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# Yield-Constrained Optimization Design Using Polynomial Chaos for Microwave Filters

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**ABSTRACT** Yield optimization aims at finding microwave filter designs with high yield under fabrication tolerance. The electromagnetic (EM) simulation-based yield optimization methods are computationally expensive because a large number of EM simulations is required. Moreover, the microwave filter design usually requires several performance objectives to be met, which is not considered by the current yield optimization methods for microwave filters. In this paper, an efficient yield-constrained optimization using polynomial chaos surrogates (YCOPCS) is employed for microwave filters considering multiple objectives. In the YCOPCS method, the low-cost and high-accuracy of polynomial chaos is used as a surrogate. An efficient yield-constrained design framework is implemented to obtain the optimal design solution. Two numerical examples demonstrate the performance of the YCOPCS method, including a coupling matrix model of a fourth-order filter with cascaded quadruplet topology and an EM simulation model of a microwave waveguide bandpass filter. The numerical results show that the YCOPCS method can obtain the filter designs with higher yield and reduce EM simulations by 80% compared to Monte Carlo-based yield optimization in all testing examples.

**INDEX TERMS** Yield optimization, polynomial chaos, microwave filters.

## I. INTRODUCTION

Manufacturing (process) variations in the structures of the microwave filters are unavoidable despite using advanced manufacturing techniques [1]–[3]. The yield is a ratio of the number of qualified products to the total number of products under fabrication tolerance [4], [5]. The manufacturing industry strives to increase the yield of microwave filter production to reduce the cost. In this case, it is important to obtain the designs with high yield before fabrication. Yield optimization method [6]–[9] focuses on this task and aims at finding designs with high yield for a certain performance specification.

Yield optimization consists of two stages: yield estimation (calculating the yield of a given set of design variable values) and optimization (searching for the design with a high yield). Yield estimation usually provides the objective of yield optimization [10]–[13]. Monte Carlo sampling (MCS) method [14] is a conventional method for yield estimation of microwave filters [15], [16]. To accurately estimate the yield, a large number of electromagnetic (EM) simulations are required, which is very time-consuming [17]. To reduce EM simulation costs, surrogate-based methods, using low-cost mathematical models to replace EM models, are often utilized in the yield estimation of microwave filters [17]–[20]. Among various methods, machine learning techniques are adopted in the surrogate-based yield estimation, e.g. artificial neural network [18] and Gaussian process regression [19]. However, these methods still require lots of EM simulation samples for surrogate modeling.

Polynomial chaos (PC) [21]–[23] is an analytical representation with an orthonormal polynomial basis that requires fewer samples than other models such as artificial neural networks and Gaussian process regression. PC is a popular

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technique in the field of uncertainty quantification, where it is typically used to replace a computationally expensive model with an inexpensive-to-evaluate polynomial function [22], [23]. The PC model obtained enables reliable estimation of the statistics of the output, provided that a suitable probabilistic model of the input is available. In recent years, the PC method has been used in microwave filter design [5], [24]. As a result, the computational cost of the PC method is significantly reduced for yield estimation than that of the MCS method.

For yield optimization, gradient-based optimizers are often utilized [4], [5], [17], [25]. In [4], three gradient-based optimizers are reviewed for the yield optimization of the microwave circuit models. To improve the search ability of gradient-based optimization methods, the modified ellipsoidal technique is designed for obtaining the design center of gradient-based optimization methods [17]. A boundary gradient search technique is used for generating a sequence of points on the boundary of the feasible region [25]. Since gradient-based optimization algorithms are easy to fall into local optimum [26], it is difficult to find the optimal solution accurately by gradient-based optimization. To partially address this issue, the objective function is transformed into a simplified form [5].

The microwave filters usually have several design specifications such as return loss, insertion loss, passband ripple in various bands. The current yield optimization methods [4], [5] only consider the maximum yield with respect to one of them. The maximization of the yield often contradicts with other specifications or performance objectives. For example, in order to reduce the insertion loss in the passband, it may be necessary to reduce bandwidth. As a result, directly optimizing the yield with respect to one specification may lead to the deterioration of other performance objectives.

To address the problems mentioned above, a yield-constrained optimization utilizing polynomial chaos surrogate (YCOPCS) for microwave filters is proposed. In the stage of yield estimation, cost-reduced PC models representing the statistics of the output of the EM models are established. We define yield functions and objective functions according to the sensitivity of all specifications with the manufacturing tolerance. The more sensitive specifications are defined as yield functions. An efficient yield-constrained global design framework is designed to obtain the optimal objective function value with yield functions as constraints. A set of numerical experiments on microwave filters are implemented to test the performance of the proposed method.

The rest of the paper is organized as follows. In section II, we introduce the basic knowledge of yield optimization. In section III, the YCOPCS method is described in detail. Test examples for the method are demonstrated in Section IV. The conclusions are drawn in Section V.

## II. BASIC KNOWLEDGE OF YIELD OPTIMIZATION

In order to clearly describe yield optimization, the acceptance index is defined for indicating whether the yield function $f_{yie}(\boldsymbol{x}, \boldsymbol{\zeta})$ satisfying the specification $u$ or not.

$$I(\boldsymbol{x}, \boldsymbol{\xi}) = \begin{cases} 1, & \text{if } f_{yie}(\boldsymbol{x}, \boldsymbol{\xi}) \le u \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where the $n$-dimensional vector $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^T$ is design variables. The $n$-dimensional vector $\boldsymbol{\xi} = [\boldsymbol{\xi}_1, \boldsymbol{\xi}_1, \ldots, \xi_n]^T$ is the process variations.

