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Z-number-based AQI in rough set theoretic framework for interpretation of air quality for different thresholds of $PM_{2.5}$ and PM_{10}

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Abstract Kolkata has a reputation for being one of the world's most polluted cities, particularly in the post-monsoon months of October, November, and December. Diwali, a Hindu festival, coincides with these months where a large number of firecrackers are set off followed by high emissions of air pollutants. As a result, the air quality index (AQI) deteriorates to "very poor" ($301 \leq AQI \leq 400$) and "poor" ($201 \leq AQI \leq 300$) categories. This situation stays for several days to a month. The present study aims to identify the thresholds for $PM_{2.5}$ and PM_{10} that cause the AQI of Kolkata to deteriorate to "very poor" and "poor." For this purpose, we have used a rough set theory-based condition-decision support system to predict the aforementioned categories of AQI. We have developed a Z-number-based novel quantification measure of semantic information of AQI to assess the reliability of the outcomes, as generated from the condition-decision-based decision rules, during post-monsoon season. The result reveals the best possible forecast of AQI with linguistic summarization of the reliability or confidence for different threshold ranges of PM_{10} and $PM_{2.5}$. Inverse-decision rules based on rough set theory are utilized to justify and validate the forecasts. The explainability of the condition-decision support system is demonstrated/visualized using a flow graph that maps rough-rule-based different decision paths between input and output with strength, certainty, and coverage. The investigation resulted in an advanced intelligent environmental decision support system (IEDSS) for air-quality prediction.

Keywords AQI · Condition-decision support system · Explainable AI · Flow graph · Machine intelligence · $PM_{2.5}$ · PM_{10} · Rough sets · Z-numbers

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Introduction

Prediction of air quality is a major challenge in early warning and control of urban air pollution. With the steep increase of urbanization and the advancement of industrialization, the problem of urban air pollution has become significantly important, as it accelerates climate change and affects human health. The post-monsoon months of October, November, and December coincide with Diwali, India's most important religious holiday. As a part of the celebration, a large amount of firing crackers get burnt starting from late evening to late night on the Diwali day, as well as before and after Diwali. Metallic chemicals such as salts of potassium, sodium, strontium, barium and copper, charcoal, iron, sulphur, manganese, and aluminum dust powder are found in fireworks. After these chemicals get burnt, gaseous pollutants are generated which are harmful for the environment and, in turn, put our health at stake.

Air pollution is one of the most serious global health and environmental concerns which is more prevalent in urban areas. It is caused by anthropological activities that degrade ecological balance and kill millions of people each year. In many cities where unplanned urbanization occurs, air pollution exceeds danger limits in terms of human health, posing a health threat or affecting life quality (Cetin & Sevik, 2016a; Singh et al., 2020). Cetin (2016) showed how the indoor CO_2 parameters influence the students' performance of activity in the examination halls. Cetin and Sevik (2016b) studied the effects of indoor plants on the concentration of CO_2 in an indoor environment under certain light conditions. Cetin et al. (2019) evaluated the air quality based on CO_2 amount and amount of particulate matter in 6 different dimensions and determined the change in sound level on a regional basis depending on the time of day and the season in different areas of Bursa City Center. In Elsunousi et al. (2021), the regional and periodic change of CO_2 and particulate matter pollution in the city of Misurata were illustrated. Climate has a direct impact on a plant's characteristics (Kaya et al., 2019; Sevik et al., 2021). Heavy metals are especially critical among the components of air pollution because they can be poisonous and deadly even at low concentrations, and even the necessary elements for organisms can be harmful at high concentrations (Jo et al., 2020). Since they are not easily decomposed, they tend to bioaccumulate, and some of them have toxic or carcinogenic effects even at low concentrations (Cetin & Jawed, 2022; Waaijers et al., 2012). Sevik et al. (2020a, b) determined the variation of Pb and Mg accumulation depending on plant species, plant organ, washing status, and traffic density in some landscape plants grown in the city center of Kastamonu. Cetin et al. (2020) used blue spruce (*Picea pungens*) tree organs to calculate the concentration of heavy metals, such as Ca, Cu, and Li in the city of Ankara. Fruits and vegetables cultivated in industrial and urban areas with significant levels of heavy metal pollution can be detrimental to public health if consumed as crops (Sevik et al., 2020a, b). The most preferred method for determining the changes in heavy metal concentrations in the atmosphere is the use of biomonitors, especially with the help of the annual growth rings of trees (Cesur et al., 2021). In this study, the changes in heavy metal concentrations in the annual rings of *Cupressus arizonica* tree growing in the city center of Kastamonu were observed. The effects of enhanced UV-B radiation on plant growth, development, biomass accumulation, yield, metabolism, and morphological characteristics of both germination and seedlings are significant (Kumar & Bhardwaj, 2019; Ozel et al., 2021). Excessive phosphorus and nitrogen pose a significant threat to the quality of freshwater lakes in Malaysia (Huang et al., 2015; Varol et al., 2022). In the worst scenarios, urban runoff contains enough pollutants to make it impossible for us to swim in or fish in our local waters. Porous plastic asphalt pavements provide an alternative technology for stormwater management (Cetin, 2013).

There have been many studies that show the impact of firework activities on air quality during the Diwali celebration in India. It has been found that fireworks cause short-term variation and degradation in air quality, as well as major changes in pollutant concentrations. In Hisar, for example, PM_{10} and TSPM concentrations are seen to increase by two to three times (Ravindra et al., 2003). Barman et al. (2008, 2009) found a considerable rise in $PM_{2.5}$ levels in Lucknow. In case of Kolkata, the mass concentrations of PM_{10} and SO_2 were found to be ~ 5 times the standard limits, as prescribed (Chatterjee et al., 2013). Mandal et al. (2022) recently examined the effect of pollution during Diwali celebrations of COVID-19 outbreak. Though most of the research has studied the change of air quality and its effect on the environment during Diwali compared to normal periods, modelling the air quality in terms of "cause-effect" relation for the purpose of predicting the probable ranges of pollutants during Diwali months remains an issue for researchers. Such modelling is helpful for environmentalists to mitigate the deterioration on air quality, and its consequences.

Intelligent decision support systems (IDSS) are developed by integrating the idea of decision support system (DSS) and artificial intelligence (AI) techniques judiciously. It often applies expert system technology to assist in the resolution of complex decision problems by adopting the knowledge of human experts with logical reasoning. An IDSS is expected, ideally, to act like a user-friendly human expert or consultant in providing support to decision-makers, e.g., by gathering and analyzing evidence, diagnosing problems, suggesting possible courses of actions, and evaluating those actions. Here, the use of AI techniques attempts to make these tasks performed efficiently by a machine with enhanced tractability to handle difficulties, while achieving close resemblance with human-like decision-making. Environmental problems are among the set of crucial domains in which inappropriate management decisions can have severe social, economic, and environmental implications. Decision provided by intelligent environmental decision support systems (IEDSSs) is significant in the interaction of humans and ecosystems as they are the instruments developed to deal with the multidisciplinary nature and high complexity of environmental challenges. In this scenario, statistical and artificial intelligence-based various data analysis tools and techniques can be judiciously integrated and used to give valuable environmental knowledge for the IEDSS decision-making process. AI techniques have been applied to environmental management problems for a long period with good results. AI tools like case-based reasoning, artificial neural networks, genetic algorithms, swarm intelligence, and decision trees were employed to develop various knowledge-based systems, expert systems, and fuzzy inference systems for this purpose (Ahmed et al., 2003; Riga et al., 2009; Dutta & Chaudhuri, 2014; Ong et al., 2015; Corominas et al., 2018; Qi et al., 2018; Abdul-Wahab et al., 2019; Zhao et al., 2019; Liu et al., 2019; Kuri-Monge et al., 2021).

Rough set theory, introduced by Pawlak (1991), provides important AI techniques for modelling complex data structures. It can effectively analyze various kinds of imprecise, inconsistent, incomplete, or imperfect information in order to find the hidden knowledge that can be used to discover a potential rule (Qu et al., 2020). Lin et al. (2011) used rule-based decision-making technique of rough set theory to predict customer churn in credit card accounts; they used a flow network graph and a path-dependent approach to infer decision rules and variables. Liou et al. (2016) used the rough set theory (RST) with flow graph in developing strategies to improve service quality. Stević et al. (2017) developed a multicriteria decision model with eight criteria and eight alternatives. Kundu and Pal (2018) introduced a new variant of rough set, namely, double bounded rough set, to quantify the tension force and proposed an algorithm based on tension measure is for link prediction. Chakraborty and Pal (2021) proposed rough set-based moving object background classification and motion uncertainty analysis with newly defined motion entropy. Rough set theory along with decision rules was employed for estimation of raw silk quality (Kar et al., 2021). The importance of the theory stems from the fact that it can show the probabilistic structure of the data being evaluated without having any prior knowledge. As a consequence, it is ideal for analyzing the probabilistic structures of data related to dynamic and non-linear meteorological phenomena. In meteorological research, the theory of rough sets is widely used. For example, Shan (2001) used a rough set-based approach to classify weather data. This theory was introduced for the prediction of drought by Liu and Qiao (2009). In addition, it was applied to assess the pollution sources of the atmospheric particulates of Jilin City both during the heating and non-heating periods (Fang et al., 2010). Chaudhuri and Dutta (2013) investigated the significance of GPT (generalized potential temperature) to define the humid condition of moist atmosphere for the predominance of the natural hazards by generating rough set theoretic “if–then” rules. Sudha (2017) proposed intelligent decision support system using rough sets and fuzzy logic for short-range rainfall prediction. A rough set-based decision support system has been used by Matarazzo (2018) for managing the air pollution in the industrial area. Rough set is found to be extremely suitable for classifying the air pollutant index in Malaysia and Singapore (Wibowo et al., 2018). Furthermore, the theory has been used to various uncertainty handling, and multicriteria decision-making applications (Kazemitash et al., 2021; Naouali et al., 2020; Pal et al., 2018; Saha et al., 2010; Suresh et al., 2012; Tang et al., 2020; Wang & Zhang, 2014; Ye et al., 2021). In our present study, we have used rough set-based “if–then” rules and inverse-decision rules for air-quality prediction and interpreting/explaining the decision-making process, respectively.

