

## Yield Condition and Propagation of Lüders' Lines in Tension-Torsion Experiments on Poly(vinyl Chloride)

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### Synopsis

We have derived from the Eyring theory of non-Newtonian flow a yield condition which is valid for an arbitrary state of stress. Experimental data obtained in simple axial compression tests show the influence of the hydrostatic stress on the yielding of poly(vinyl chloride). This fact confirms the proposed condition and disproves the von Mises criterion. Tension-torsion tests performed on thin tubes lead to results which fit our condition fairly well. The pattern of Lüders' lines appearing on the surface of thin tubes subjected to simple tension, simple shear, and tension-torsion are parallel to the direction where the value of the normal stress is equal to the hydrostatic stress.

### Introduction

In a previous paper<sup>1</sup> we have proposed a condition which defines the stress level at which significant plastic deformation starts. Our treatment was derived from the Eyring theory of non-Newtonian flow<sup>2</sup> where deformation is a rate process. The yield condition takes into account the influence of the hydrostatic stress and consists in a generalization of the von Mises criterion.

Experimental data obtained in tensile and compressive tests fit the theory fairly well.<sup>1</sup> It is the purpose of this paper to study the validity of our yield condition for combined tension and torsion tests and to observe the directions of the Lüders lines.

### Criterion

In the case where an arbitrary state of stress is applied, plastic deformation starts when  $W$ , a critical value of the energy, is reached.  $W$  depends on temperature and strain rate and means the mechanical energy a segment of macromolecule needs to jump from one equilibrium position to another.

We have given previously<sup>1</sup> the following expression for  $W$ :

$$W = v_0 \gamma_0 \sqrt{\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2} + v_0 p (\Delta v_0 / v_0) \quad (1)$$

where  $v_0$  is the volume of the segment of macromolecule,  $\Delta v_0$  is the volume increase occurring as a segment jump,  $\gamma_0$  is the elementary strain,  $p$  is the

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hydrostatic stress, and  $\tau_{xy}$ ,  $\tau_{yz}$ ,  $\tau_{zx}$  are the components of the deviator of the stress.

We believe that condition (1) is expressed even more simply by using Nadai's concept of the octahedral shearing stress  $\tau_0$ .<sup>3</sup> Equation (1) may therefore be expressed by writing:

$$\tau_0 + Ap = f(\dot{\epsilon}, T) \quad (2)$$

where  $A$  is a constant, and  $T$  and  $\dot{\epsilon}$  are temperature and strain rate, respectively. At a given temperature and strain rate, plastic deformation starts when:

$$\tau_0 + Ap = \text{constant} \quad (3)$$

This condition is reduced to von Mises' criterion when the hydrostatic stress vanishes.

According to eq. (3), the yield stress in tensile tests  $\sigma_t$  must differ from the yield stress in compressive tests  $\sigma_c$ :

$$\frac{\sigma_t}{\sigma_c} = \frac{\sqrt{2} - A}{\sqrt{2} + A} \quad (4)$$

If the von Mises condition were valid for high polymers,  $\sigma_t$  would equal  $\sigma_c$ .

For a tension-torsion test at given temperature and strain rate, condition (3) must be written:

$$^{1/3} \sqrt{6\tau^2 + 2\sigma^2} + A(\sigma/3) = C \quad (5)$$

where  $\sigma$  and  $\tau$  are the applied stresses and where:

$$A = \frac{\sqrt{2}(\sigma_c - \sigma_t)}{\sigma_c + \sigma_t} \quad (6)$$

$$C = \frac{2\sqrt{2}\sigma_c\sigma_t}{3(\sigma_c + \sigma_t)}$$

In this case, the components of the plastic strain may be evaluated if one calculates the maximum energy dissipation for a given value of the first strain invariant, according to our previous paper.<sup>1</sup> If  $\epsilon$  and  $\gamma$  are the tensile and shear strain respectively, the first invariant of the strain is:

$$I_1 = \frac{\epsilon^2(3 + \epsilon)}{1 + \epsilon} + \gamma^2 + 3 \quad (7)$$

$$\simeq 3\epsilon^2 + \gamma^2 + 3$$

The energy dissipation per unit volume is:

$$W_{dis} = \sigma\epsilon + \tau\gamma \quad (8)$$

On taking into account eq. (7), eq. (8) becomes:

$$W_{dis} = \sigma\epsilon + \tau \sqrt{I_1 - 3 - 3\epsilon^2} \quad (9)$$

It follows that  $W_{dis}$  is maximum when:

$$\sigma/3\epsilon = \tau/\gamma \quad (10)$$

From condition (10) we may derive the relationship between the applied stresses  $\sigma$  and  $\tau$ , and the elongation rate  $\dot{\epsilon}$  and the shear rate  $\dot{\gamma}$  at the yield limit:

$$\sigma\dot{\gamma}/3 = \tau\dot{\epsilon} \quad (11)$$

The yield value depends on the strain rate. Two different rates of strain  $\dot{\Gamma}_1$  and  $\dot{\Gamma}_2$  can be compared if one considers the time  $t$  necessary to produce a plastic strain related to a given value of  $I_1$ . It follows from eq. (7) that:

$$t = \sqrt{I_1 - 3}/\sqrt{\dot{\gamma}^2 + 3\dot{\epsilon}^2} \quad (12)$$

thus:

$$\begin{aligned} \dot{\Gamma}_1/\dot{\Gamma}_2 &= t_2/t_1 \\ &= \sqrt{\dot{\gamma}_1^2 + 3\dot{\epsilon}_1^2}/\sqrt{\dot{\gamma}_2^2 + 3\dot{\epsilon}_2^2} \end{aligned} \quad (13)$$

### Experimental

To determine whether our relationship (3) is valid, we have chosen poly(vinyl chloride) (Solvic 227, from Solvay et Cie) because this glassy polymer possesses a stress-strain curve with a well definite yield point and exhibits fine Lüders' lines.

We carried out our tests with thin tubes. The shape and dimensions of the test pieces used in simple tension and in tension-torsion tests are shown in Figure 1.

Compression tests were made on hollow cylindrical specimens 2 mm high, which were cut from the central part of the test pieces shown in Figure 1.

Tensile and compression curves were obtained with an Instron testing machine at room temperature. Both tests were made at the same rate of strain,  $\dot{\epsilon} = 10\%/min$ . (This value was calculated from the crosshead speed because at the yield point the rate of change of stress is zero although the strain is increasing at a constant rate.) The imposed strain was parallel to the axis of the specimens.

Tension-torsion stress-strain curves were obtained with an Instron testing machine combined with a classical torsion apparatus (see Fig. 2). The imposed elongation was parallel to the axis of the specimen, the elongation rate  $\dot{\epsilon}$  was chosen equal to  $10\%/min$ . The applied shearing stress  $\tau$  produced by hanging weights was normal to the axis of the specimen. The shear rate  $\dot{\gamma}$  depending on  $\tau$  was measured by using an Instron marker control. Whenever the angle of torsion varied from a constant value, a "pip" appeared on the stress-strain curve.

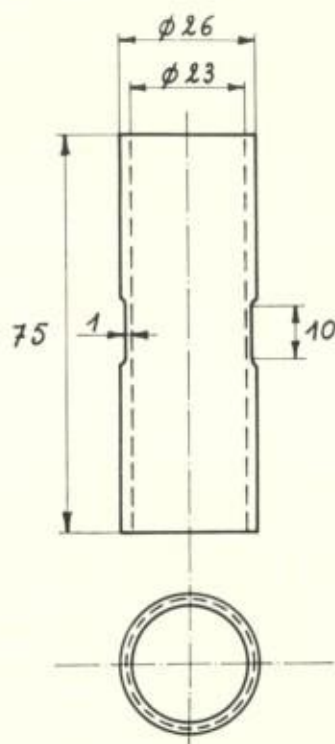


Fig. 1. The test piece used in simple tension and in tension-torsion tests. Unit: 1 mm.

An example of a stress-strain curve is given in Figure 3; the distance between two adjacent pips corresponds to a variation of shear strain  $\Delta\gamma = 4.4 \times 10^{-3}$ .

### Results and Discussion

Tensile and compression tests performed at the same temperature (23°C) and the same rate of strain (10%/min) give the following value:

$$\sigma_c / \sigma_t = 1.30 \quad (14)$$

$\sigma_c$  as well as  $\sigma_t$  corresponds to the average of four experimental values. The fact that  $\sigma_c$  differs from  $\sigma_t$  proves the influence of the hydrostatic stress and gives some confidence in the validity of our treatment.

According to eqs. (5); (6), and (14), yielding in a tube subjected to tension-torsion should start when

$$2.3\sqrt{3\tau^2 + \sigma^2} + 0.3\sigma = 2.6\sigma_t \quad (15)$$

On plotting  $\sigma$  and  $\tau$  as rectangular coordinates, the expression above gives an ellipse. This ellipse is shown in Figure 4; the ellipse corresponding to the von Mises criterion is also given for comparison.

