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Yaroslavl International Conference on Discrete Geometry (dedicated to the centenary of A. D. Alexandrov)[1]

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The Yaroslavl International Conference on Discrete Geometry dedicated to the centenary of A. D. Alexandrov was organized by the International B.N. Delaunay Laboratory "Discrete and Computational Geometry" and took place from August 13 to 18, 2012 at the P.G. Demidov Yaroslavl State University. The purpose of this note is to highlight the main results presented at the conference and to discuss the role of the meeting in the development of the field of Discrete and Computational Geometry in Yaroslavl.

The article is published in the author's wording.

Discrete Geometry in Yaroslavl

Discrete Geometry is a fast developing branch of modern mathematics, with applications in many areas, ranging from entertainment to manufacturing, and from medicine to facility planning. It is even more important in the new century, which is centered around innovation and technology. The Government of the Russian Federation supported the creation of a laboratory of Discrete and Computational Geometry at the Demidov Yaroslavl State University under the leadership of Herbert Edelsbrunner, one

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of the foremost experts in the area who combines scientific leadership with experience in commercial applications. In 2011, the International Delaunay Laboratory on Discrete and Computational Geometry was established.

The Russian schools of geometry and topology are recognized world wide for their academic leadership and the wealth and significance of their results in the fields. In spite of this leadership role, the gap between pure mathematical investigations and their commercial exploitation has remained. One of the fundamental principles of the Laboratory is the development of mutual interactions between mathematics, computations, and applications. To mention an example, the Laboratory promotes and develops persistent homology, a branch of Computational Topology that originates in a paper by H. Edelsbrunner, D. Letscher and A. Zomorodian [1]. Being based on ideas of Morse Theory (a classic field in Mathematics), persistent homology addresses the question of topological significance of data, and provides a method to separate noise from features. While the method is primarily algebraic, it has fast algorithms (Computation) that have effective implementations, giving detailed mathematical analyzes of large datasets in a matter of seconds or minutes. The results of these analyzes are used in disciplines that include cosmology, medicine, cartography, and data visualization (Applications).

At the Laboratory, a group of well-known specialists from Russia (Demidov Yaroslavl State University, Lomonosov Moscow State University, Steklov Mathematical Institute), Austria (Institute of Science and Technology in Vienna), and the USA (University of Texas in Brownsville), join forces to pursue common research, to popularize Discrete and Computational Geometry, and to educate for the purpose of creating a modern word-class center in the field. The organization of a representative international conference on theoretical and practical topics related to the research program is important for the development of this Laboratory, which is named after B. N. Delaunay, an outstanding Soviet geometer and a corresponding member of the Academy. The first Yaroslavl International Conference on Discrete Geometry is dedicated to a student of B. N. Delaunay, the academician A. D. Alexandrov, one of the originators of modern Discrete Geometry, whose centenary is celebrated in 2012.

Remembering A. D. Alexandrov

Born on the 4th of August 1912 in Volyn', the family estate in the Ryazan region, Alexander Danilovich Alexandrov was one of the greatest geometers of the Twentieth Century, dying on the 27th of July 1999. As member of a noble family, his father, Daniil Alexandrov graduated from Saint Petersburg University, rejected an offer to enter graduate school, and went to teach in a local school. Belonging to a Polish noble family, Alexandrov's mother, Elizaveta Bartoshevich also taught at that school after she graduated from a pedagogical institute.

A. D. Alexandrov entered Leningrad State University in 1929 to study physics, and graduated with a specialization in theoretical physics. His scientific advisor was the outstanding physicist Vladimir Fok. At the same time, Alexandrov entered a geometric crystallography seminar led by the outstanding mathematician Boris Delaunay. Alexandrov had to divide his time between physics and geometry, but being a member of the Physics Institute, he devoted most of his time to physics. His Master Diploma was dedi-

Figure 1. Alexandrov in 1952

cated to a link between group theory and quantum mechanics. However, the influence of Delaunay turned out to be stronger than that of Fok. In 1934, the interests of Alexandrov shifted towards mathematics, and in the beginning to Geometric Crystallography. In the same year, Alexandrov co-authored a book with B. Delaunay and N. Padurov on the "Mathematical Foundations of Structural Analysis of Crystals", and he wrote an important paper on 4-dimensional parallelohedra, in which he proved a celebrated theorem on convex polyhedra with centrally symmetrical faces. In 1935, Alexandrov defended his Candidate Dissertation.

