\mathcal{G}} \mathcal{G}''_{ij} \mathcal{G}'^{i} \mathcal{G}'^{j} = 0 \tag{5.21}$$

$$D'^{\alpha i} \mathcal{G}'_{i} = - \tfrac{1}{2} D^{\alpha} \tag{5.22}$$

Thus Eq. (5.19) simplifies as follows

$$\mathrm{Tr}\, \mathcal{M}^2_{1/2} = -2\, e^{-\mathcal{G}} - 5 (D^{\alpha})^2 + 2 (D^{\alpha})^2 \mathcal{G}'^{i} \mathcal{G}'_{i}$$
$$+ 16\, D'^{\alpha i} D'^{\alpha}{}_{i} + 8\, \mathcal{G}''^{ij} \mathcal{G}''_{ij}\, e^{-\mathcal{G}} \tag{5.23}$$

The gravitino contribution to the fermion squared mass matrix is

$$\mathrm{Tr}\, \mathcal{M}^2_{3/2} = 4\, e^{-\mathcal{G}} \tag{5.24}$$

Therefore we finally get

$$\text{Super Trace } \mathcal{M}^2 = \sum_{J=0}^{3/2} (-)^{2J} (2J+1)\, m_J^2 =$$

$$= \mathrm{Tr}\, \mathcal{M}_0^2 + \mathrm{Tr}\, \mathcal{M}_1^2 - \mathrm{Tr}\, \mathcal{M}_{1/2}^2 - \mathrm{Tr}\, \mathcal{M}_{3/2}^2 =$$

$$= (N-1)\left(2 m_{3/2}^2 - \varkappa^2 D^\alpha D^\alpha\right) + 2 \tilde{g}_\alpha D^\alpha \,\mathrm{Tr}\, T^\alpha \tag{5.25}$$

Equation (5.25) reproduces the known results of flat supersymmetry derived in Ref. 11) by setting

$$m_{3/2} = 0 \quad ; \quad \varkappa = 0 \tag{5.26}$$

For $N = 1$ it reproduces the result of Ref. 6) and for arbitrary N and $D^\alpha = 0$ it reproduces the results of Ref. 9).

We note that the last term in Eq. (5.25) is possible only if $\mathrm{Tr}T^\alpha \neq 0$, i.e., for Abelian $U(1)$ factors of G. The sum rules given by Eq. (5.25) are model-independent in the sense that they do not depend on the superpotential function $g(z)$ nor on the gauge group G.

We also note that the condition of the vanishing cosmological constant in (5.20) implies that

$$6 m_{3/2}^2 \geqslant \varkappa^2 (D^\alpha)^2 \tag{5.27}$$

Therefore the first term in Eq. (5.25) could become negative in the range

$$2 m_{3/2}^2 < \varkappa^2 (D^\alpha)^2 < 6 m_{3/2}^2 \tag{5.28}$$

provided a local minimum for V exists in this range of values for $m_{3/2}$ and D^α.

The former mass matrix (5.16), as well as the sum rules (5.25), describe the most general situation of the coupled Yang-Mills-matter-supergravity system in the case in which both the superHiggs effect and the normal Higgs effect for the gauge group G simultaneously occur.

We would like to comment on the possible implications of these mass relations. As previously stated, if $M_{Pl} \to \infty$, we get back the flat space result, i.e.,

$$\text{Supertrace } \mathcal{M}^2 = 2\,\tilde{g}_\alpha D^\alpha \, \mathrm{Tr}\, T^\alpha \tag{5.29}$$

On the contrary, if the scale of supersymmetry breaking is $m \simeq \sqrt{\mu M_{Pl}}$ where $\mu \sim O(100\ \mathrm{GeV} - 1000\ \mathrm{GeV})$, then the gravitational corrections, e.g., the gravitino mass term, cannot be neglected any more and one gets a deviation from the mass formulae of global supersymmetry[11]. The gauge part can give a negative contribution to Eq. (5.25). However, most of the realistic models have $D^\alpha = 0$ so that this term does not contribute in this situation. Equation (5.25) shows that in spontaneously broken local supersymmetry, with supergravity broken at an intermediate scale, $m \sim \sqrt{\mu M_{Pl}}$, one can envisage model building with a correct tree level mass spectrum without need of a Fayet-Iliopoulos term[4]. Broken supergravity naturally provides masses for scalar particles while keeping the matter-fermions (quarks, leptons) massless. An illustrative example was given in Ref. 9) and further arguments for an intermediate supersymmetry breaking in local supergravity were given by Barbieri et al.[7] and also in Ref. 23). This appears to be an attractive possibility for solving in a simple way the problem of mass splittings in spontaneously broken supersymmetric models of electroweak and strong interactions.

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