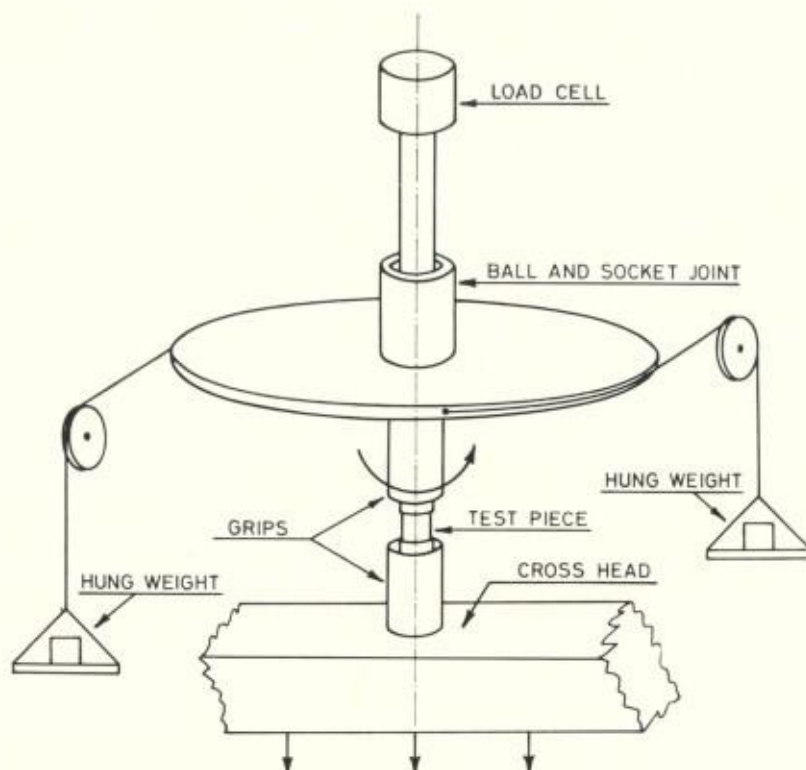


Fig. 2. Apparatus for combined tension and torsion tests.

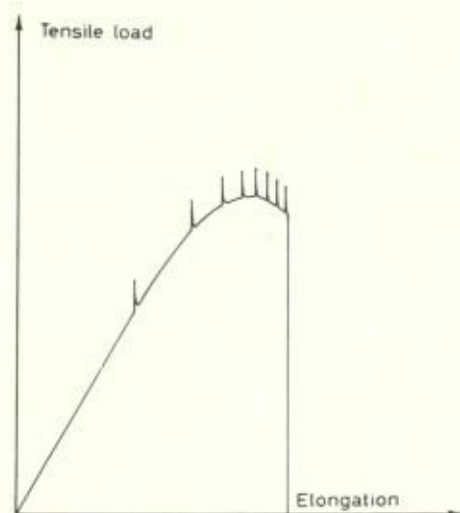


Fig. 3. Example of stress-strain curve obtained in a tension-torsion test. The "pips" on the curve allow evaluation of the shear rate  $\dot{\gamma}$ .

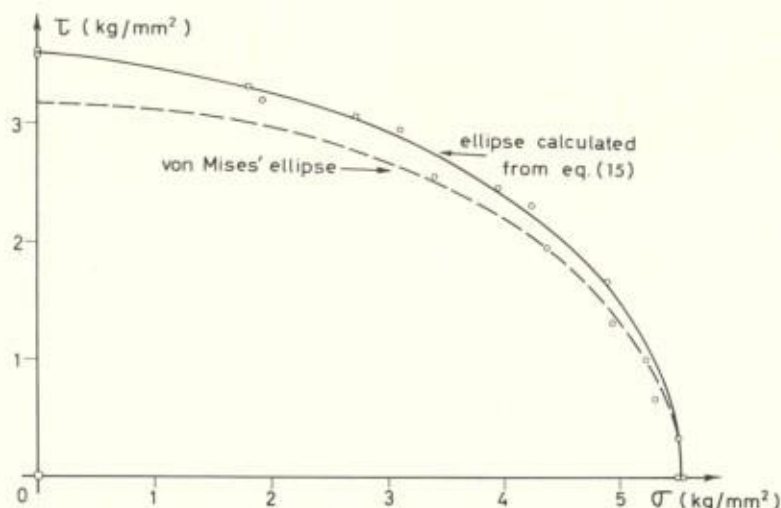


Fig. 4. Plot of  $\tau$  vs.  $\sigma$  at the yield limit for tension-torsion tests. Experimental data are compared to the ellipse corresponding to eq. (15) and to the von Mises ellipse.