Advised by B. Delaunay, Alexandrov moved to the systematic study of convex polyhedra and surfaces. He experienced a period of stormy activities and soon uncovered deep results on mixed volumes, defending his Doctoral Dissertation in the Spring of 1937. He was not yet 25 years old. Soon after, Alexandrov moved on to the H. Weyl problem about the realization of a regular Riemannian metric of positive curvature on a sphere in 3-dimensional Euclidean space. In the late 1930s, Alexandrov solved this problem under conditions on the metric that were substantially weaker than originally proposed. The

method of solution was deep and surprising, being based on considerations of a similar problem about non-negative polyhedral metrics. Along this way, Alexandrov discovered and proved one of the most amazing theorems in geometry, namely a criterion for an unfolding to be that of a convex polyhedron. Here, an unfolding is a collection of flat polygons glued to each other along matching sides. Alexandrov proved, that if the total sum of angles meeting at each vertex is smaller than or equal to 2π, and the unfolding is homeomorphic to a sphere, then it is the unfolding of a unique convex polyhedron. This statement is very surprising, but the method he used to prove it was even more impressive and unexpected. In particular, Alexandrov found a way to recognize when a map of one manifold *into* another manifold is a map *onto* the entire second manifold. By means of this method, he proved a series of results on the existence and uniqueness of polyhedra with given conditions. This method had exerted a strong influence on further investigations of his outstanding followers and students. During this period of his life, Alexandrov has made major contributions to the intrinsic geometry of convex surfaces. Just to state one example, he proved the Gluing Theorem for convex surfaces, which along with the theorem on the realization of convex metrics, has become a cornerstone of the modern theory of bending convex surfaces.

In 1964, Alexandrov followed an invitation of the head of the Siberian branch of the Academy of Sciences, Academician M. A. Lavrentiev, and moved to Novosibirsk (Academgorodok), where he worked for more than 20 years. In 1986 Alexandrov returned back to Saint Petersburg, where he worked in the St. Petersburg Department of Steklov institute of Mathematics until his death in July 1999.

Alexandrov has left a lasting legacy on the academic life by founding his famous Geometry School, with A. V. Pogorelov, Yu. G. Reshetnyak, Yu. F. Borisov, Yu. E. Borovskikh, Yu. D. Burago, A. K. Gutz, A. V. Kuzminykh, R. I. Pimenov, A. V. Shaidenko, A. L. Verner, V. A Zalgaller, A. M. Zamorzaev among his direct students and followers. In 1946, Alexandrov was elected a corresponding member, and in 1964, he became full member of the USSR Academy of Science. Among other distinctions, he was awarded the Stalin Prize in 1942, the International Lobachevski Prize in 1951, the Euler Golden Medal of Russian Academy of Science in 1991.

From 1952 to 1964, Alexandrov was the rector of Leningrad State University. His rectorship featured by distinguished and sometimes courageous decisions. To mention an outstanding example, he kept Genetics alive as a scientific discipline within the University, at a time this subject was under attack from the Communist powers for ideological reasons. Any resistance to the official Communist ideology, even any open dissent said loudly, was cruelly persecuted and extremely dangerous. All this demanded great civil courage. In the late 1960s, Alexandrov sheltered Vadim Delaunay, a grandson of Boris Delaunay, in his house in Academgorodok near Novosibirsk for almost a year. Vadim was a famous dissident, who came to Alexandrov after his first release from prison. Alexandrov loved him and helped him in every way humanly possible. When Vadim was sent to prison a second time, Alexandrov visited him twice in a forced labor camp in the Tyumen' region. Alexandrov had an independent mind and character, and he resisted the cruel and unfair political system to the extent possible.