Decision-making in real-world problems is generally performed based on information that is often uncertain, ambiguous, incomplete, and/or imprecise. Fuzzy set theory of Zadeh (1965) is reputed in dealing with this kind of information. The effectiveness of the theory has been demonstrated in different domains, e.g., engineering, meteorology, economics, biological, social sciences, and computer science. In 2011, he defined Z-numbers (Zadeh, 2011), another new concept/measure concerning computing with words (CWW). This number provides a more general framework or structure for modelling/representing the uncertain information arising in real-world phenomena by incorporating the reliability of information. Since then, Z-number has been widely used in different fields like psychological research (Aliev & Memmedova, 2015), medical diagnosis (Wu et al., 2017), marketing (Alizadeh & Serdaroglu, 2016), multicriteria game model (Peng et al., 2019b), investment risk analysis (Peng et al., 2019a), natural language understanding (Banerjee & Pal, 2015), video tracking (Pal et al., 2019), and safety analytics (Das et al., 2020). Hendiani and Bagherpour (2019) used the concept of Z-numbers to model a possibilistic approach with the purpose of calculating the sustainability index in the context of reliability. A nice review on Z-numbers with different applications since its inception in 2011 has been recently reported (Banerjee et al., 2021). Z-numbers have not been used in the prediction of air quality so far. The present study is an attempt to demonstrate the significance of Z-numbers for quantifying the air quality in terms of linguistic abstraction. This concept is unique.

National air quality standards may vary depending on the approach adopted for balancing health risks, technological feasibility, economic considerations, and various other political and social factors, which, in turn, is influenced by other things such as the level of development and national capability in air quality management (Beig et al., 2010). Air quality index (AQI) is aimed at making decisions about how to protect the health of people by optimizing the short-term exposure to air pollution and controlling the activity levels when pollution levels are high. It assesses the current air quality, which is dependent on the specific level of concentration of an individual air pollutant. One may note that the air quality deteriorates during the Diwali festival in October–November, and the AQI remains high for several days to a month. For the post-monsoon months of October, November, and December, we consider AQI categories of "poor" (AQI within 201–300) and "very poor" (AQI within 301–400).

The present article describes the development of an intelligent environmental decision support system. It demonstrates the effectiveness of rough set theory-based decision rules, inverse-decision rules, flow graphs, and z-numbers in doing so. Interpretability is the ability to identify the relationships and counterfactuals between input and output, and the ability to search for evidence in the data that supports a particular outcome (Kovács et al., 2021). The use of the proposed inverse-decision rules provides interpretation/explanation of the decisions made on air-quality prediction. Flow graph that maps the rough-rule-based different paths between input and output with strength, certainty, and coverage enables visualization of the explainability of the condition-decision support system. Inverse rules in conjunction with Z-numbers thus enable interpretability of the proposed decision support system.

The novelty of the paper mainly lies in the following:

1. Modelling the AQI in terms of particulate matter ($PM_{2.5}$ and PM_{10}) using rough set theoretic "cause-effect"/ "if–then" relation for prediction of air quality during Diwali months
2. Developing a Z-number-based AQI that quantifies the abstraction of decision-making information concerning AQI prediction, which is first of its kind
3. Using rough set-based inverse-decision rules for determining the possible ranges of particulate matter ($PM_{2.5}$ and PM_{10}) that are responsible for a given air quality; this demonstrates the explainability of the system towards a decision
4. Designing an information flow graph between input and output that clearly demonstrates/visualizes the explainability of the condition-decision support system

All these novel features precisely illustrate how to integrate the merits of AI tools such as rough set-based condition-decision support system, inverse-decision rules with flow graph having explainability, and Z-numbers for quantification of semantic information to define AQI so as to design an advanced IEDSS for prediction and quantification of air quality during Diwali.

Material and methods

Study area

Kolkata (erstwhile Calcutta) is one of the large metropolitan cities of India. It is in the Ganges Delta at 22°33′North and 88°20′East, along the eastern side of the Hooghly River at an elevation of about 9 m. Kolkata is a densely populated city with a population of 14.4 million. It is included in the world's twenty-five severely polluted cities including ten others from India (Bera et al., 2020) facing rising air pollution and multi-pollutant crisis. The city's economic and industrial expansion, as well as different industries (paper and pulp, rubber, iron, plastic, textile, and food), vehicular emissions, dust from construction sites, solid waste burning, wind-blown dust from open lands, and thermal power plants, all contribute significantly to air pollution in Kolkata. As a result, the air quality in this area has deteriorated considerably. According to a study conducted at major traffic intersections in Kolkata, the levels of significant pollutants, such as PM_{10}, NO_2, CO, SO_2, and lead, are found to be substantially higher than the permitted values (Ghose et al., 2004). Kolkata, like the rest of India, celebrates Diwali together along with "Kali Puja" festival with eagerness and devotion. A large number of crackers and sparklers of different kinds and intensities get burnt on the day of Diwali and also the adjacent days before and after.

Data collection and data pre-processing

The location of Kolkata and the data from twenty monitoring stations are shown in Fig. 1. These twenty monitoring stations are Dunlop Bridge, Rabindrabharati, Shyambazar, Victoria Memorial, Ultadanga, Beliaghata, Moulali, Salt Lake, Minto Park, Topsia, BITM, Hyde Road, Gariahat, Jadavpur, Fort William, Rabindra Sarobar, Tollygunge, Mominpore, Behala Chowrasta, and Baishnabghata. We have conducted our study based on the data and observations of 7 years from 2015 to 2021 over Kolkata, India, during October, November, and December. The data of daily air quality of Kolkata city is obtained from the West Bengal Pollution Control Board (WBPCB). Our present study considers two pollutants, namely, $PM_{2.5}$ and PM_{10} (particulate matter of size 10 μm or less and 2.5 μm or less, respectively).

After collecting the data, pre-processing is executed to improve the data quality. Missing data is a common problem for most air pollution monitoring stations due to instrument failure, data entry error, maintenance, and other unmanageable factors. Though there are a few missing values in our data record, to fill the empty values, a linear interpolation technique is used.

Methodology: tools and definitions

Here, we provide some basic definitions and AI tools which are employed in formulating the proposed methodology for air-quality prediction. These include information table, decision table, and characteristics of decision rules in the framework of rough set theory. These are followed by the concept of flow graph and Z-numbers.

Information system and decision table

An information system (Pawlak, 2004) can be viewed as a data table (matrix) where columns denote different attributes, the rows denote different objects of interest, and the entries of the table represent attribute values. This is represented as a pair of sets as (Pawlak, 2004)

$$S = (U, A) \tag{1}$$

Here, U and A represent the universe of objects and the set of attributes, respectively. These are non-empty finite sets. $a : U \rightarrow V_a$, where V_a is the set denoting all values of a, referred to as the domain of a, for each $a \in A$. Any subset B of A characterizes/determines a binary relation $I(B)$ on U. This is also known as an indiscernibility relation, described as (Pawlak, 2004)

$$(x, y) \in I(B) \text{ iff } a(x) = a(y) \text{ for every } a \in A \tag{2}$$

Here, $a(x)$ denotes the value of the attribute a corresponding to element x. $I(B)$ is an equivalence relation. $U/I(B)$ or simply by U/B denotes the family of equivalence classes caused by $I(B)$, i.e., partitions on U as determined by B. $B(x)$ represents the block of the partition U/B that contains x.

If $(x, y) \in I(B)$, then x and y are said to be B-indiscernible (indistinguishable) with respect to attribute-subset B.

If we arrange an information system in two distinct classes of attributes, namely, condition and decision attributes, then it is called a decision system, defined by (Pawlak, 2004)

$$S = (U,\ C,\ D) \tag{3}$$

where C and D are two disjoint sets of condition and decision attributes. $C \cup D = A$. $C(x)$ and $D(x)$ denote the condition class and decision class as introduced by x, respectively.

