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ABSTRACT

Sixty Japanese children ranging in age from 3 years 4 months to 7 years 5 months were individually interviewed with three Piagetian tasks. Children's levels of representation were assessed by asking for a graphic representation of 4 dishes, 6 pencils, 8 small blocks etc. A conservation-of-number task was then given to assess children's level of abstraction. It was found (a) that there is a close relationship between children's levels of abstraction and of representation, and (b) that children can represent at or below their level of abstraction but not above this level. (Author)

YOUNG CHILDREN'S REPRESENTATION OF GROUPS OF OBJECTS: THE RELATIONSHIP BETWEEN ABSTRACTION AND REPRESENTATION

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Abstract: Sixty Japanese children ranging in age from 3 years 4 months to 7 years 5 months were individually interviewed with three Piagetian tasks. Children's levels of representation were assessed by asking for a graphic representation of 4 dishes, 6 pencils, 8 small blocks, etc. A conservation-of-number task was then given to assess children's level of abstraction. It was found (a) that there is a close relationship between children's levels of abstraction and of representation, and (b) that children can represent at or below their level of abstraction but not above this level.

There is a common belief in mathematics education that children progress from the "concrete" to the "semiconcrete" level of pictures and then to the "abstract" level of numerals and mathematical symbols. However, Piaget and his collaborators (Furth, 1981; Greco, 1962; Piaget & Inhelder, 1948/1956) made a distinction between abstraction and representation and showed that pictures can be used at a high or low level of abstraction, and that numerals, too, can be used at a high or low level of abstraction. The purpose of this study was to investigate the relationship between abstraction and representation by building on Sinclair, Siegrist, and Sinclair's research (1983).

Method

The subjects were 60 Japanese children ranging in age from 3 years 4 months to 7 years 5 months. The 60 consisted of 15 each of the following four groups: 3 years 4 months to 4 years 5 months; 4 years 6 months to 5 years 5 months; 5 years 6 months to 6 years 5 months; and 6 years 6 months to 7 years 5 months. The children were randomly selected from class lists of two private day-care centers and two public elementary schools in Fukuyama and Okayama (without any consideration of gender). All came from middle-class families.

Three tasks were administered in individual interviews. The children's responses were categorized by three researchers using the criteria for each task as described below. The reliability coefficient was found to be .86 for the first task, .95 for the second, and 1.00 for the third. The researchers discussed disagreements until consensus was reached.

Representation-of-groups-of-objects task

The child was given a sheet of paper and a black marker. Four small dishes were then aligned, and the interviewer asked the child to "draw/write what's here so that your mother will be able to tell what I showed you." (In spoken Japanese, the words for "draw" and "write" sound exactly the same.) The interviewer was careful not to use words like "number" and "how many" which could have suggested quantities. When a child asked whether to draw or to write, the response was "*You* decide which way you like." The same procedure was followed with (a) 3 spoons, (b) 6 pencils, and (c) 8 small blocks, with a new sheet of paper each time.

The children's responses were categorized using three of the six types of representation conceptualized by Sinclair, Siegrist, and Sinclair (1983).

Level 1: Global representation of numerical quantity (absence of one-to-one correspondence). For example, the child drew 8 lines to represent 6 pencils or 12 shapes to represent 8 blocks.

Level 2: Representation of all the sets with one-to-one correspondence. Examples are: 6 lines for 6 pencils and "1234" for 4 dishes.

Level 3: Representation with one numeral indicating the total quantity. For example, writing "4" to represent 4 dishes.

Conservation-of-number task

This task was given to assess the child's level of abstraction. (According to Piaget, children construct number through *constructive* abstraction, which he also called *reflective* or *reflecting* abstraction.) With 20 each of red and blue counters, the child was asked (a) to make a row that had the same number (as the eight that had been aligned) and (b) (after the one-to-one correspondence had been destroyed) whether there were as many in one row as in the other, or more in one row or more in the other, and "How do you know that?"

The responses were categorized according to the following levels:

Level 1: Absence of one-to-one correspondence

Level 2: One-to-one correspondence without conservation

Level 3: One-to-one correspondence with conservation

To be categorized at Level 3, the child had to give one of the following logical explanations:

a. You didn't add or take away anything (identity).
b. I can put them back to the way they were before, and you'll see that it's still the same amount (reversibility).
c. This line is longer, but it has lots of space in between (compensation).

Writing-of-numerals task

This task was given to find out if, in the representation-of-groups-of-objects task, children used the numerals they knew how to write. Without the presence of any objects, the interviewer asked, in random order, "Can you write a *three* (then *four*, *eight*, *six*, and so on)?"

The responses were categorized into the following three levels:

Level 1: No knowledge of any numeral.

Level 2: Some knowledge of some numerals.

Level 3: Knowledge of all the numerals.

Results

Ten of the 60 children were excluded from the analysis because their representation of the objects in Task 1 did not include a quantitative aspect. For example, they drew only one dish for 4 dishes, representing only the qualitative aspect. As can be seen in Table 1, these children were found at the lowest level of abstraction as well as at the highest level.