In this work, we assume that $\boldsymbol{x}$ is uniformly distributed in the bounded domain and $\boldsymbol{\xi}$ follows a Gaussian distribution with a probability density function $\rho(\boldsymbol{\xi})$. The yield for $\boldsymbol{x}$ is defined as the probability of $I(\boldsymbol{x}, \boldsymbol{\xi}) = 1$, which is defined by

$$Prob(I(\boldsymbol{x}, \boldsymbol{\xi}) = 1 \mid x) = \mathrm{E}[I(\boldsymbol{x}, \boldsymbol{\xi})] = \int_{-\infty}^{\infty} I(\boldsymbol{x}, \boldsymbol{\xi})\rho(\boldsymbol{\xi})d\boldsymbol{\xi} \tag{2}$$

Since it is difficult to calculate $prob(I(\boldsymbol{x}, \boldsymbol{\xi}) = 1|\boldsymbol{x})$ directly [27], yield estimation is used to obtain the approximate solution of (2). Two classical methods, namely yield estimation using the Monte Carlo sampling (MCS) method and yield estimation using the PC model, are introduced in Section II.A and Section II.B, respectively. Based on the yield estimation, the yield optimization framework is described in Section II.C.

### A. YIELD ESTIMATION USING MCS METHOD

Yield estimation using the MCS method is a conventional method for obtaining the yield of a given design. It includes simulating manufacturing errors of the design variables, calculating the acceptance index of the samples with manufacturing errors, and estimating yield. The details of yield estimation using the MCS method are as follows:

Firstly, the manufacturing errors are simulated with uniform distribution. The $k$-th simulated manufacturing error $\boldsymbol{\xi}_k$ is defined as

$$\boldsymbol{\xi}_k = \boldsymbol{r}_k \circ \boldsymbol{s} \tag{3}$$

where $\boldsymbol{r}_k \circ \boldsymbol{s}$ represents Hadamard product [28] between $\boldsymbol{s}$ and $\boldsymbol{r}_k$. $\boldsymbol{s}$ is the difference between the lower bound and upper bound of the manufacturing error. $\boldsymbol{r}_k$ is a uniformly distributed random vector with the element between $-1$ and 1.

Secondly, EM simulation is performed at the sample $\boldsymbol{x} + \boldsymbol{\xi}_k$ and the acceptance index of $\boldsymbol{x} + \boldsymbol{\xi}_k$ is calculated using equation (1).

Finally, the yield estimation is approximated by the ratio of acceptable samples to the total samples, defined by

$$Prob(I(\boldsymbol{x}, \boldsymbol{\xi}) = 1 \mid \boldsymbol{x}) = \frac{\sum_{k=1}^{K} I\left(\boldsymbol{x} + \boldsymbol{\xi}_k\right)}{K} \tag{4}$$

where $K$ is the number of simulated samples in process variations.

For the microwave filters, in order to estimate the yield accurately, a large number of EM simulations are required. Therefore, yield estimation using the MCS method is time-consuming.

### B. YIELD ESTIMATION USING THE PC MODEL

Compared with yield estimation using the MCS method, yield estimation using the PC model is more efficient because it adopts a low-cost PC surrogate model replacing the expensive EM simulations. PC can accurately describe the stochastic process of the random variables in most distribution [29]. The function $f(\boldsymbol{\xi})$ of random variables $\boldsymbol{\xi}$ can be represented by a spectral expansion as follows

$$f(\boldsymbol{\xi}) \approx f^{PC}(\boldsymbol{\xi}) = \sum_{\alpha=0}^{p} c_\alpha \Phi_\alpha(\boldsymbol{\xi}) \tag{5}$$

where $p$ is the order of PC expansion, the multivariate polynomials $\Phi_\alpha(\boldsymbol{\xi})$ is given by the product of corresponding one-dimensional polynomials $\phi_\alpha^{(i)}(\xi_i)$

$$\Phi_\alpha(\boldsymbol{\xi}) = \prod_{i=1}^{n} \phi_\alpha^{(i)}(\xi_i) \tag{6}$$

Various polynomials $\phi_\alpha^{(i)}(\xi_i)$ have been proposed [30], they are selected based on the probability density distributions of random variables. For the uniformly independent random variables $\boldsymbol{\xi}$, the spectral expansion of Legendre polynomials (7) are used.

$$\begin{cases} \phi_0^{(i)}(\xi_i) = 1 \\ \phi_1^{(i)}(\xi_i) = \xi_i \\ \phi_{\alpha+1}^{(i)}(\zeta_i) = \left(\frac{2\alpha+1}{\alpha+1}\right)\xi_i\phi_\alpha^{(i)}(\xi_i) - \left(\frac{\alpha}{\alpha+1}\right)\xi_i\phi_{\alpha-1}^{(i)}(\xi_i), \\ \qquad for\ \alpha \geq 1 \end{cases} \tag{7}$$

For the Gaussian distributed random variables $\boldsymbol{\xi}$, the spectral expansion of Hermite polynomials (8) are used.

$$\phi_\alpha^{(i)}(\boldsymbol{\xi}_i) = (-1)^\alpha e^{(\xi_i)^2/2} \frac{d^\alpha}{d\xi_i^\alpha} e^{-(\xi_i)^{2/2}} \tag{8}$$

The total number of basis functions $N_p$ in polynomials (7) and (8) is the function of the dimension of design variable $n$ and order of PC polynomial $p$.

$$N_p = \frac{(n+p)!}{n!p!} \tag{9}$$

Once the PC model is established, it replaces the EM simulation models to perform the yield estimation.

### C. YIELD OPTIMIZATION FRAMEWORK

Yield optimization is to find the design variable values with maximal yield. Two common optimization frameworks include direct yield optimization and yield-constrained optimization. The first uses yield as the objective function directly [5], [19]. It only considers the yield with respect to one performance specification. The optimization problem is formulated in equation (10). However, it does not consider other performances.

$$\boldsymbol{x}^* = \arg\max_x \mathrm{Prob}(I(\boldsymbol{x}, \boldsymbol{\xi}) = 1 \mid \boldsymbol{x}) \tag{10}$$

where $\boldsymbol{x}^*$ is the optimal design.