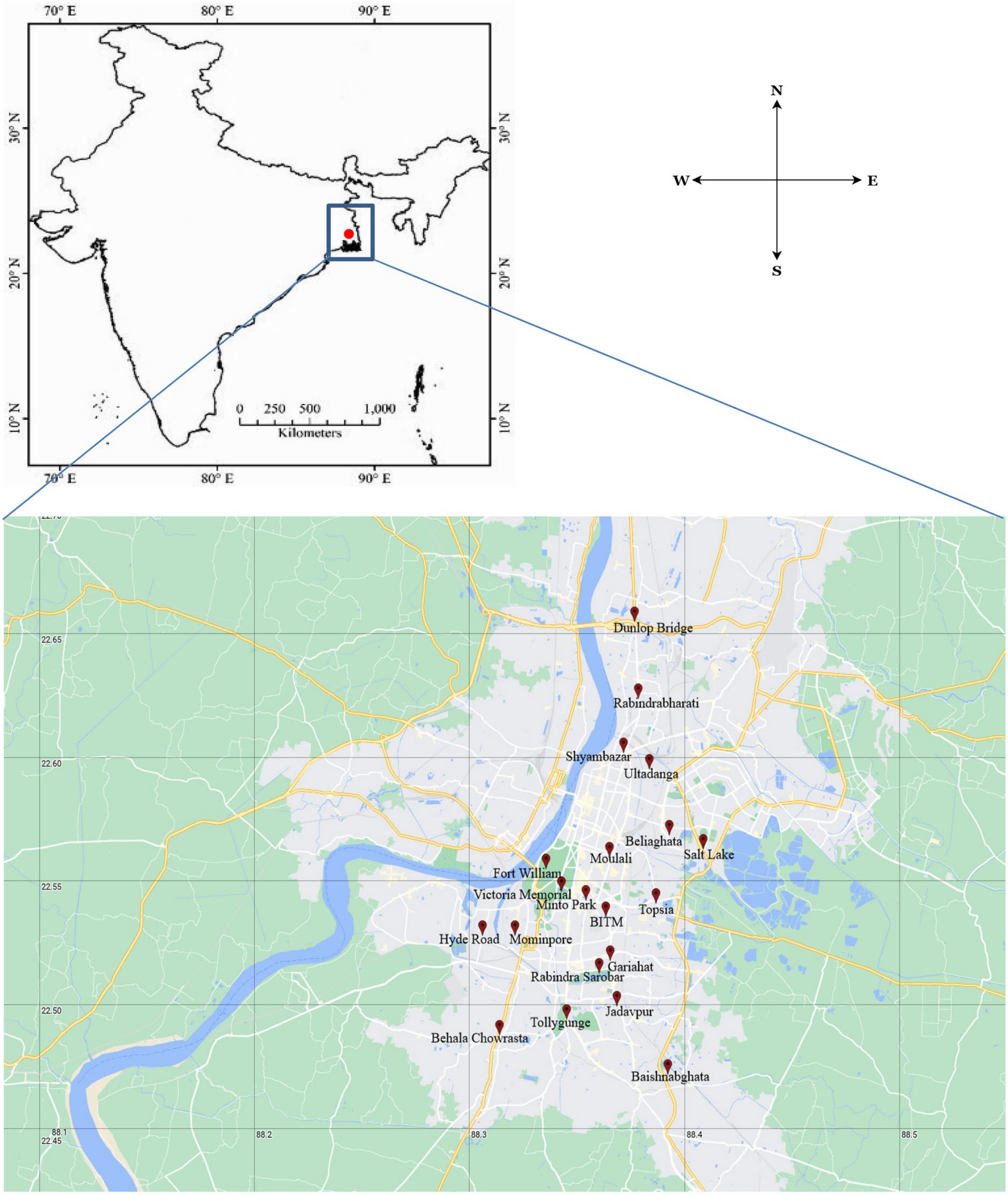


Fig. 1 Location of the Kolkata metropolitan area and air monitoring stations

A decision system may also be referred as a decision table that represents various decisions (e.g., actions and results) made when some conditions get satisfied. That means, each row of the table represents a decision rule that characterizes decisions in terms of conditions.

Decision rules: characteristic features (Pawlak, 2004)

Let $S = (U, C, D)$ represent a decision table as mentioned above. Let every $x \in U$ determine a sequence given by

$c_1(x), ..., c_n(x), d_1(x), ..., d_m(x)$

where $\{c_1, ..., c_n\} = C$ and $\{d_1, ..., d_m\} = D$.

The sequence (Eq. 4) may be called a decision rule induced by x (in S) and represented as

$$c_1(x), ..., c_n(x) \rightarrow d_1(x), ..., d_m(x) \tag{4}$$

In brief, $C \underset{x}{\rightarrow} D$.

This decision rule is characterized by the following features:

(a) Support of the decision rule $C \underset{x}{\rightarrow} D$

Support is a number defined as

$$Supp_x(C,\ D) = |A(x)| = |C(x) \cap D(x)| \tag{5}$$

(b) Strength of the decision rule $C \underset{x}{\rightarrow} D$

$$\sigma_x(C,\ D) = \frac{Supp_x(C, D)}{|U|}, \tag{6}$$

where |.| denotes the cardinality of a set ε.ε.

(c) Certainty factor of $C \underset{x}{\rightarrow} D$

$$cer_x(C,\ D) = \frac{|C(x) \cap D(x)|}{|C(x)|} = \frac{supp_x(C, D)}{|C(x)|} = \frac{\sigma_x(C, D)}{\pi[C(x)]}, \tag{7}$$

$$\text{where}[C(x)] = \frac{|C(x)|}{|U|}. \tag{8}$$

If $cer_x(C, D) = 1$, then $C \underset{x}{\rightarrow} D$ indicates a certain decision rule; otherwise not.

(d) Coverage factor of $C \underset{x}{\rightarrow} D$

$$cov_x(C,\ D) = \frac{|C(x) \cap D(x)|}{|D(x)|} = \frac{supp_x(C, D)}{|D(x)|} = \frac{\sigma_x(C, D)}{\pi[D(x)]}, \tag{9}$$

$$\text{where } D(x) \neq 0 \text{ and } \pi[D(x)] = \frac{|D(x)|}{|U|}. \tag{10}$$

(e) Inverse-decision rule of $C \underset{x}{\rightarrow} D$

If $C \underset{x}{\rightarrow} D$ represents a decision rule, then $D \underset{x}{\rightarrow} C$ represents its inverse-decision rule.

Inverse-decision rules have been utilized in our proposed prediction methodology to provide explanations (reasons) or justification behind any decision taken.

Information flow graph

Information flow graph (Pawlak, 2005) maps the decision paths from input to output of a rough rule base. Accordingly, it provides a special kind of database in which statistical features of different objects are described in terms of information flow distribution rather than the raw data about individual objects. This type of data representation provides new insights into data structures and enables data analysis in more intelligent way.

A flow graph is a directed acyclic finite graph defined as (Pawlak, 2005)

$$G = (N,\ \mathcal{B},\ \varphi). \tag{11}$$

Here, N denotes a set of nodes, $\mathcal{B} \subseteq N \times N$ represents a set of directed branches, and $\varphi : \mathcal{B} \rightarrow R^+$ denotes a flow function where R^+ is the set of non-negative reals. Each node of the flow graph represents an attribute of the information system.

The basic concepts (Pawlak, 2005) of the flow graph G are as follows:

- If $(x, y) \in \mathcal{B}$, then x is an input of y, and y is an output of x.
- If $x \in N$, then $I(x)$ and $O(x)$ denote the sets of all x's inputs and outputs.
- Input and output of G are defined as

$$\mathrm{I}(G) = \{\mathrm{x} \in \mathrm{N} : \mathrm{I}(\mathrm{x}) = \varnothing\} \text{ and } \mathrm{O}(G) = \{\mathrm{x} \in \mathrm{N} : \mathrm{O}(\mathrm{x}) = \varnothing\} \tag{12}$$

Every node x in G is associated with its inflow and outflow as (Ramanna & Chitcharoen, 2013)

$$\varphi + (x) = \sum\nolimits_{y \in I(x)} \varphi(y, x) \tag{13}$$

$$\varphi - (x) = \sum\nolimits_{y \in O(x)} \varphi(x, y) \tag{14}$$

Similarly, the inflow and outflow for the whole flow graph G are represented as

$$\varphi + (G) = \sum\nolimits_{x \in I(G)} \varphi - (x) \tag{15}$$

$$\varphi-(G)=\sum_{x\in O(G)}\varphi+(x) \tag{16}$$

The inflow and outflow of an internal node of a graph are supposed to be the same, and hence of the graph G (Pal & Chakraborty, 2017).

A normalized flow graph can be characterized by $G^{*}=(N,\mathcal{B},\sigma)$.

Here, N and $\mathcal{B}$ are as in Eq. (11).

σ denotes the strength of a branch (x,y) in G^{*}, where

$$\sigma(x,y)=\frac{\varphi(x,y)}{\varphi(G)},\ 0\le\sigma(x,y)\le 1 \tag{17}$$

σ of a branch indicates the percentage of the total flow through it. Its higher value means larger percentage of flow.

Certainty (cer) and coverage (cov) factors of a branch (x, y) in G^{*} are expressed as

$$cer(x,y)=\frac{\sigma(x,y)}{\sigma(x)} \tag{18}$$

$$cov(x,y)=\frac{\sigma(x,y)}{\sigma(y)} \tag{19}$$

Z-numbers

Z-number (Zadeh, 2011) is based on fuzzy set theory. This has been useful in computing with words (CWW) (Zadeh, 1996). In CWW, the perceptions are encoded in the words and phrases that are used to describe various events. This phenomenon is inspired by the exceptional ability of the human brain in making perception-based decisions. The underlying concept of Z-number characterizes the certainty of information (Pal et al., 2019).

A Z-number has two tuples, and is defined as

$$Z=\langle A,\ B\rangle \tag{20}$$

Tuple A, which is a constraint, is allowed to take the values of X (a real-valued uncertain variable, interpreted as the subject of Y). Tuple B is a measure of reliability of A. They are usually fuzzy numbers representing words or clauses in a natural language (Banerjee & Pal, 2013).

Let us consider a statement Y = Tropical cyclones are *likely* to become severe.

Here,

X = intensity of tropical cyclones, A = severe, B = likely, and the Z-based information of the statement is,

$$Z=\langle\text{severe},\ \mathit{likely}\rangle. \tag{21}$$

Thus, the uncertainty and fuzziness that occur in linguistic terms best suit the restriction and the reliability metric of Z-numbers. Z-number in terms of linguistic terms provides a natural way of interpretation and enhances the flexibility and reliability of decision-making systems. A detailed review on Z-numbers with its various applications made so far since its inception in 2011 is available in Banerjee et al. (2022). In our present study, we have employed Z-numbers as the linguistic measure of the reliability of the condition-decision support system to characterize the different criteria of the AQI.