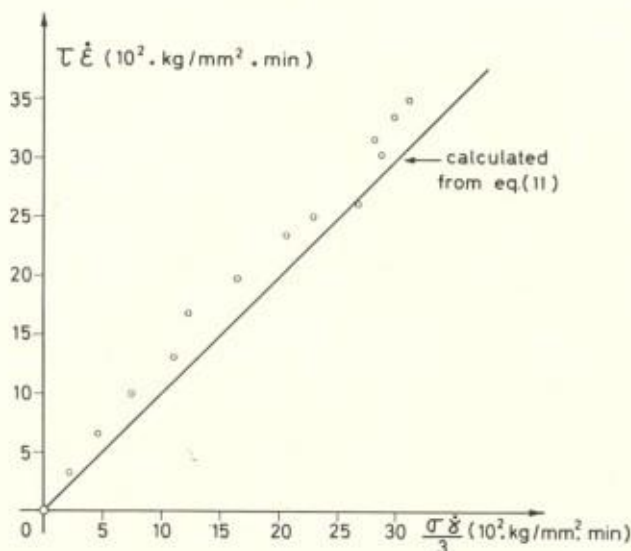


Fig. 5. Plot of  $\tau \dot{\epsilon}$  vs.  $\sigma \dot{\gamma}/3$ . Experimental data are compared to the straight line corresponding to eq. (11).

Our tests were made at room temperature, which varied between 22.1°C and 23.9°C. As in our equipment the shear stress was imposed,  $\dot{\gamma}$  depended on  $\tau$ . In order to compare our tests points with the ellipses shown on Figure 4, the measured values of  $\sigma$  and  $\tau$  have to be corrected to take into account the changes in temperature and shear rate.

We have studied previously<sup>4</sup> the variation of the yield stress of poly(vinyl chloride) with temperature and strain rate. From these results

we know that the stress is 1.5% higher when temperature is lowered from 1°C and 3% higher when strain rate is doubled. We have reduced our data to a constant temperature (23°C).

From eq. (13) we may compare strain rates in order to correct the values of  $\sigma$  and  $\tau$ . All the data have been reduced to the strain rate of the tensile test  $\dot{\epsilon} = 10\%/min$ . This correction is significant for high shear loads; it exceeds 11% in the present case and may not be neglected.

Tests points are plotted in Figure 4; they follow very closely the upper ellipse corresponding to eq. (15), disproving von Mises' criterion.

On plotting  $\tau\dot{\epsilon}$  versus  $\sigma\dot{\gamma}/3$ , eq. (11) gives a straight line which is drawn in Figure 5. The experimental points lie about 10% above the theoretical straight line, but it is not possible to draw conclusions because the accuracy of our measurements of  $\dot{\gamma}$  at the yield point is also about 10%.

### Luders' Lines

Several theories based on different concepts have been proposed<sup>1,5-7</sup> to explain the inclination of the Lüders' lines observed on thin specimens of high polymers or metals strained in tensile tests.

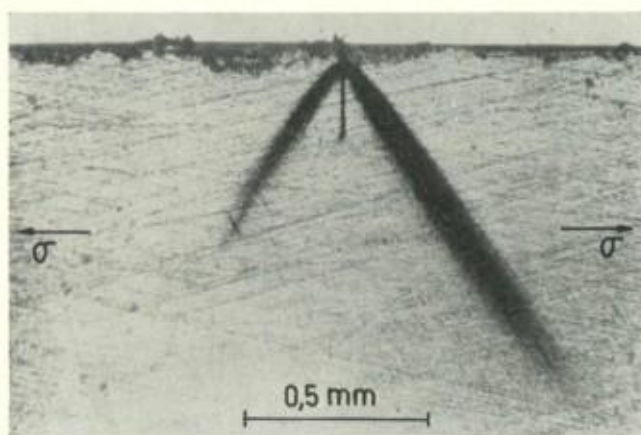


Fig. 6. Lüders' lines appearing on the surface of a specimen subjected to simple tension  $\sigma$ .

According to these theories, Lüders' lines must make with the direction of the tensile stress an angle  $\theta$  defined by:

$$\cos \theta = \frac{1}{\sqrt{3}} \quad (16)$$

In the direction which fits condition (16) the normal stress has the value of the hydrostatic stress.