Alexander Danilovich was a person of many talents – in philosophy and the methodology of science, in poetry, in music, and in mountain climbing – he had fantastic energy,

and an explosive temperament. He was a great lecturer and speaker. Alexandrov left an unforgettable impression in everybody who had the pleasure to know him personally. In 2012, three conferences dedicated to the centenary of A. D. Alexandrov have been organized in major Russian centers of Discrete Geometry where his students are now active: in Saint Petersburg, in Novosibirsk, and in Yaroslavl.

The Organization

The Program Committee consisted of members of the Delaunay Laboratory as well as other leading specialists in Discrete and Computational Geometry: H. Edelsbrunner (Chair), V. Buchstaber, N. Dolbilin, V. Dolnikov, S. Glyzin, A. Ivanov, O. Musin, S. Novikov, Yu. Reshetnyak, and E. Shchepin. The Local Organizing Committee took on the responsibility for planning and scheduling. The Chair, M. Nevskii, and the members, A. Garber, D. Glyzin, A. Maksimenko, A. Nikolaev, E. Nevskaya, P. Parfenov, A. Ukhalov, and O. Yakimova did an excellent job that laid the foundations for the success of the conference, which also benefited from the gracious and experienced assistance of Alexander Rusakov, Rector of Demidov Yaroslavl State University, and his staff. The Conference was funded by the Government of the Russian Federation, under Grant 220, Contract 11.G34.31.0053.

The Program

The international program committee strived to cover topics in pure as well as in practical areas of Discrete and Computational Geometry. For this reason, the conference represented contributions from a wide range of branches, which we now categorize and list.

Computational Topology has become a separate field of research after the seminal paper by Edelsbrunner, Letscher and Zomorodian [1]; see also the textbook by Edelsbrunner and Harer [2]. It investigates methods for recognizing topological properties of geometric and physical objects from experimental data, such as the coordinates of points, values of functions, and other measurable characteristics. Using theoretical ideas of Morse theory and algebraic topology as well as modern techniques from computer science, it has exciting applications in various fields in the sciences. This branch of topology is not well known in Russia, which makes it all the more important to represent it at the conference. The talks within this subfield presented at the conference are:

- Y. Bazaikin and I. Taimanov, *Morse theory and numerical algorithms for the computation of topological characteristics of three-dimensional bodies*;
- S. Glyzin, *Finite-dimensional models of diffusion chaos*;
- M. Mrozek, *Homological persistence of maps*;
- V. Manturov, *The parity bracket and reducing questions about knots to questions about their representatives*;
- A. Akopyan, *Cutting the same fraction of several measures*;

- B. Wang, *Local homology computation with Vietoris-Rips complexes*;
- B. T. Fasy, *Geometry of Gaussian mixture models*;
- R. Karasev, *Projective ham sandwich theorems*;
- G. Sharygin, *The Euler class of a combinatoric circle bundle.*

The **Theory of Polyhedra** is a classic branch of Discrete Geometry, going back to work in ancient Greece. L. Euler, A. Cauchy, H. Minkowski, and A. D. Alexandrov contributed much to this field. Today, the theory of polyhedra attracts the interest of many specialists, not only because of the beauty and depth of the results, but also because of applications in medicine, biology, and computer science. The talks presented at the conference are:

- I. Pak, *The quantitative Steinitz problem*;
- C. Nara, *Continuous flattening of Archimedean polyhedra*;
- I. Sabitov, *New classes of rigid polyhedra and their volume polynomials*;
- A. Gaifullin, *Sabitov polynomials for volume of* 4*-dimensional polyhedra*;
- N. Erokhovets, *Buchstaber invariants of simple polytopes*;
- M. Kozachok, *Perfect prismatoids and the conjecture concerning face numbers of centrally symmetric polytopes*;
- A. Magazinov, *Convex hull of a Poisson point process in the Clifford torus*;
- A. Maximenko, *Estimating the number of facets of a* 2*-neighborly polytope*;
- A. Nikolaev, *Vertices of the cut polytope, relaxations, and* 3*-uniform hypergraphs*;
- V. Volodin, *Geometric realizations of gamma-vectors of* 2*-truncated cubes.*

Motivated by the growing importance of computers, **Computational Geometry** has grown in size and importance during the last decades, complementing Discrete Geometry by covering related algorithmic questions. It deals with computational representation of geometric objects and the calculation of their geometric characteristics. In spite of its application-oriented character, Computational Geometry is based on deep results in mathematics. The talks within this subfield presented at the conference are:

- M. Kerber, *The dual complex of hyper-rectangular partitions*;
- M. Belkin, *Machine learning and differential geometry of data*;
- B. Atamanyuk, *About space tetrizations of Delaunay*;
- L. Mestetskiy, *Skeleton of binary image versus binary image of skeleton*;
- N. Dyshkant, *Efficient localization of mesh nodes in Delaunay triangulations*;

- J. Phillips, *Epsilon-samples for kernels*;
- E. Timofeev, *Nearest neighbor searching on sequence space*;
- O. Yakimova, *A new algorithm for the generalization of linear cartographic objects.*

Questions about **Geometric Arrangements** include problems on packings, tilings, lattices, coverings, and crossings. They belong to classic topics of Discrete Geometry and go back to works of I. Newton and D. Gregory, J. Kepler and C. F. Gauss. Problems of this kind arising naturally and are simple to formulate, but they are often exceedingly difficult and deep while having important applications in coding theory, logistics, design, and other areas. The field attracts the interest of many specialists all over the world. The talks within this subfield presented at the conference are:

- A. Bezdek and W. Kuperberg, *Unavoidable crossings in thinnest coverings of the plane/sphere with congruent convex disks*;
- E. Bannai, *Various kinds of t-designs and tight t-design*;
- I. Barany, *On a question of V. I. Arnold*;
- R. Erdal, *Minkowski sums of Voronoi polytopes and commensurate Delaunay tilings*;
- H. Nozaki, *Oriented graphs and complex spherical codes*;
- A. Gavrilyuk, *Local liftability of tilings.*

Geometric Optimization deals with critical points of geometric functionals. Examples include geodesics, minimal networks, minimal surfaces, harmonic surfaces, and constant curvature surfaces. Problems in this field demand a synthetic approach that combines methods of discrete and differential geometry, combinatorics, differential equations, graph theory, and convex analysis. Many geometric optimization problems have important applications, such as the classic Steiner problem on the shortest connection of a finite point set in a metric space. The talks within this subfield presented at the conference are:

- H. Martini, *Recent results in Minkowski geometry*;
- J.-I. Itoh, *Closed geodesics on a regular dodecahedron*;
- M. Nevskii, *On homothetic copy of a simplex which contains a convex body*;
- D. Ilyutko and I. Nikonov, *Extreme networks and the Maxwell formula in normed spaces.*

Combinatorial Methods are of great importance in Discrete Geometry and Computational Topology. One reason is that many natural problems in these fields lead to huge enumerations of structures, another that geometric and topological approaches often have non-trivial geometric interpretations which lead to progress on combinatorial problems. The talks within this subfield presented at the conference are:

- L. Montejano, *A new topological Helly theorem and some transversal results*;
- I. Shkredov, *Additive structures in subgroups*;
- A. Antonov and V. Bondarenko, *Graphs of polytopes for the complete bipartite subgraph problem*;
- G. Chelnokov, *On transversals of quasi-algebraic families of sets*;
- I. Shnurnikov, *On the number of complement regions in submanifold arrangements*.