Formulation of methodology

Different ranges of pollutants

The air quality index or AQI is a daily air quality measure which can be used to quantify concentration of pollutants. It is a measurement of how air pollution affects a person's health with a small duration. The air quality index is composed of eight pollutants, i.e., particulate matters $PM_{2.5}$ (size 2.5 μm or less) and PM_{10} (size 10 μm or less), sulphur dioxide (SO_2), ammonia (NH_3), nitrogen dioxide (NO_2), carbon monoxide (CO), lead (Pb), and ozone (O_3). The Central Pollution Control Board (CPCB) used the following methodology to calculate the AQI:

The general equation for the sub-index (I_p) for pollutant P for a given pollutant concentration (C_p) is

$$I_p=\frac{(I_{HI}-I_{LO})}{BP_{HI}-BP_{LO}}(C_p-BP_{LO})+I_{LO} \tag{22}$$

Here,

I_p = sub-index for pollutant P

C_p = rounded concentration of pollutant P

BP_{HI} = breakpoint that is greater than or equal to C_p

BP_{LO} = breakpoint that is less than or equal to C_p

I_{HI} = AQI value corresponding to BP_{HI}

I_{LO} = AQI value corresponding to BP_{LO}

The highest sub-index, I_p, represents the AQI of the location.

The sub-indices for individual pollutants are calculated using its 24-hourly average concentration value (8-hourly in case of CO and O_3) at a monitoring location. It is not always possible that all the eight pollutants to be monitored in all the locations. The overall AQI is determined only if the data for at least three pollutants are available, one of which must be either $PM_{2.5}$ or PM_{10}. Otherwise, the data is considered insufficient for computing the AQI. For sub-index calculation, a minimum of 16 h of data is required. Even if the data are insufficient to determine the AQI, the sub-indices for monitored pollutants are computed and disseminated. In that case, the individual pollutant-wise sub-index will provide the air quality status for that pollutant. The AQI is provided in real-time basis through the web-based system. It is an automated system that collects data from continuous monitoring stations without the need for human interaction and displays the AQI based on running average values (for example, AQI at 6 a.m. on a day will incorporate data from 6 a.m. on previous day to the current day). For manual monitoring stations, an AQI calculator has been developed where data can be manually entered to obtain AQI value.

The AQI classifications used below are provided by the CPCB of India. Here, AQI is measured on a six-point scale: severe, very poor, poor, moderate, satisfactory, and good. Table 1 illustrates different categories of AQI and the corresponding health impacts. The AQI values and corresponding ambient concentrations (health breakpoints) for the identified eight pollutants are presented in Table 5.

Due to Diwali festival during October–November, AQI deteriorates to "very poor" and "poor" categories and remains high for about a month. In our present study, we have dealt with the cases of two AQI categories, viz., "very poor" (AQI within 301–400) and "poor" (AQI within 201–300) during the post-monsoon month of October, November, and December. For the purpose of the identification of the influence of the burning of firecrackers on AQI, we have considered two pollutants, viz., $PM_{2.5}$ and PM_{10}, as these are most important to affect the air quality.

The air quality data of the year 2015–2019 are used as design set and that of 2020–2021 are used for validation (validation set). The concentration of $PM_{2.5}$ during the post-monsoon months is found to be in the range of (91–270) μg/m^3 having AQI categories "poor" and "very poor" with our 7 years of study period. The concentration values of $PM_{2.5}$ are divided into four parts using normal probability distribution as follows: $91 \leq PM_{2.5} \leq 135$, $136 \leq PM_{2.5} \leq 180$, $181 \leq PM_{2.5} \leq 225$, and $226 \leq PM_{2.5} \leq 270$. Similarly, the concentration of PM_{10}, having the range of (143–334) μg/m^3 with AQI categories "poor" and "very poor" is categorized using normal probability distribution as follows: $143 \leq PM_{10} \leq 190$, $191 \leq PM_{10} \leq 238$, 239-$\leq PM_{10} \leq 286$, and $287 \leq PM_{10} \leq 334$.

Framing of decision rules

The decision rules are constructed in terms of "condition–decision," "cause–effect," or "if–then" relations using rough set theory. The effectiveness of these rules is demonstrated by confirming the ranges of PM_{10} and $PM_{2.5}$ for the predominance of the "poor" and "very poor" AQI. The decision rules with different conditions are framed as follows:

Fact 1: If ($91 \leq PM_{2.5} \leq 135$), then (AQI is poor or very poor)
Fact 2: If ($136 \leq PM_{2.5} \leq 180$), then (AQI is poor or very poor)
Fact 3: If ($181 \leq PM_{2.5} \leq 225$), then (AQI is poor or very poor)

Table 1 Different categories of national air quality index (AQI) with health impact (CPCB, 2014)

AQI	Remark	Possible health impacts
0–50	Good	Minimal impact
51–100	Satisfactory	Minor breathing discomfort to sensitive people
101–200	Moderate	Breathing discomfort to the people with lung, heart disease, children and older adults
201–300	Poor	Breathing discomfort to people on prolonged exposure
301–400	Very poor	Respiratory illness to the people on prolonged exposure
> 400	Severe	Respiratory effects even on healthy people

Fact 4: If ($226 \le PM_{2.5} \le 270$), then (AQI is poor or very poor)
Fact 5: If ($143 \le PM_{10} \le 190$), then (AQI is poor or very poor)
Fact 6: If ($191 \le PM_{10} \le 238$), then (AQI is poor or very poor)
Fact 7: If ($239\text{-} \le PM_{10} \le 286$), then (AQI is poor or very poor)
Fact 8: If ($287 \le PM_{10} \le 334$), then (AQI is poor or very poor)

Flow graphs are generated corresponding these two sets of rules. These graphs depict different paths between input and output representing different decision rules with their respective strength, certainty, and coverage values.

Z-number-based AQI

In our investigation, the concept of Z-numbers is employed to develop metrics (indices) characterizing the air quality in the month of October, November, and December during Diwali time. As stated before, our objective is to identify different threshold ranges of $PM_{2.5}$ and PM_{10} responsible for AQI value to be "poor" and "very poor." Rough set theoretic condition–decision support system is used to quantify the information associated with decision rules. Every decision rule has certainty and coverage factors. The certainty value describes the conditional probability that an object belongs to the decision class indicated by the decision rule as well as it meets the condition of the rule. In contrary, coverage value illustrates the conditional probability of justifications behind a particular decision (Chaudhuri & Dutta, 2013). Based on the results obtained using rough set theory, the Z-based information of B is provided (Eq. 20). A of the Z-number is defined as a group of classes having similar criteria that define AQI. Here, we only consider two criteria of AQI, namely, poor and very poor. That is, $A = \{\text{poor, very poor}\}$ where "poor" denotes AQI value within the range of 201–330 and "very poor" denotes AQI value within the range of 301–400. B is a measure of the degree of sureness regarding the value of A. As mentioned, while defining the Z-numbers above, A and B are usually fuzzy numbers denoting words or phrases. Determination of B is done with the help of the certainty and coverage values obtained from Eqs. (7) and (9). B is the set of reliability values, e.g., $B = \{$*Certainly*, *Most Likely*, *Likely*, *Less Likely*, *May be*, *Unlikely*$\}$.

Table 2 Ranges of certainty and coverage and corresponding linguistic description using Z-number

Ranges of certainty and coverage	Z-number-based information
$0.70 \le \text{Certainty} \le 1$ and $0.70 \le \text{Coverage} \le 1$	*Certainly*
$0.60 \le \text{Certainty} \le 1$ and $0.50 \le \text{Coverage} \le 1$ or $0.60 \le \text{Coverage} \le 1$ and $0.50 \le \text{Certainty} \le 1$	*Most Likely*
$0.50 \le \text{Certainty} < 70$ and $0.50 \le \text{Coverage} < 70$	*Likely*
$0.20 \le \text{Certainty} < 0.50$ and $0.20 \le \text{Coverage} < 0.50$	*Less Likely*
$0.60 \le \text{Certainty} \le 1$ and $0.01 \le \text{Coverage} < 50$ or $0.60 \le \text{Coverage} \le 1$ and $0.01 \le \text{Certainty} < 50$	*May be*
$0.00 \le \text{Certainty} < 0.20$ and $0.00 \le \text{Coverage} < 0.20$	*Unlikely*

The criteria for Z-number are defined as in Table 2. For example, if certainty is within the range of $0.70 \le \text{Certainty} \le 1$ and coverage also lies within the range of $0.70 \le \text{Coverage} \le 1$ or vice versa, then it is labeled as "*Certainly*." If the said AQI is "poor," then the Z-based information can be written as $Z = <\text{poor}, \textit{Certainly}>$. Thus, this Z-based metric can also be used as an index to determine whether an AQI is "poor" or "very poor." The label "*May be*" denotes an unusual range where the certainty lies within the range of $0.60 \le \text{Certainty} \le 1$ and the coverage lies within the range of $0.01 \le \text{Coverage} < 50$, or vice versa.

Validation and inverse-decision rules

We validate the decision rules using the post-monsoon air quality data of the years 2020 and 2021. As mentioned earlier, inverse-decision rules are framed during validation time with the validation set. These inverse-decision rules are used to provide justifications (reasons) for given decisions. These are as follows:

Inverse Fact 1: If AQI is (poor or very poor), then ($91 \le PM_{2.5} \le 135$)

Inverse Fact 2: If AQI is (poor or very poor), then ($136 \leq PM_{2.5} \leq 180$)
Inverse Fact 3: If AQI is (poor or very poor), then ($181 \leq PM_{2.5} \leq 225$)
Inverse Fact 4: If AQI is (poor or very poor), then ($226 \leq PM_{2.5} \leq 270$)
Inverse Fact 5: If AQI is (poor or very poor), then ($143 \leq PM_{10} \leq 190$)
Inverse Fact 6: If AQI is (poor or very poor), then ($191 \leq PM_{10} \leq 238$)
Inverse Fact 7: If AQI is (poor or very poor), then ($239\text{-} \leq PM_{10} \leq 286$)
Inverse Fact 8: If AQI is (poor or very poor), then ($287 \leq PM_{10} \leq 334$)

The same criteria for Z-number as mentioned in Table 2 are used while implementing inverse rules. The overall research framework is illustrated in a block diagram (Fig. 2).