Another method adopts the yield as the constraint [27] and employs other performances as the objective function $f_{obj}(\boldsymbol{x}, \boldsymbol{\xi})$. This method considers both the yield function and objective function and we call this method yield-constrained optimization. It is defined by

$$\begin{aligned} &\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} f_{obj}(\boldsymbol{x}, \boldsymbol{\xi}) \\ &s.t. Prob\,(I\,(\boldsymbol{x}, \boldsymbol{\xi}) = 1\,|\boldsymbol{x}) \geq 1 - \varepsilon \end{aligned} \tag{11}$$

where $\varepsilon$ is a user-defined requirement of yield.

## III. YCOPCS METHOD

An efficient yield-constrained optimization using polynomial chaos surrogate (YCOPCS) is implemented for microwave filters considering multiple objectives. YCOPCS method uses a sampling reduction strategy for reducing the cost of the PC Model for yield estimation as described in Section III.A. The yield-constrained optimization implemented in the YCOPCS method is introduced in Section III.B. The summary of the YCOPCS method is described in Section III.C. The discussion of the YCOPCS method is presented in Section III.D.

### A. SAMPLING REDUCTION STRATEGY FOR PC MODELS

In the YCOPCS method, the yield function $f_{yie}(\boldsymbol{x}, \boldsymbol{\xi})$ is employed for the constraint. $f_{obj}(\boldsymbol{x}, \boldsymbol{\xi})$ is used for the objective function. The PC models of $f_{yie}(\boldsymbol{x}, \boldsymbol{\xi})$ and $f_{obj}(\boldsymbol{x}, \boldsymbol{\xi})$ are established using Legendre polynomials (7) and Hermite polynomials (8) as expressed in equations (12) and (13), respectively.

$$f_{obj}(\boldsymbol{x}, \boldsymbol{\xi}) \approx \sum_{\beta=0}^{p}\sum_{\alpha=0}^{p} h_{\alpha,\beta}\Phi_\alpha(\boldsymbol{x})\psi_\beta(\boldsymbol{\xi}) \tag{12}$$

$$f_{yie}(\boldsymbol{x}, \boldsymbol{\xi}) \approx \sum_{\beta=0}^{p}\sum_{\alpha=0}^{p} c_{\alpha,\beta}\Phi_\alpha(\boldsymbol{x})\psi_\beta(\boldsymbol{\xi}) \tag{13}$$

where $\Phi_\alpha(\boldsymbol{x})$ and $\psi_\beta(\boldsymbol{\xi})$ are basis functions of uniformly distributed design variables $\boldsymbol{x}$ and the Gaussian distributed process variation $\boldsymbol{\xi}$, respectively. $h_{\alpha,\beta}$ and $c_{\alpha,\beta}$ are coefficients of $f_{obj}(\boldsymbol{x}, \boldsymbol{\xi})$ and $f_{yie}(\boldsymbol{x}, \boldsymbol{\xi})$, respectively.

The mean value of $f_{yie}(\boldsymbol{x}, \boldsymbol{\xi})$ is approximated by

$$E_\xi\left(f_{yie}(\boldsymbol{x}, \xi)\right) \approx \sum_{\alpha=0}^{p} c_{\alpha,0}\Phi_\alpha(\boldsymbol{x}) \tag{14}$$

And the variance of $f_{yie}(\boldsymbol{x}, \boldsymbol{\xi})$ is approximated by

$$Var_{\boldsymbol{\xi}}\left(f_{yie}(\boldsymbol{x}, \boldsymbol{\xi})\right) \approx \sum_{\beta=1}^{p}\left(\sum_{\alpha=0}^{p-\beta} c_{\alpha,\beta}\Phi_\alpha(\boldsymbol{x})\right)^2 \tag{15}$$

A stochastic collocation method [31] is used to generate samples. Firstly, $M_1$ quadrature points for design variables and $M_2$ quadrature points for process variations are determined using the sparse grid approach [32]. Then, in order to reduce the EM simulation samples for modeling, the joint quadrature samples for both design variables $\boldsymbol{x}$ and process variations $\boldsymbol{\xi}$ are obtained through a process involving

$$\min_{\boldsymbol{x}_k, \boldsymbol{\xi}_k, w_k} \sum_{j_1=0}^{N_{2p}}\sum_{j_2=0}^{N_{2p-j_1}}\left(\sigma_{0_{j_1}}\sigma_{0_{j_2}} - \sum_{k=1}^{M}\Phi_{j_1}(\boldsymbol{x}_k)\psi_{j_2}(\boldsymbol{\xi}_k)w_k\right)^2 \tag{16}$$

where

$$\sigma_{0\,j_1}\sigma_{0\,j_2} = \begin{cases} 1 & j_1 = j_2 = 0 \\ 0 & \text{otherwise} \end{cases}$$

Based on (16), joint quadrature points and their weights $\{\boldsymbol{x}_k, \boldsymbol{\xi}_k, w_k\}_{k=1}^{M}$ are obtained. The number of joint quadrature points $M$ satisfies $N_p \leq M \leq N_{2p}$. According to quadrature points and weights, $h_{\alpha,\beta}$ and $c_{\alpha,\beta}$ can be well computed by

$$c_{\alpha,\beta} \approx \sum_{k=1}^{M} f_{yie}\left(\boldsymbol{x}_k, \boldsymbol{\xi}_k\right) \Phi_\alpha\left(\boldsymbol{x}_k\right) \psi_\beta\left(\boldsymbol{\xi}_k\right) w_k \quad (17)$$

$$h_{\alpha,\beta} \approx \sum_{k=1}^{M} f_{obj}\left(\boldsymbol{x}_k, \boldsymbol{\xi}_k\right) \Phi_\alpha\left(\boldsymbol{x}_k\right) \psi_\beta\left(\boldsymbol{\xi}_k\right) w_k \quad (18)$$

