

## NOTE

### Yield and Purification in Multistage Countercurrent Separation Systems

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#### Abstract

The approaches of Klinkenberg and Rony offer a route to a complete analytical solution of the problem of multistage countercurrent separation systems, but some revisions of their equations are required to express the true maximal separative power of multistage systems.

Although solutions for special cases had appeared earlier, Klinkenberg (1) was the first to publish a complete analytical theory of multistage countercurrent separation systems, and Rony (2) followed his approach. The latter presented his results in a different way and employed different symbols, but it should clearly be possible to relate the two sets of equations. Our interest was stimulated by finding that the equations of Rony and Klinkenberg did not give concordant results.

Klinkenberg's key equation (Eq. 25 of Ref. 1) may be written:

$$R = \frac{(r_e + 1)\phi^{n+m} - r_e\phi^{n+m-1} - \phi^m}{\phi^m + r_w\phi - (r_w + 1)} \quad (1)$$

where  $R$  = ratio of  $\frac{\text{net outflow of solute in Phase 2}}{\text{net outflow of solute in Phase 1}}$

$r_e$  and  $r_w$  are reflux ratios defined as follows:

$$\frac{\text{total outflow in Phase 2}}{\text{solute returned (at same end) in Phase 1}} = \frac{r_w + 1}{r_w}$$

$$\frac{\text{total outflow in Phase 1}}{\text{solute returned (at same end) in Phase 2}} = \frac{r_e + 1}{r_e}$$

$n$  and  $m$  are numbers of ideal stages in the extraction and washing sections respectively,

$\phi$  is the distribution ratio of the solute considered, defined in the simplest case as (concentration in Phase 2)/(concentration in Phase 1). But see also the discussion in Ref. 2. (In this note we do not discuss the case where  $\phi = 1$ , when certain of the equations become indeterminate.  $\phi$  corresponds to  $E$  in Ref. 1 and to  $\dot{K}_i$  in Ref. 2).

In Ref. 1 the value of the coefficient of  $\phi^m$  in the denominator is given as  $(r_w + 1)$ , whereas it should be simply 1.

With  $R$  defined as above, the net fractional yield of solute emerging in Phase 1 is (Ref. 2, eq. 33, rearranged and transcribed into the present symbols):

$$(Y)_1 = \frac{1}{1 + R} = \frac{1 + \frac{1}{r_w} \frac{\phi^m - 1}{\phi - 1}}{1 + \frac{r_e}{r_w} \phi^{n+m-1} + \frac{1}{r_w} \frac{\phi^{n+m} - 1}{\phi - 1}} \quad (2)$$

If  $r_e/r_w$  is allowed to retain a finite value as  $r_w$  approaches infinity, the limit is

$$(Y)_1 = \frac{1}{1 + \frac{r_e}{r_w} \phi^{n+m-1}} \quad (3)$$

Rony's result for a system with total reflux (Ref. 2, Eq. 25) was

$$(Y)_1 = \frac{1}{1 + \phi^{n+m-1}} \quad (4)$$

and Eq. (3) shows that this is actually valid only for the special case where  $r_e = r_w$  as both approach infinity, which is a quite unnecessary restriction. Clearly the value of  $(Y)_1$  can be manipulated at will by varying the ratio of  $r_e$  and  $r_w$ , even when both are very large.

Klinkenberg (1) was not consistent in his use of  $R$ . When discussing a system with total reflux, he defined  $R$  so as to represent the ratio of total outflows, which is a very different thing from the ratio of net outflows. One would not, for example, contemplate using the ratio of total outflows in the case of partial reflux. Thus, while Klinkenberg's result (Eq. 6, below) is correct, it is misleading if the significance of  $R$  is taken to be always the same.

There is a conceptual difficulty over interpreting what is meant by a ratio of net outflows at total reflux, but nevertheless this is the quantity required for comparison with  $R$  values at finite reflux, and the correct form is

$$R = \frac{r_e}{r_w} \eta^{n+m-1} \quad (5)$$

rather than

$$R = \eta^{n+m-1} \quad (6)$$

(as given in Ref. 1). Equation (5) expresses, of course, a limiting value of  $R$ .

Obviously, the maximum possible performance of a multistage separation system (countercurrent extraction, distillation, fractional crystallization) is much better than would be deduced from Eqs. (4) and (6).

#### REFERENCES

1. A. Klinkenberg, *Chem. Eng. Sci.*, 1, 86 (1951).
2. P. R. Rony, *Sep. Sci.*, 5, 1 (1970).

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