Results and discussion

Variation in $PM_{2.5}$ and PM_{10} during Diwali

On a normal, pre-Diwali, Diwali, or post-Diwali day, the diurnal variation (6–6 a.m.) of PM_{10} and $PM_{2.5}$ is illustrated in Fig. 3 based on the Diwali data during 2019, as an example. It is observed that the daytime PM_{10} and $PM_{2.5}$ concentrations on normal and pre-Diwali days are clearly similar and lower than those on Diwali and post-Diwali days. However, the concentrations of PM_{10} and $PM_{2.5}$ were consistently greater at night than during the day time. During the Diwali, as well as before (pre) and after (post) Diwali, the night-time concentrations of PM_{10} and $PM_{2.5}$ peaked between 8 p.m. and 3 a.m., indicating that the majority of the firework activity took place during that time. The influence of firework activity during the night lasted until the next day morning, as we found that the concentrations of these two pollutants increase on the Diwali and post-Diwali days during 6 a.m.–1 p.m. as compared to pre-Diwali and normal days. The pollution levels remained high from several days to about a month instead of subsiding after the Diwali. PM, generated during firecracker burning, has an important part in the environment since various ions and particles adsorbed by them. Both PM_{10} and

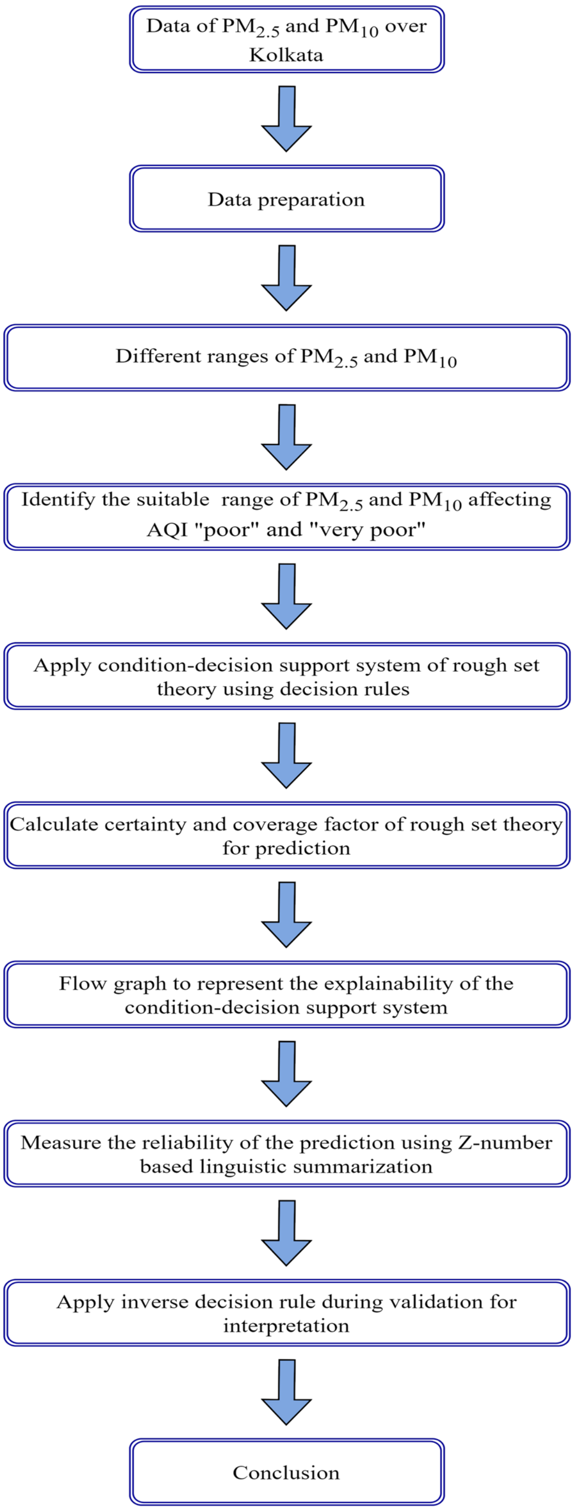


Fig. 2 Block diagram showing proposed research framework

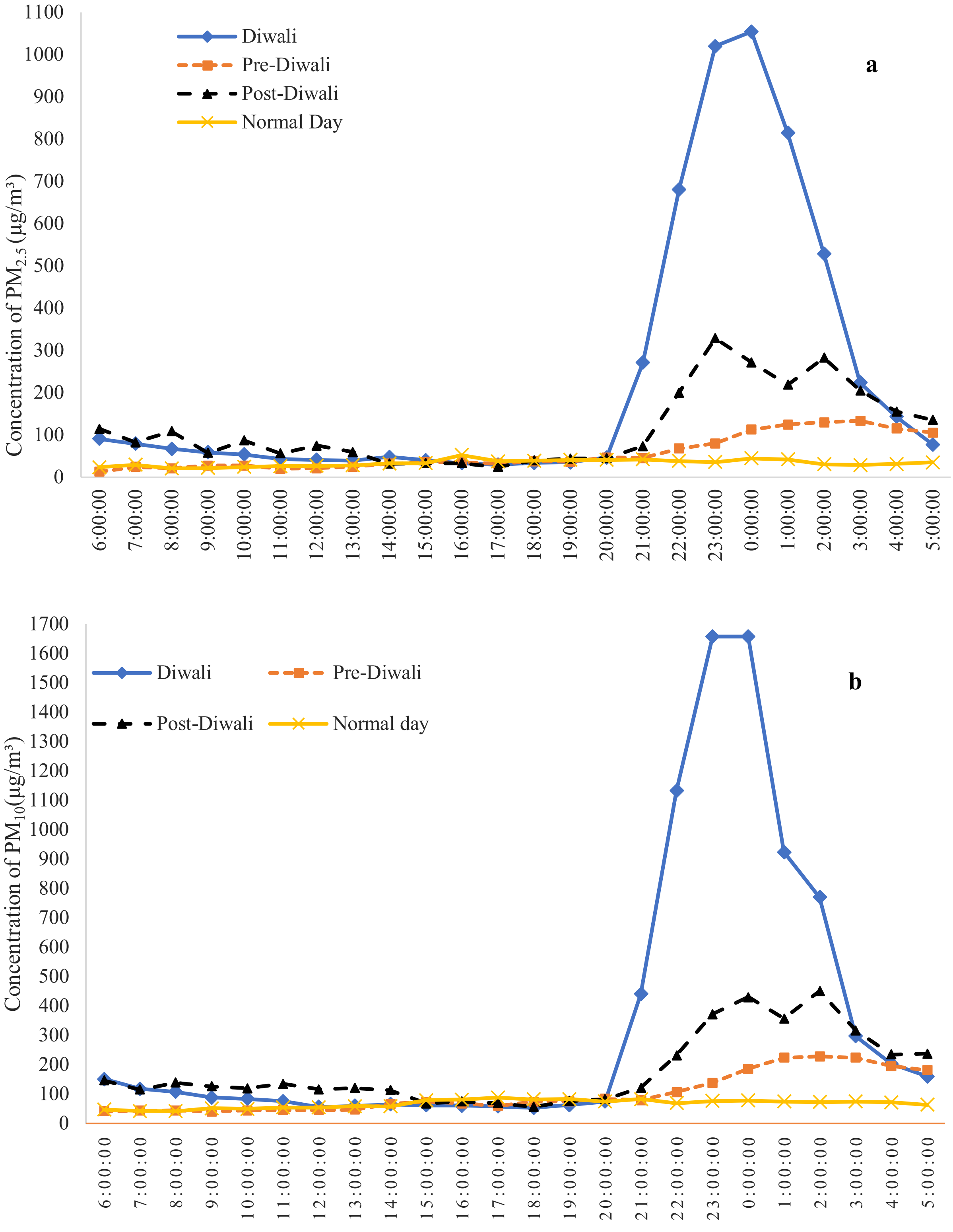


Fig. 3 Diurnal variation of **a** $PM_{2.5}$ and **b** PM_{10} during Diwali, pre-Diwali, post-Diwali, and normal day

$PM_{2.5}$ can penetrate the respiratory system; however, atmospheric particulate matter $PM_{2.5}$ which is about 3% the diameter of a human hair is more harmful because it can travel deep into the lungs and cause a wide range of respiratory and cardiovascular diseases, as well as cancer.

Flow graph and explainability

As stated earlier, a flow graph is associated with a decision table. It is a directed acyclic graph in which a directed branch x connects the input and output nodes (viz., $C(x)$ and $D(x)$, respectively) corresponding to a decision rule $C \underset{x}{\rightarrow} D$. The through flow of a branch is represented by the decision rule's strength factor (Pawlak, 2004). Figures 4 and 5 show graphical representation of two flow graphs corresponding to $PM_{2.5}$ and PM_{10}. As mentioned earlier, the concentrations of $PM_{2.5}$ and PM_{10} are divided into four ranges, so there are four roots for each of $PM_{2.5}$ and PM_{10} satisfying the decisions — "poor" and "very poor" AQI. Every root covers some strength, certainty, and coverage values which are visualized easily with the help of flow graph. This representation enables determining the probable roots for each prediction with priority.

The term "explainable AI" or "interpretable AI" refers to human being able to understand the path that artificial intelligence technology took to make a decision using dynamically generated graphs or textual descriptions. Flow graphs in Figs. 4 and 5 that represent two decision tables provide a clear insight into the decision-making process with strength, certainty, and coverage values; thereby demonstrating the interpretation ability or explanation ability of the proposed condition-decision support system to make air-quality prediction.