### B. YIELD-CONSTRAINED OPTIMIZATION

Yield-constrained optimization framework is adopted in this paper. The probabilistic constraint of yield is expressed as

$$Prob_{\boldsymbol{\xi}}\left(f_{yie}\left(\boldsymbol{x}, \boldsymbol{\xi}\right) \leq u\right) \geq 1 - \varepsilon \quad (19)$$

where 1-$\varepsilon$ is the desired yield by users (or called chance), and $\varepsilon \epsilon$ [0,1] is the risk level that users can accept. Formula (21) is also known as the chance constraint [27], [33]–[36] used for obtaining the optimal solution in different yield levels.

The probabilistic constraint (19) is transformed into a deterministic constraint (20). Taking advantage of the mean (14) and the variance (15) of the yield function, equation (20) becomes a convex second-order cone constraint [37].

$$\sqrt{\frac{(1-\varepsilon)}{\varepsilon}} \mathrm{Var}_{\boldsymbol{\xi}}\left(f_{\mathrm{yie}}\left(\boldsymbol{x}, \boldsymbol{\xi}\right)\right) + E_{\boldsymbol{\xi}}\left(f_{\mathrm{yie}}\left(\boldsymbol{x}, \boldsymbol{\xi}\right)\right) \leq u \quad (20)$$

Then, we combine the yield constraint and the objective function into the following yield-constrained optimization problem.

$$\min_{\boldsymbol{x}} E_{\boldsymbol{\xi}}\left(f_{obj}(\boldsymbol{x}, \boldsymbol{\xi})\right)$$
$$s.t. \sqrt{\frac{(1-\varepsilon)}{\varepsilon}} Var_{\boldsymbol{\xi}}\left(f_{yie}\left(\boldsymbol{x}, \boldsymbol{\xi}\right)\right) + E_{\boldsymbol{\xi}}\left(f_{yie}\left(\boldsymbol{x}, \boldsymbol{\xi}\right)\right) \leq u \quad (21)$$

Substituting equations (15), (16), (17), and (18) into equation (21), we obtain

$$\min_{\boldsymbol{x}} \sum_{\alpha=0}^{p} h_{\alpha,0} \Phi_\alpha\left(\boldsymbol{x}\right)$$
$$s.t. \sqrt{\frac{(1-\varepsilon)}{\varepsilon}} \sqrt{\sum_{\beta=1}^{p}\left(\sum_{\alpha=0}^{p-\beta} c_{\alpha,\beta}\Phi_\alpha\left(\boldsymbol{x}\right)\right)^2} + \sum_{\alpha=0}^{p} c_{\alpha,0}\Phi_\alpha\left(\boldsymbol{x}\right) \leq u \quad (22)$$

Since the square-root terms are difficult to optimize [38], equation (22) is transformed into equation (23) to reduce the complexity of the problem.

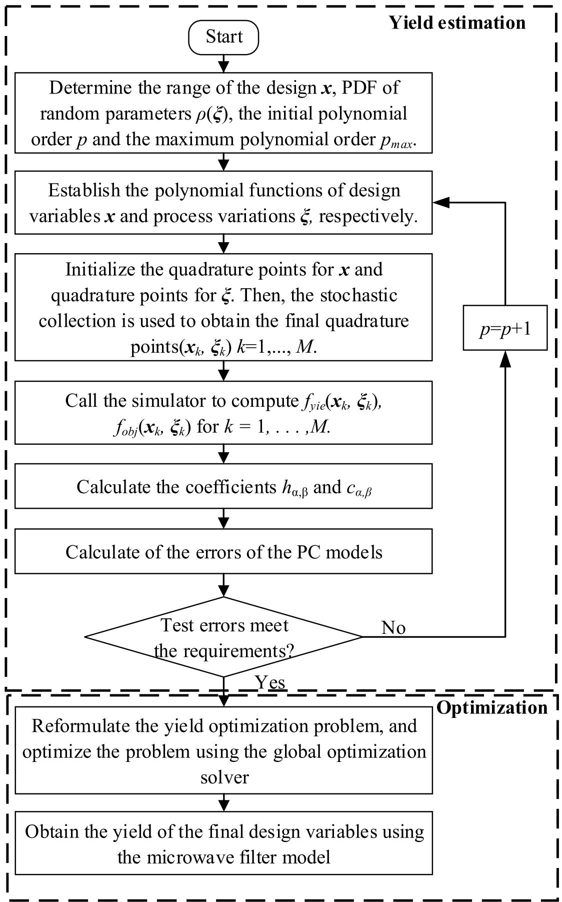


**FIGURE 1. Flowchart of the proposed YCOPCS method.**

$$\min_{x\in\chi} \sum_{\alpha=0}^{p} h_{\alpha,0} \Phi_\alpha\left(\boldsymbol{x}\right)$$
$$s.t. \frac{(1-\varepsilon)}{\varepsilon} \sum_{\beta=1}^{p}\left(\sum_{\alpha=0}^{p-\beta} c_{\alpha,\beta}\Phi_\alpha\left(\boldsymbol{x}\right)\right)^2 \leq \left(u - \sum_{\alpha=0}^{p} c_{\alpha,0}\Phi_\alpha\left(\boldsymbol{x}\right)\right)^2 \quad (23)$$

where $\chi$ is the design region of the design variables $\boldsymbol{x}$.