Complementing the presentations of research results, an **Open Problems Session** reminded us of important old and interesting new open problems in Discrete Geometry. These were presented by R. Karasev, O. Musin, M. Kerber, I. Barany, L. Montejano, A. Bezdek, H. Martini, C. Nara, J.-I. Itoh, and other participants. The statements of some of these problems are available on the website of the Laboratory (`http://dcglab.uniyar.ac.ru`). In addition, we had a special session devoted to the memory of **Alexandr Danilovich Alexandrov**. Nikolay Dolbilin and Idzhad Sabitov shared their memories of Alexandr Danilovich as an outstanding scientist, principled leader, remarkable teacher, cheerful man, and a Knight of the Truth.

Concluding Remarks

The six days of the conference featured 16 plenary talks of 40 minutes each, 10 plenary talks of 25 minutes, and 16 sectional talks of 25 minutes, with a total of more than 50 participants from 10 countries. The conference brought together world-leading experts in Discrete and Computational Geometry and Topology. They presented results on the state of the art and recent progress in the field. We feel it is important to underline the participation of many young mathematicians. Many of the talented young scientists attracted to the field of Discrete Geometry by the International Delaunay Laboratory presented their results at the conference. Among them there were A. Gaifullin, who presented his outstanding theorem on 4-dimensional bellows, R. Karasev, who introduced his beautiful ham sandwich theorems, V. Manturov, who spoke about new knot representations, and I. Shkredov, who talked about applications of additive group structures. We also mention A. Akopyan, N. Erokhovets, A. Maksimenko, A. Nikolaev, I. Nikonov, G. Chelnokov, A. Gavrilyuk, M. Kozachok, A. Magazinov, V. Volodin, and I. Shnurnikov for their participation in the conference. The young foreign participants worked primarily in the field of the Computational Topology. Among them were B. Wang from Utah, who discussed the application of persistent homology to the reconstruction of stratified spaces, B. Fasy from North Carolina and Austria, who talked about the geometry of Gaussian mixture models, M. Kerber from Austria, who presented results on the geometric realization of complexes dual to partitions with hyper-rectangles, and J. Phillips from Utah, who introduced ε-samples as applied to the study of uncertainty in data. The book of abstracts is already published [3], and this volume contains a subset of the presented papers.

Conferences of moderate format like this one can be more effective with more impact than large congresses. All participants can enjoy the intimate atmosphere and communicate with each other freely, form new connections, lead fruitful discussions, and start collaborations with like-minded or complementing researchers. All this contributes to the progress of the subject to which this conference is devoted. We hope that the further development of the International Delaunay Laboratory at the Demidov State University as a center of Discrete and Computational Geometry gives an opportunity to make such conferences in Yaroslavl a long-lasting tradition.

References

1. H. Edelsbrunner, D. Letscher and A. Zomorodian, "Topological persistence and simplification," *Discrete Comput. Geom.* **28**, 511–533, 2002.

2. H. Edelsbrunner, J. Harer, *Computational Topology: An Introduction*, Amer. Math. Soc., Providence, Rhode Island, 2010.

3. Yaroslavl International Conference "Discrete Geometry", Dedicated to the Centenary of A. D. Alexandrov. August 13–18, 2012. Abstracts, Yaroslavl State University, Russian Federation, 2012.

Ярославская международная конференция «Дискретная геометрия», посвященная 100-летию А. Д. Александрова

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Ярославская международная конференция «Дискретная геометрия», посвященная 100-летию А. Д. Александрова, была организована и проведена Международной лабораторией «Дискретная и вычислительная геометрия» им. Б. Н. Делоне с 13 по 18 августа 2012 года в Ярославском государственном университете им. П.Г. Демидова. Целью настоящей статьи является освещение основных результатов, представленных на конференции, и обсуждение ее роли в развитии дискретной и вычислительной геометрии в Ярославле. Статья публикуется в авторской редакции.

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