Results of condition-decision support system using design set

In this study, we apply rough set theory to identify the threshold ranges of $PM_{2.5}$ and PM_{10} that make degradation of air quality in the months of October, November, and December during Diwali over Kolkata. We consider the certainty and coverage values of a rough set theoretic rule for a specific decision with different conditions. These values were obtained using Eqs. (7) and (9), respectively. Higher values of certainty and coverage mean better prediction. Every decision rule has two parts — condition (i.e., different ranges of pollutants) and decision (i.e., AQI categories). Figure 6 displays "poor" and "very poor" AQI which are caused by different threshold ranges of $PM_{2.5}$. With the range (91–135) μg/m^3, i.e., $91 \leq PM_{2.5} \leq 135$ (*Fact 1*), the higher values of certainty (0.70) and coverage (1.00) are obtained when AQI is "poor" (Fig. 6a). Here, the certainty factor of 0.70 for the said rule means, 70% of the days fulfil the condition and the decision of the same rule, and 30% are missed predictions. Furthermore, the certainty factor of 1.00 for the aforementioned rule indicates that 100% of the days which fulfil the decision also fulfil the condition of the rule. These two values together led to overall superior prediction by that rule. The results further imply that the certainty of occurrence of the "very poor" AQI is maximum (1.00) when $PM_{2.5}$ remains within $136 \leq PM_{2.5} \leq 180$, $181 \leq PM_{2.5} \leq 225$, and $226 \leq PM_{2.5} \leq 270$ (*Facts 2, 3, and 4*). However, coverage values in those cases are found to be 0.57, 0.18, and 0.01, respectively, thereby giving an overall moderate prediction. For the other range $91 \leq PM_{2.5} \leq 135$, the certainty and coverage values for "very poor" air quality are seen to be 0.30 and 0.24, respectively, thereby indicating poor prediction (Fig. 6b).

Figure 7 depicts the values of the certainty and coverage factors for different threshold ranges of PM_{10} for the prevalence of "poor" and "very poor" air quality during the post-monsoon season. Here, the certainty for the occurrence of "poor" air quality is 0.90 and its coverage is 0.54 when PM_{10} remains within the range of $143 \leq PM_{10} \leq 190$ (Fig. 7a). This leads to an overall good prediction. For the same air quality, the certainty and coverage values are 0.35 and 0.46 respectively for the range $191 \leq PM_{10} \leq 238$. The optimum value of certainty factor, i.e., 1.00, is found for the air quality being "very poor" in the range of ($239\text{-} \leq PM_{10} \leq 286$) and ($287 \leq PM_{10} \leq 334$), as shown in Fig. 7b (*Facts 7 and 8*). However, coverage values here are obtained as 0.34 and 0.15, respectively, implying that 66% and 85% of predictions are missed, indicating moderate prediction. Furthermore, certainty and coverage factors are 0.65 and 0.48 respectively for the AQI being "very poor" in the range of $191 \leq PM_{10} \leq 238$. This leads to overall moderately good prediction. Again, these two factors are seen to be 0.10 and 0.03 respectively for the range

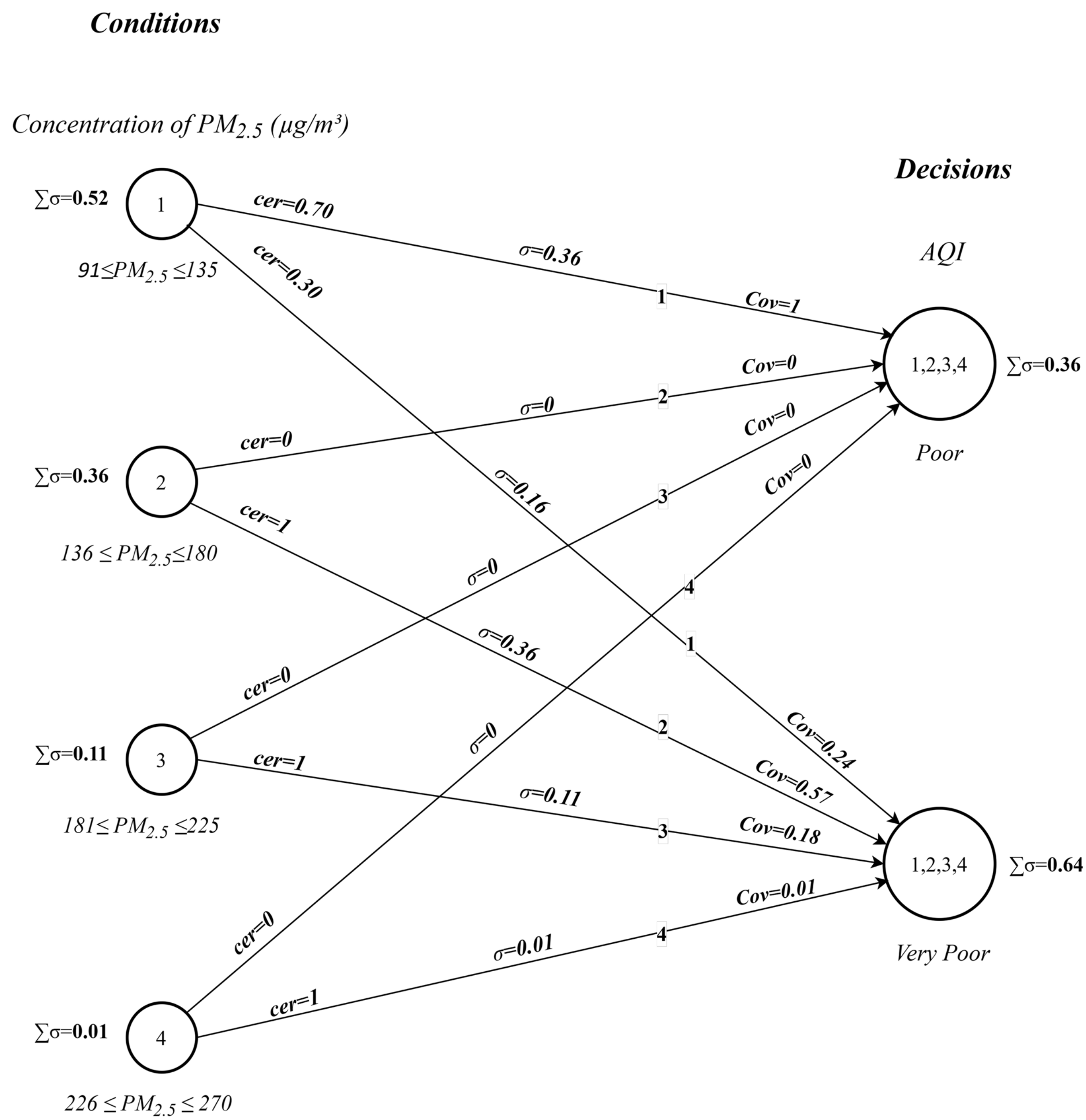


Fig. 4 Flow graph for the decision algorithm of $PM_{2.5}$

$143 \leq PM_{10} \leq 190$, indicating bad prediction with huge number of false alarms.

Z-number-based information

Since unreliability is an indissoluble aspect of real-world data, precise decision-making necessitates accurate information. The idea of Z-number deals with this unreliability and vagueness by incorporating them into mathematical calculations. The use of Z-numbers to obtain a linguistic description of an AQI has various implications. Certainty and coverage values and corresponding linguistic descriptions using Z-number for different rules are presented in Table 2. Table 3 depicts Z-number describing AQI criteria, i.e., “poor” and “very poor,” for different threshold ranges of $PM_{2.5}$