Relationship between Abstraction and Representation

Table 1 shows the relationship between children's levels of *representation* revealed by the representation-of-groups-of-objects task and their levels of *abstraction* demonstrated in the conservation task. It can be seen in this table that most of the children (36/50, or 72%) showed a perfect relationship between the two variables in the diagonal cells. Eight (16%) were found to be at Level I on both tasks, 14 (28%) at Level II, and 14 (28%) at Level III on both tasks. In other words, 8 (16%) could not make a one-to-one correspondene in the conservation task (Level I) and drew an incorrect number of circles or sticks in the representation task (Level I). Fourteen (28%) made a one-to-one correspondence without conservation (Level II) and drew the correct number of pictures in the representation task (Level II). Another 14 (28%) conserved number (Level III) and wrote one numeral for the total number in the set (Level III). Below the diagonal in Table 1 are 14 children (4 + 10) who represented at a lower level than their level of abstraction. Four of them were at Level II in abstraction but at Level I

Table 1. Relationship Between Levels of Abstraction and of Representation

Levels of Abstraction	Levels of Representation			Representation Only of Qualitative Aspect
	I	II	III	
I	8	0	0	2
II	4	14	0	0
III	0	10	14	7

in representation. Ten were at Level III in abstraction but at Level II in representation. However, *no one was found to be at a higher level of representation than of abstraction* (above the diagonal in Table 1). These were significant findings both statistically and theoretically and will be discussed further below.

Relationship between knowledge of numerals and their use

As can be seen in Table 2, 33 (1 + 18 + 14) of the 50 children knew how to write all the numerals and were categorized at Level III of the writing-of-numerals task. However, only 14 of them used this knowledge in the representation-of-groups-of-objects task. The majority of those who knew how to write numerals used pictures and tally marks (Level II of the representation task). The significance of this phenomenon, too, will be discussed below.

Table 2. Relationship Between Knowledge of Numerals and Levels of Representation

Knowledge of numerals	Levels of representation		
	I	II	III
I	10	4	0
II	1	2	0
III	1	18	14

Discussion

Piaget (1977) pointed out that when children represent reality, they do not represent reality itself. They represent what they *think* about reality. If they do not yet have number in their heads and look at a set of objects, they cannot think about the objects with numerical precision. They therefore represent the set they are looking at at Level I, with a vaguely quantitative idea like "a bunch" or "more than one."

As they construct number (through constructive abstraction), children become able to see each set with more numerical precision. They then become able to represent each set at Level II, with numerical accuracy.

Level-II representations indicate that children are still thinking about individual objects. By contrast, Level-III representations show that children are now thinking about the total quantity as a higher-order unit. For these Level-III children, "4" seems better suited to represent 4 dishes than "oooo" or "1234."

Children's writing "1234" for 4 dishes is especially instructive because it shows that children use numerals, too, at their respective levels of abstraction. If they are still thinking about individual objects, they use numerals, too, in a way that allows them to represent their thinking.

It is also significant to note in Table 2 that most of the children who knew how to write numerals did not use this knowledge. They preferred to use pictures or tally marks that permitted them to represent each object in the set. This finding, too, supports Piaget's view that when children represent reality, they represent what they *think* about reality.

It was pointed out in connection with Table 1 that children sometimes represented at a level *lower* than their level of abstraction, but never at a level *higher* than their level of abstraction. This finding supports Piaget's view, elaborated clearly by Furth (1981), that children can represent their knowledge *at* or *below* their level of abstraction but not *above* this level. At Level I of the conservation task, for example, children cannot represent number concepts that are not yet in their heads. At Level II of the conservation task, they cannot represent a total quantity that is not yet one solid unit.

Mathematics educators often make statements about counting such as the following: "Children should learn that the last number named represents the last object as well as the total number of objects in the collection (NCTM, 2000, p. 79)." This statement reflects an erroneous assumption that numerals represent number. Representation is what human beings do. Neither numerals nor pictures represent. Therefore, children should not be taught that the last number named represents the total number of objects in the collection. Without any teaching, by the time they are at Level III of the conservation task, children become able to use numerals to represent the total number of objects.

This study shows the inadequacy of the belief that children progress from the "concrete" to the "semiconcrete" level of pictures and then to the "abstract" level of numerals and mathematical symbols. In the conservation-of-number task, we saw that children can think about concrete objects at a high or low level of abstraction. (At Level II of abstraction, they cannot conserve, but at Level III, they conserve the equality of the two sets.) Pictures and tally marks, too, can be drawn at a higher or lower level of abstraction. (At Level I of the representation task, children revealed their prenumerical thinking by drawing an inaccurate number of objects. The number became correct at Level II of representation, reflecting a higher level of abstraction.) Spoken and written numerals, too, can be used at a higher or lower level of abstraction. (The children who wrote "1234" were still thinking about individual objects. Those at a higher level of abstraction wrote "4" because they thought about the set of objects as a unit.)

The representation task in this study asked for children's *productions*. The relationship between abstraction and representation is more transparent in their productions than in their *reading* of numerals and mathematical symbols. For example, when first graders see "4 + _ = 6," some write "2" in the blank, but many write "10." This is because when children see mathematical writing, they represent to themselves the relationship that they are capable of making (abstraction). Those who can make a part-whole relationship represent this relationship to themselves and know that 4 is part of 6. Those who cannot make a part-whole relationship merely represent "4," "+," and "6" to themselves.

Older children may know symbols such as "0.321," "0.2," and "6/8 x 2/3" at some level. However, different children at different levels of abstraction give different meanings to these symbols in various contexts. The implication of this study for older children is that educators need to focus their efforts more sharply on the *mental* relationships children are making (abstraction) rather than simply believing that children understand or do not understand *the meaning* of "0.321," "0.2," and "6/8 x 2/3." The meaning is in the child's head, not in the symbols.

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