In the case of simple tensile tests, Lüders' bands inclined at an angle of 55° with respect to the tensile stress appear on our test pieces; this value fits eq. (16) fairly well (see Fig. 6).

In the case of plane stresses, as in tension-torsion tests, the direction where the normal stress has the value of the hydrostatic stress makes with the direction of the major stress  $\sigma_1$  an angle  $\theta$  defined by:

$$\cos^2 \theta = (\sigma_1 - 2\sigma_2) / [3(\sigma_1 - \sigma_2)] \quad (17)$$

where  $\sigma_2$  is the minor stress.

We have compared the values of  $\theta$  obtained from eq. (17) with the inclination of Lüders' lines appearing on the thin tubes subjected to tension-torsion. In this case, eq. (17) may be written:

$$\cos^2 \theta = \frac{3\sqrt{\sigma^2 + 4\tau^2} - \sigma}{6\sqrt{\sigma^2 + 4\tau^2}} \quad (18)$$

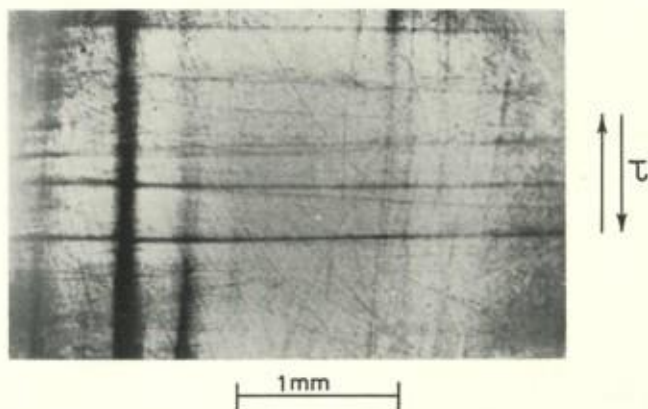


Fig. 7. Lüders' lines pattern on a specimen subjected to simple shear stress. The two sets of lines are, respectively, parallel and perpendicular to the direction of  $\tau$ .

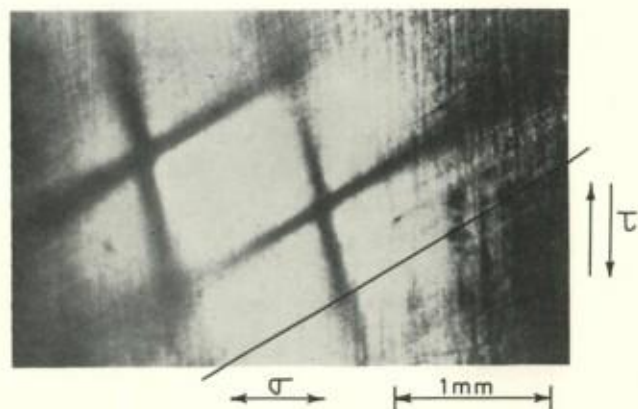


Fig. 8. Lüders' lines pattern on a specimen subjected to tension-torsion ( $\sigma/2\tau = 0.968$ ). The direction where the normal stress has the value of the hydrostatic stress is given on the micrograph.

In simple shear, Lüders' lines are two sets of straight lines parallel and normal, respectively, to the applied shear stress (see Fig. 7). These directions fit the values of  $\theta$  calculated from eq. (18) in this special case.

In Figure 8 we give an example of Lüders' lines appearing on the surface of a test piece subjected to tension-torsion. The angle calculated from eq. (18) is  $\theta = 52^\circ$ . The two sets of Lüders' lines must form an angle  $2\theta$ ; the value measured on the micrograph is  $2\theta = 105^\circ \pm 2^\circ$ . According to our results, Lüders' lines seem to form in the direction defined by eq. (18).

### Conclusions

The yield stress corresponding to simple tension tests differs from the yield stress corresponding to simple compression tests. This experimental fact shows the influence of the hydrostatic stress on the yielding of high polymers and is in agreement with the proposed yield condition.

The yield stress measured in tension-torsion tests fit our yield condition fairly well, disproving the von Mises criterion.

The observed inclination of the Lüders' lines appearing on thin tubes subjected to simple tension, simple shear, and tension-torsion, are parallel to the direction where the normal stress has the value of the hydrostatic component of the stress.

Our yield condition implies that plastic deformation is a rate process. This condition consists in a generalization of the von Mises criterion and is based on a physical concept.

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