Both the simplified constraints and objective function are polynomials. We can obtain the optimal design variables using any global polynomial solvers. Therefore global optimization solver [39] is used to obtain the optimal design in (23) in this paper.

### C. SUMMARY OF THE YCOPCS METHOD

For yield estimation, the stochastic collocation method is used to reduce the EM simulation samples for the PC modeling, and an order adaptive strategy is designed for improving the accuracy of the PC model. For optimization, the yield-constrained optimization (also known as the chance constraints technique) is employed to obtain optimal designs with yield as a constraint. The probabilistic objective function and constraints are transformed into an equivalent polynomial optimization problem to reduce complexity. The flowchart of the algorithm is shown in Fig. 1. The YCOPCS method is summarized in the following steps.

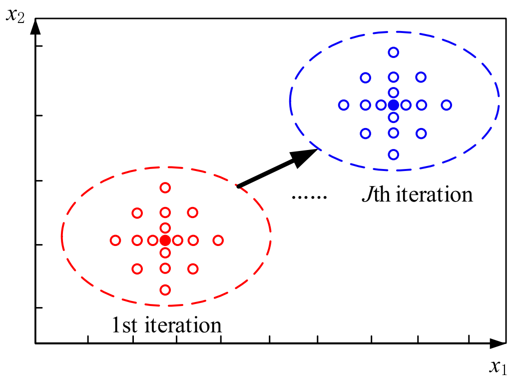


**FIGURE 2. Schematic of the yield optimization with continuous updating of the PC model [5].**

*Step 1):* Determine the range of the design $\boldsymbol{x}$, the probability distribution function of stochastic parameters $\rho(\boldsymbol{\xi})$, the initial polynomial order $p = 2$, and the maximum polynomial order $p_{max}$.

*Step 2):* Establish polynomial functions (7) and (8) for design variables $\boldsymbol{x}$ and process variations $\boldsymbol{\xi}$, respectively.

*Step 3):* Initialize the quadrature points for $\boldsymbol{x}$ and quadrature points for $\boldsymbol{\xi}$. Then, the stochastic collection is used to obtain the quadrature points $(\boldsymbol{x}_k, \boldsymbol{\xi}_k)k = 1, \ldots, M$.

*Step 4):* Call the simulator to compute $f_{obj}(\boldsymbol{x}_k, \boldsymbol{\xi}_k)$ and $f_{yie}(\boldsymbol{x}_k, \boldsymbol{\xi}_k)$ for $k = 1, \ldots, M$.

*Step 5):* Calculate the coefficients $h_{\alpha,\beta}$ and $c_{\alpha,\beta}$ of the polynomials, respectively.

*Step 6):* Calculate the errors of the PC models (the yield PC model and the objective PC model). If the test errors meet the requirements, perform Step 7. Otherwise, $p = p + 1$, perform Step 2.

*Step 7):* Set up the yield-constrained optimization problem, and optimize the design variables using a global optimization solver to obtain the final design.

*Step 8):* Obtain the yield of the final design using the MCS method using the microwave filter model.

### D. DISCUSSION ON THE YCOPCS METHOD

#### 1) THE EFFICIENCY OF THE YCOPCS METHOD

We compare our proposed method with an existing yield optimization method [5] for microwave filters. It continuously updates the new PC model according to the design center obtained in each iteration, as shown in Fig. 2. The required modeling samples are $J \cdot Np$, where $J$ is the number of iterations in the optimization process.

In our proposed method, we establish the PC model in the entire design space considering both the design variables and the process variations. According to Section III.A, the number of modeling samples is $M(N_p < M < N_{2p})$ in our proposed method. $M = N_{2p}$ is the upper bound of EM simulation samples and can be expanded with $N_p$ term as the following form.

$$M = \frac{(2p+n)\,(2p+n-1)\cdots(n+p+1)}{2}\,\frac{(n+p)!}{n!p!} \tag{24}$$

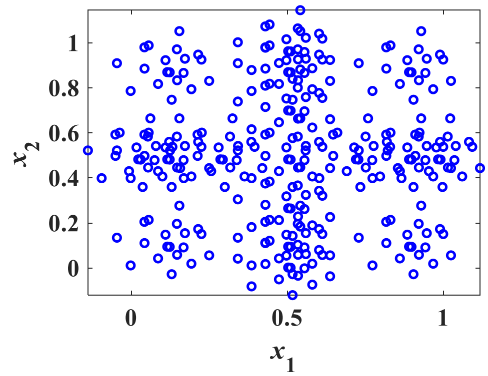


**FIGURE 3. Samples for modeling without sampling reduction strategy.**

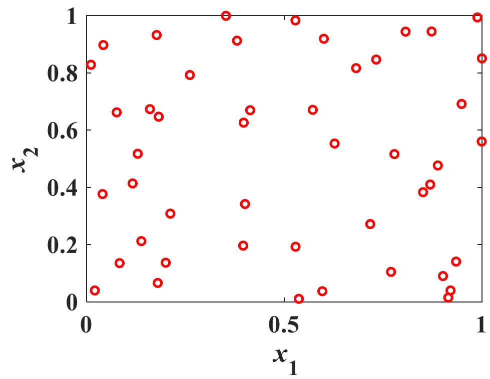


**FIGURE 4. Samples for modeling with sampling reduction strategy.**

Substituting (9) into (24) we can find that the number of EM simulation samples $J \cdot N_p$ using the method in [5] is larger than our proposed YCOPCS method if $J$ satisfies $J > \frac{(n+2p)(n+2p-1)\cdots(n+p+1)}{2}$.