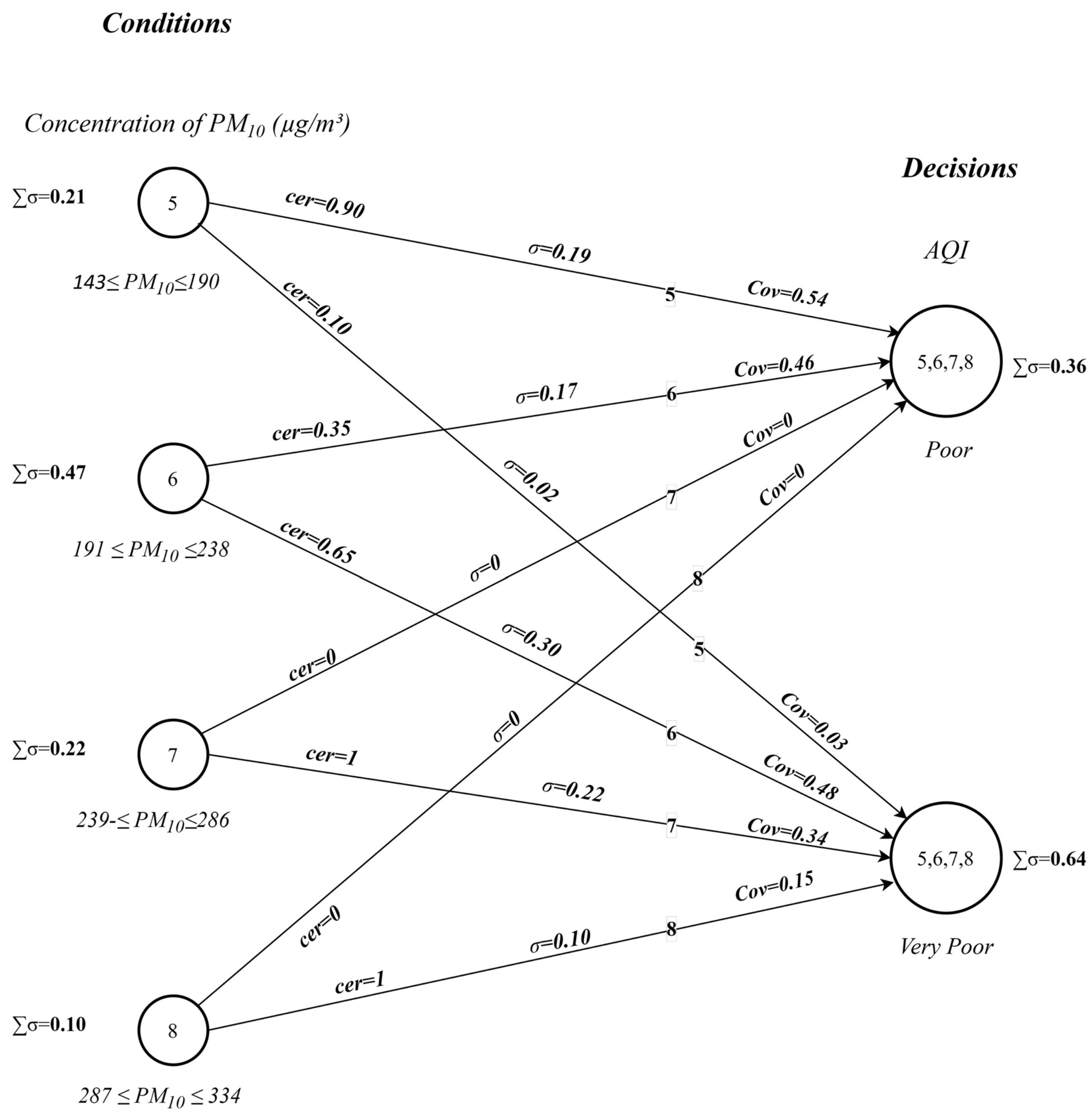
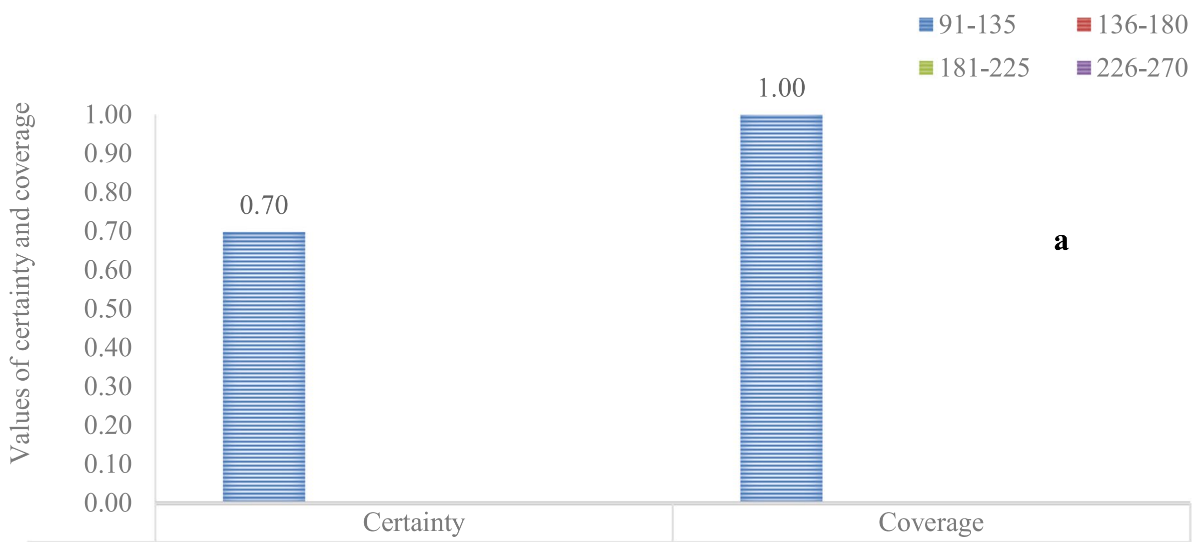
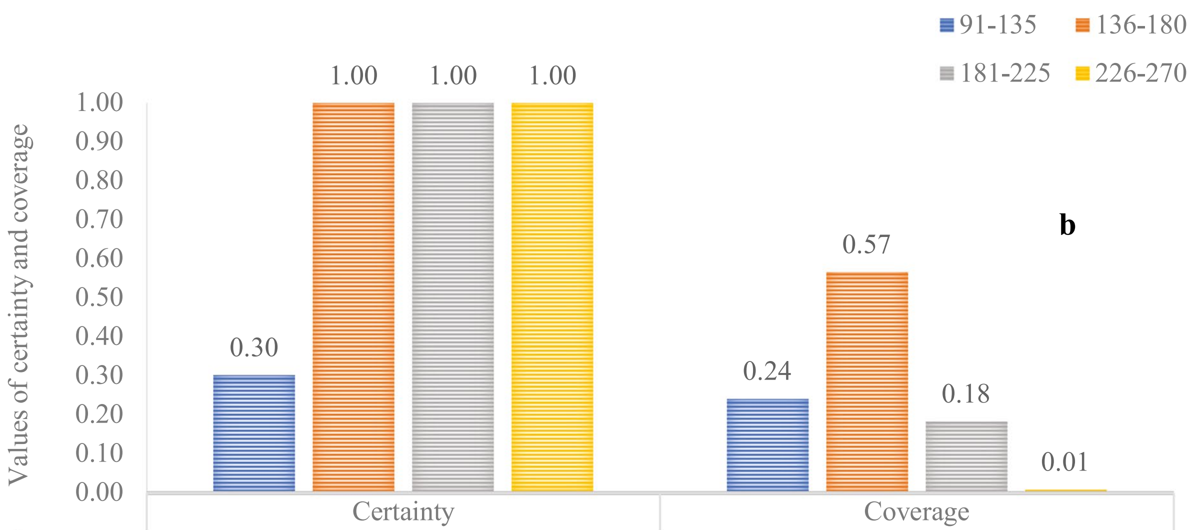


Fig. 5 Flow graph for the decision algorithm of PM_{10}

and PM_{10} with decision rules. For example, when $PM_{2.5}$ is within the range of $91 \leq PM_{2.5} \leq 135$, Z-based information of AQI can be computed as $Z = <$poor, *Certainly*$>$ and $Z = <$very poor, *Less Likely*$>$. That means, with the aforesaid range of $PM_{2.5}$, the reliability of "poor" AQI is "*Certainly*" having certainty and coverage values 0.70 and 1.00, respectively, and the reliability of "very poor" AQI is "*Less Likely*" with certainty and coverage values 0.30 and 0.24, respectively. Similarly, in the range of $143 \leq PM_{10} \leq 190$, the measurement of Z-based AQI can be written as $Z = <$poor, *Most Likely*$>$ and $Z = <$very poor, *Unlikely*$>$. One may note that, when both certainty and coverage values are equal to or below 0.20, their Z-numbers would contain reliability value as "*Unlikely*." Likewise, for all the threshold ranges of



	Certainty	Coverage
91-135	0.70	1.00
136-180	0.00	0.00
181-225	0.00	0.00
226-270	0.00	0.00



	Certainty	Coverage
91-135	0.30	0.24
136-180	1.00	0.57
181-225	1.00	0.18
226-270	1.00	0.01

Fig. 6 The variations of the certainty and coverage factors with the facts corresponding to the different ranges of $PM_{2.5}$ as condition and occurrences of **a** poor and **b** very poor AQI as decision

Fig. 7 The variations of the certainty and coverage factors with the facts corresponding to the different ranges of PM_{10} as condition and occurrences of **a** poor and **b** very poor AQI as decision

Table 3 Linguistic description of AQI using Z-number-based information obtained from decision rules for design set

Decision rules	Z-number-based information of air quality
Fact 1: If ($91 \leq PM_{2.5} \leq 135$), then (AQI is poor or very poor)	Z = <poor, *Certainly*> Z = <very poor, *Less Likely*>
Fact 2: If ($136 \leq PM_{2.5} \leq 180$), then (AQI is poor or very poor)	Z = <poor, *Unlikely*> Z = <very poor, *Most Likely*>
Fact 3: If ($181 \leq PM_{2.5} \leq 225$), then (AQI is poor or very poor)	Z = <poor, *Unlikely*> Z = <very poor, *May be*>
Fact 4: If ($226 \leq PM_{2.5} \leq 270$), then (AQI is poor or very poor)	Z = <poor, *Unlikely*> Z = <very poor, *May be*>
Fact 5: If ($143 \leq PM_{10} \leq 190$), then (AQI is poor or very poor)	Z = <poor, *Most Likely*> Z = <very poor, *Unlikely*>
Fact 6: If ($191 \leq PM_{10} \leq 238$), then (AQI is poor or very poor)	Z = <poor, *Less Likely*> Z = <very poor, *May be*>
Fact 7: If ($239\text{-} \leq PM_{10} \leq 286$), then (AQI is poor or very poor)	Z = <poor, *Unlikely*> Z = <very poor, *May be*>
Fact 8: If ($287 \leq PM_{10} \leq 334$), then (AQI is poor or very poor)	Z = <poor, *Unlikely*> Z = <very poor, *May be*>

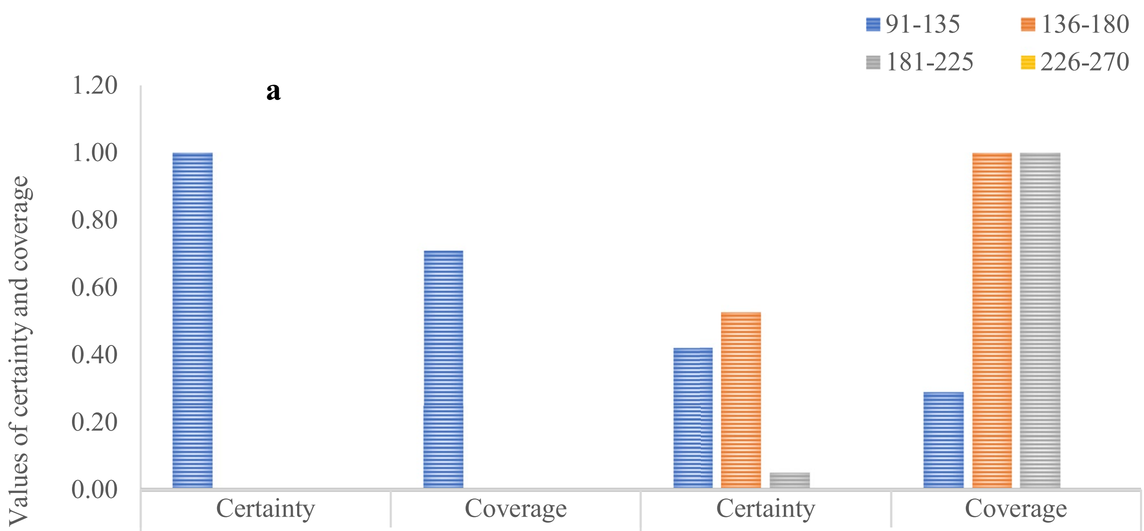
PM_{10} and $PM_{2.5}$, linguistic summarization of AQI has been carried out (Table 3). From this, one can infer that the reliability of our prediction of different criteria of air quality (AQI), viz., "poor" and "very poor," can be represented efficiently with the help of Z-numbers.