In order to further demonstrate the sampling reduction strategy described in Section III.A of YCOPCS, the $p = 3, n = 2$ situation in design space [0, 1] is taken as an example. $M_1 = 17$ quadrature points for the design variables and $M_2 = 19$ quadrature points for process variations are determined using the sparse grid approach. The total number of initial sample points is $M_1 \cdot M_2 = 323$, as shown in Fig. 3. After the sampling reduction strategy, the number of samples is reduced to only 47 in the design variables space, as shown in Fig. 4. It is shown that the sampling reduction strategy of the proposed YCOPCS method largely reduces the number of samples. Therefore, it achieves significant computational cost savings for expensive EM simulation.

#### 2) THE EFFECTIVENESS OF THE YCOPCS METHOD

The effectiveness of the YCOPCS method depends on the accuracy of PC models. In this work, an order adaptive strategy is designed for improving the accuracy of the PC models. Besides, the valid design region is the key to guarantee that the optimal yield design can be found. The valid design region is defined by $[\boldsymbol{x}_0 - r\tau, \boldsymbol{x}_0 + r\tau]$, where $\boldsymbol{x}_0$ is the initial design, $\tau$ is the standard deviation of the manufacturing error, and $r$ determines the size of the design space. In [5], the difference between the initial design and the optimal yield design is smaller than 2.5 $\boldsymbol{\tau}$. In this work, $r > 4$ is recommended,

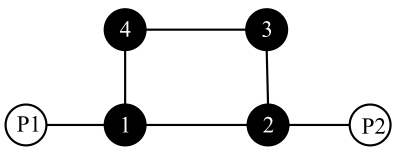


**FIGURE 5.** Topology of the fourth-order filter with cross-coupling.

so that our algorithm works in a large enough design region to ensure the optimal yield can be found. In the two examples in Section IV, $r > 4.7$ for all design variables, and successfully optimized to the optimal yield.

## IV. VERIFICATION EXAMPLES

In this section, in order to test the performance of the YCOPCS method, it is compared with MC-based yield optimization [4] by solving two examples including a bandpass filter design problem based on coupling matrix, and a waveguide filters based on the EM simulation. Both methods are implemented in the MATLAB 2019a platform. The software is run on a desktop computer with the configuration of Intel Core i5-6500 3.20 GHz and 20 GB RAM.

### A. EXAMPLE 1: A COUPLING MATRIX MODEL OF A FOURTH-ORDER FILTER WITH CROSS-COUPLING

A coupling matrix model representing a fourth-order filter with cross-coupling is used as a mathematical model to verify the YCOPCS method [40]. The topology of the bandpass filter is shown in Fig. 5. The filter is operating at the center frequency of 11 GHz with a bandwidth of 300MHz. Two finite transmission zeros are assigned at 10.7 GHz and 11.3 GHz, respectively.

The normalized coupling matrix is written as:

$$[\boldsymbol{M}] = \begin{bmatrix} 0 & M_{S1} & 0 & 0 & 0 & 0 \\ M_{S1} & 0 & M_{12} & 0 & M_{14} & 0 \\ 0 & M_{12} & 0 & M_{23} & 0 & 0 \\ 0 & 0 & M_{23} & 0 & M_{12} & 0 \\ 0 & M_{14} & 0 & M_{12} & 0 & M_{S1} \\ 0 & 0 & 0 & 0 & M_{S1} & 0 \end{bmatrix} \quad (25)$$

The $S$-parameters of the microwave filter can be calculated as

$$\boldsymbol{S}_{11} = \pm\left(1 - 2A_{11}^{-1}\right) \quad (26)$$

$$\boldsymbol{S}_{21} = 2\boldsymbol{A}_{21}^{-1} \quad (27)$$

where $S_{11}$ is the reflection coefficient at the port attached to the first resonator, and $S_{12}$ is the transmission coefficient between the ports attached to the first and second resonators. The immittance matrix $\boldsymbol{A} = \boldsymbol{R} + \boldsymbol{U} - j\boldsymbol{M}$, where $\boldsymbol{U} = \begin{bmatrix} \boldsymbol{u} & 0 \\ 0 & 0 \end{bmatrix}$ and $\boldsymbol{R} = \begin{bmatrix} 0 & 0 \\ 0 & \boldsymbol{r} \end{bmatrix}$.$\boldsymbol{u}$ is a $4 \times 4$ identity matrix and $\boldsymbol{r}$ is a $2 \times 2$ identity matrix.

The yield function and the objective function are shown in (28) and (29).

$$f_{yie}(\boldsymbol{x}, \boldsymbol{\xi}) = \max |S_{11}|_{\text{dB}} (\boldsymbol{x}, \boldsymbol{\xi}, f) \quad (28)$$

$$f_{obj}(\boldsymbol{x}, \boldsymbol{\xi}) = \max \left(|S_{12}|_{\text{dB}} (\boldsymbol{x}, \boldsymbol{\xi}, f_1), |S_{12}|_{\text{dB}} (\boldsymbol{x}, \boldsymbol{\xi}, f_2)\right) \quad (29)$$

**TABLE 1.** Optimized design variables for the fourth-order filter with cross-coupling.

| | $\varepsilon$=0.05 | $\varepsilon$=0.1 |
|---|---|---|
| $M_{S1}$ | 1.14521 | 1.14521 |
| $M_{12}$ | 0.97984 | 0.97402 |
| $M_{23}$ | -0.23350 | -0.24209 |
| $M_{41}$ | 0.86163 | 0.86163 |

**TABLE 2.** Yield results of different methods for the fourth-order with cross-coupling.

| Algorithms | Yield (%) | Objective function value (dB) | Number of samples |
|---|---|---|---|
| Initial point | 89.00 | -41.14 | — |
| *Our proposed* ($\varepsilon$=0.05) | 95.00 | -33.66 | 375 |
| *Our proposed* ($\varepsilon$=0.10) | 94.00 | -39.40 | 375 |
| MC-based | 92.50 | -38.82 | 2000 |

where $f$ is in the frequency band of [10.85GHz 11.15GHz], $f_1$ is 10.7 GHz and $f_2$ is 11.3 GHz. Because the topology is symmetric, the design variables are $\boldsymbol{x} = [M_{S1}, M_{12}, M_{23}, M_{41}]^T$. The initial design variable is $\boldsymbol{x}_0 = [M_{S1}, M_{12}, M_{23}, M_{41}]^T = [1.11185, 0.95130, -0.23504, 0.83654]^T$. The design variables $\boldsymbol{x}$ admits a uniform distribution in $[1.034025, 0.88471, -0.21858, 0.77798]^T \times [1.14521, 0.97984, -0.24209, 0.86163]^T$ and the uncertain parameter $\boldsymbol{\xi}$ follows a Gaussian distribution with 0.005 standard deviation. The specification of yield for this filter is given by $|S_{11}|_{\text{dB}} < -20$ dB from 10.85 GHz to 11.15 GHz. The final yield optimization results of the proposed YCOPCS method are verified by the MCS method using 200 mathematical function evaluations (number of samples), and 200 mathematical function evaluations are used in each iteration of MC-based yield optimization with 10 iterations.

The errors of the obtained objective and yield PC models with respect to the mathematical model are 7.61e$^{-2}$ and 5.96e$^{-2}$, respectively. The errors are smaller than the predefined constant (0.10) so that the objective and yield PC models are suitable for optimization. The optimized design variables are shown in Table 1 for the risk levels $\varepsilon = 0.05$ and $\varepsilon = 0.10$. Table 2 lists the final yields of the design solution using the proposed method with the two risk levels and MC-based yield optimization, as well as the required number of mathematical function evaluations. It is seen from Table 2 that the optimization results of the YCOPCS method are better than MC-based yield optimization on accuracy and simulation cost. The proposed YCOPCS method obtains the optimal solution with a 95 % yield using 375 mathematical function evaluations. MC-based yield optimization obtains 92.50 % yield using 5 times more mathematical function evaluations than our proposed method. Fig. 6 shows the yield before and after optimization using the proposed method for the fourth-order bandpass filter.

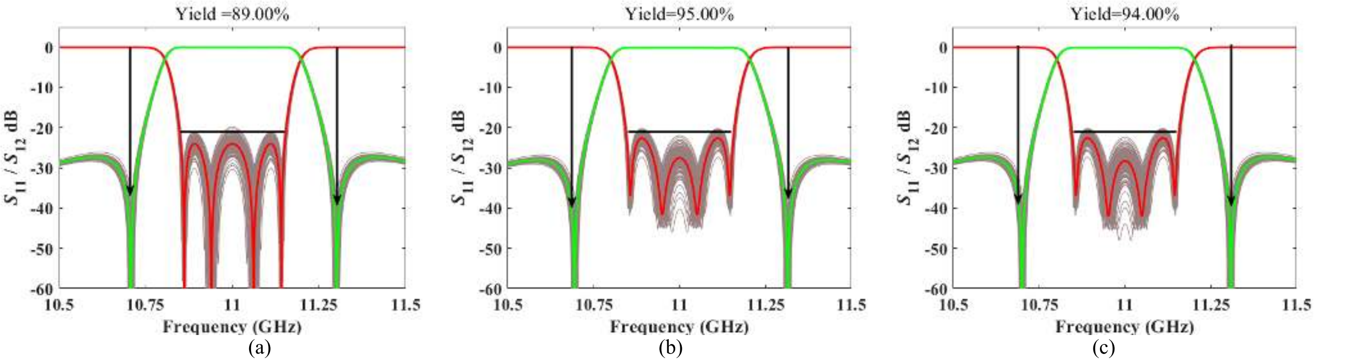


**FIGURE 6.** Yield optimization results of the fourth-order filter with cross-coupling using MCS method on mathematical model evaluation (a) initial design, (b) our proposed ($\varepsilon = 0.05$) and (c) our proposed ($\varepsilon = 0.10$). Gray lines: 200 mathematical model evaluation samples. Red line: $S_{11}$ response evaluated at the optimal design solution. Green line: $S_{12}$ response evaluated at the optimal design solution.

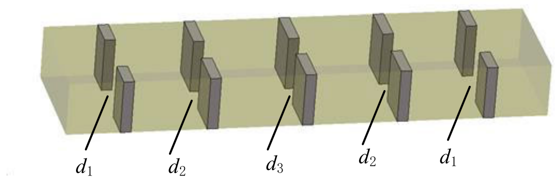


**FIGURE 7.** Geometry of the fourth-order bandpass waveguide filter with process variation of metal diaphragms $x = [d_1, d_2, d_3]^T$.

### B. EXAMPLE 2: A FOURTH-ORDER BANDPASS WAVEGUIDE FILTER WITH PROCESS VARIATIONS OF METAL DIAPHRAGMS

A waveguide bandpass filter [41] is used to test the performance of the proposed YCOPCS method. The geometry of the structure is shown in Fig. 7. The design parameters are $\boldsymbol{x} = [d_1, d_2, d_3]^T$(mm), where $d_1$, $d_2$, and $d_3$ represent the distances between the first, second, and third pairs of metal diaphragms, respectively. The section dimensions of the waveguide are $a = 22.86$ mm and $b = 10.16$ mm (WR-90). The specification for the yield of this filter is given by $|S_{11}|_{\text{dB}} \leq -19$ dB in the frequency range from 10.88 GHz to 11.16 GHz. The objective function is the maximum value of $|S_{12}|_{\text{dB}}$ at 10.7 GHz and 11.3 GHz. The yield function and the objective function are formulated in (30) and (31), respectively.

$$f_{yie}(\boldsymbol{x}, \boldsymbol{\xi}) = \max |S_{11}|_{\text{dB}} (\boldsymbol{x}, \boldsymbol{\xi}, f) \quad (30)$$

$$f_{obj}(\boldsymbol{x}, \boldsymbol{\xi}) = \max (|S_{12}|_{\text{dB}} (\boldsymbol{x}, \boldsymbol{\xi}, f_1), |S_{12}|_{\text{dB}} (\boldsymbol{x}, \boldsymbol{\xi}, f_2)) \quad (31)$$

where $f$ is in the frequency band of [10.88 GHz 11.16 GHz], $f_1$ is 10.7 GHz and $f_2$ is 11.3 GHz. The design variables $\boldsymbol{x}$ satisfy a uniform distribution in $[10.3940, 6.6389, 6.0855]^T \times [10.6040, 6.7731, 6.2085]^T$, and the process variations $\boldsymbol{\xi}$ follow a Gaussian distribution with 20$\mu$m standard deviation. The initial design variable value is $\boldsymbol{x}_0 = [10.499, 6.706, 6.147]^T$. The final yield optimization results are verified by the MCS method using 200 EM simulations (CST). As a comparison, the MC-based yield optimization is performed with 10 iterations and 200 EM simulations (CST) at each iteration.

**TABLE 3.** Optimized design variable values for the fourth-order bandpass waveguide filter with process variation of metal diaphragms /mm.

| | $\varepsilon$=0.05 | $\varepsilon$=0.10 |
|---|---|---|
| $d_1$ | 10.546 | 10.550 |
| $d_2$ | 6.721 | 6.734 |
| $d_3$ | 6.157 | 6.175 |

**TABLE 4.** Yield results of different methods for the fourth-order bandpass waveguide with process variations of metal diaphragms.

| Algorithms | Yield (%) | Objective function value (dB) | Number of samples |
|---|---|---|---|
| Initial point | 68.00 | -23.00 | — |
| *Our proposed* ($\varepsilon$=0.05) | 91.50 | -23.19 | 248 |
| *Our proposed* ($\varepsilon$=0.10) | 83.50 | -23.19 | 248 |
| MC-based | 82.50 | -22.98 | 1400 |

The errors of the obtained objective and yield PC models are 8.72e$^{-2}$ and 7.91e$^{-2}$, respectively. The errors are smaller than the predefined constant (0.10) so that the objective and yield PC models are suitable for optimization. The optimized design variables for the risk level $\varepsilon = 0.05$ and $\varepsilon = 0.10$ are shown in Table 3. The final yield of the proposed YCOPCS method with the two risk levels and MC-based yield optimization and the number of required EM simulations are listed in Table 4. From Table 4, we see that the optimization results of the proposed YCOPCS method are better than MC-based yield optimization in both accuracy and EM simulation cost. Using only 248 EM simulations, the optimal solutions obtained by our proposed method achieve the yields of 91.5 % and 83.5 % for $\varepsilon = 0.05$ and $\varepsilon = 0.10$, respectively. As a comparison, MC-based yield optimization obtains an optimal solution with a yield of 82.5 % using 5.67 times more EM simulations than the YCOPCS method. Fig. 8 shows the yield before and after optimization using the YCOPCS method for the fourth-order bandpass waveguide filter with process variation of metal diaphragms.

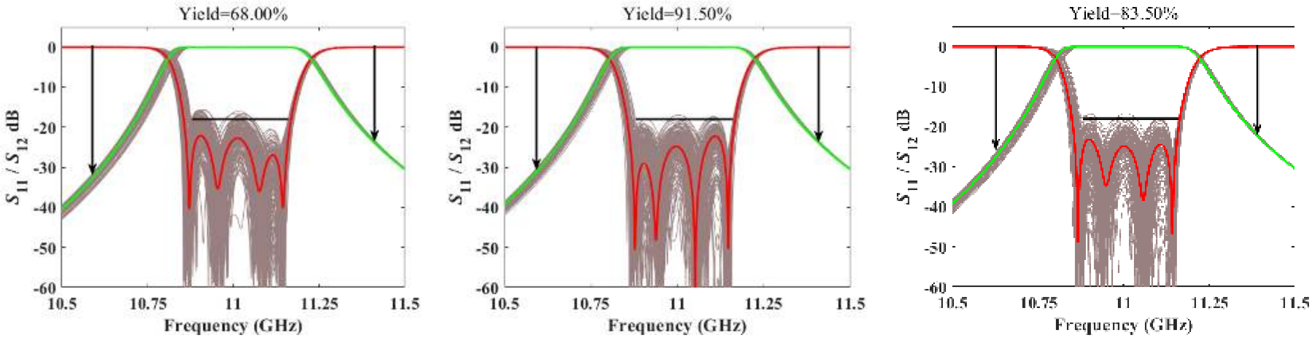


**FIGURE 8.** Yield optimization results of the fourth-order bandpass waveguide filter with process variation of metal diaphragms using MCS method on EM simulations (a) initial design, (b) our proposed ($\varepsilon = 0.15$) and (c) our proposed ($\varepsilon = 0.20$). Gray lines: 200 EM simulations. Red line: $S_{11}$ response evaluated at the optimal design solution. Green line: $S_{12}$ response evaluated at the optimal design solution.

## V. CONCLUSION

In this paper, a novel yield-constrained optimization method based on the polynomial chaos (PC) model for microwave filters is presented for fast and accurately obtaining the design solution with high yield and good performances. Numerical experiments show that the proposed method achieves high-accuracy yield estimation and good yield optimization results with only a fraction of the sampling cost that other methods require. In the future, the yield-constrained optimization method is used to solve the complex microwave filters with high dimension.

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