Validation based on inverse-decision rules

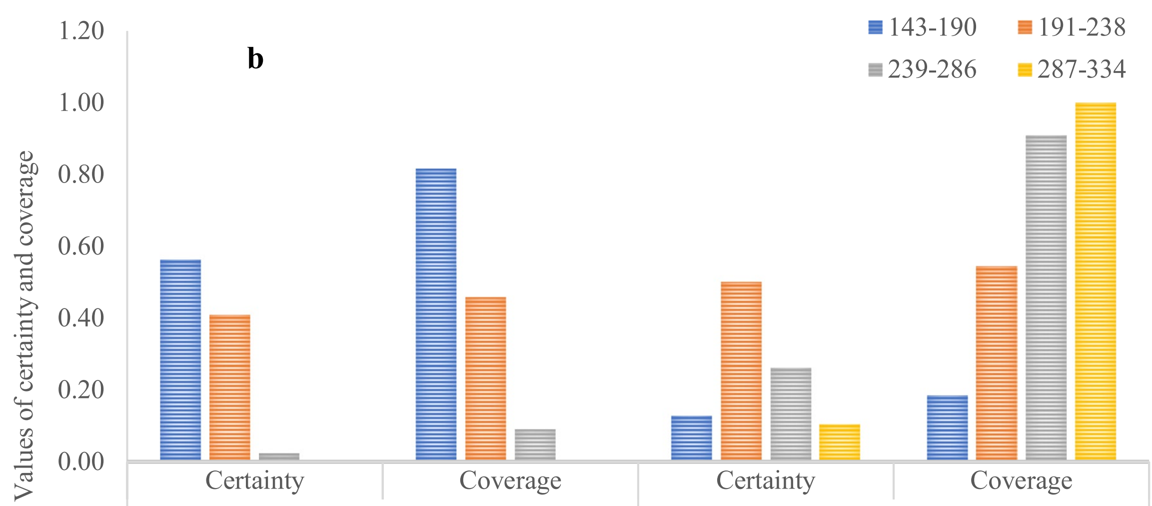
Inverse-decision rules are described earlier. These are used for validating the decisions of the decision rules of design set with explanations for air-quality prediction. Every inverse-decision rule has two parts — condition (i.e., AQI categories) and decision (i.e., ranges of pollutants). That means, for a given prediction, say AQI is "poor," we just traverse back to the inverse direction, i.e., from output towards the input side, and compute the certainty and coverage values of that inverse-decision rule using Eqs. (7) and (9). The rule having higher certainty and coverage values gives a better prediction.

Figure 8 depicts the certainty and coverage values of the AQI categories, viz., "poor" and "very poor," for the prediction of the appropriate range of $PM_{2.5}$ and PM_{10} using inverse-decision rules. Given that AQI is "poor," the maximum value of the certainty factor here is 1.00, and the coverage factor is 0.71 for the prediction of $PM_{2.5}$ concentrations when $PM_{2.5}$ lies within the range of $91 \leq PM_{2.5} \leq 135$. This indicates that for this range of $PM_{2.5}$, one gets excellent prediction of air quality being "poor" (Fig. 8a). The certainty and coverage values for all other threshold ranges are seen to be 0.00 when AQI is "poor." Thus, it is clear that during the post-monsoon months of October, November, and December, the range of $PM_{2.5}$, i.e., $91 \leq PM_{2.5} \leq 135$, is alone contributing to air quality being "poor." These results also validate the decisions obtained from the design set (Fig. 6a). Similarly, in the case of PM_{10}, when AQI is "poor," the optimum value of the certainty is 0.56 and that of the coverage factor is 0.81 corresponding to the range $143 \leq PM_{10} \leq 190$, thereby inferring that a good prediction comes because of this range (Fig. 8b). On the other hand, for the same air quality "poor," the certainty factor for PM_{10}, lying within the range of $191 \leq PM_{10} \leq 238$, is 0.41 and the coverage factor for the same range is 0.46. This means, the range $191 \leq PM_{10} \leq 238$ of PM_{10} is responsible for moderate prediction of "poor" air quality. With the range $239\text{-} \leq PM_{10} \leq 286$, these two factors are 0.03 and 0.09 respectively causing a bad prediction. Thus, the range $143 \leq PM_{10} \leq 190$ contributes mostly to air quality being "poor" which validates the result obtained in the case of design set.

Figure 8a depicts that when AQI is "very poor," the maximum values of certainty (=0.53) and coverage (=1.00) are found for $PM_{2.5}$, lying in the range of $136 \leq PM_{2.5} \leq 180$. That means, $PM_{2.5}$ is responsible at most for good prediction of "very poor" air quality, and this too is possible for the range $136 \leq PM_{2.5} \leq 180$. For PM_{10}, with the same AQI category, the certainty and



	PM2.5 <poor>		PM2.5 <very poor>	
	Certainty	Coverage	Certainty	Coverage
91-135	1.00	0.71	0.42	0.29
136-180	0.00	0.00	0.53	1.00
181-225	0.00	0.00	0.05	1.00
226-270	0.00	0.00	0.00	0.00



	PM10 <poor>		PM10 <very poor>	
	Certainty	Coverage	Certainty	Coverage
143-190	0.56	0.81	0.13	0.19
191-238	0.41	0.46	0.50	0.54
239-286	0.03	0.09	0.26	0.91
287-334	0.00	0.00	0.11	1.00

Fig. 8 The variations of the certainty and coverage factors of inverse-decision rules for **a** $PM_{2.5}$ and **b** PM_{10} during validation

coverage factors for the range $143 \leq PM_{10} \leq 190$ are 0.13 and 0.19, and for the range of $239\text{-} \leq PM_{10} \leq 286$ are 0.26 and 0.91, respectively. For the other two ranges, viz., $287 \leq PM_{10} \leq 334$ and $191 \leq PM_{10} \leq 238$, these two factors are found to be 0.11 and 1.00, and 0.50 and 0.54, respectively (Fig. 8b). From these, it can be inferred that PM_{10} is responsible at most for moderate prediction of "very poor" air quality and that would happen for its range $191 \leq PM_{10} \leq 238$.

Validation of Z-number-based information

Table 4 depicts Z-numbers describing AQI criteria of different inverse-decision rules. Inverse-decision rules are just reverse to that of decision rules and provide interpretability of the same. Validation is done to validate the decision rules of the design set. So, a comparative study of Z-based information is made with respect to the design set during validation. For example, when $PM_{2.5}$ is "poor," the Z-measure for the range $91 \leq PM_{2.5} \leq 135$ is *Z*= <poor, *Certainly*> and *Z*= <very poor, *Less Likely*> (*Inverse Fact 1*), which are exactly the same as Z-measures obtained from the *Fact 1*. Thus, it can be said that when the value of $PM_{2.5}$ lies within the range of $91 \leq PM_{2.5} \leq 135$, the air quality is <poor, *Certainly*> and there is a chance of breathing discomfort to people on prolonged exposure (Table 1). In this way, this study correlates air pollution measurement with possible health impacts. Except a few deviations, the Z-measures obtained from the inverse-decision rules are exactly similar to those of the decision rules and also validate the same. Below are examples of some deviations though negligible. Z-measure for *Fact 4* is <very poor, *May be*>, while *Inverse Fact 4* for the same is *Z*= <very poor, *Unlikely*>. Z-measure for decision rule 6 (*Fact 6*) is <very poor, *May be*>, while the same for the inverse-decision rule 6 (*Inverse Fact 6*) is <very poor, *Likely*>. The confusion in reliability lies mostly between two adjacent metrics (linguistic hedges). In this context, one may note that the metric "*May be*" corresponds to an unusual case where the certainty for the prediction lies within the range of $0.60 \leq$ Certainty ≤ 1 while the coverage lies within the range of $0.01 \leq$ Coverage < 50, or vice versa, and this makes the conclusion on a prediction very difficult.

Further, it may be mentioned that the certainty and coverage factors of the rules (and hence their linguistic matrices) depend to some extent on the size of the dataset. In the aforesaid results, the sizes of the design set and validation set are different. In a part of the investigation, to corroborate this, we have divided the design set into two equal parts and obtained the decision rules separately. Interestingly, the rules 4 and 6 obtained from both the parts are seen to match the corresponding inverse-decision rules in terms of Z-based information.

The AQI values and corresponding ambient concentrations (health breakpoints) are presented in

Table 4 Linguistic description of AQI using Z-number-based information obtained from inverse-decision rules for validation set

Inverse-decision rules	Z-number-based information of air quality
Inverse Fact 1: If AQI is (poor or very poor), then ($91 \leq PM_{2.5} \leq 135$)	*Z*= <poor, *Certainly*> *Z*= <very poor, *Less Likely*>
Inverse Fact 2: If AQI is (poor or very poor), then ($136 \leq PM_{2.5} \leq 180$)	*Z*= <poor, *Unlikely*> *Z*= <very poor, *Most Likely*>
Inverse Fact 3: If AQI is (poor or very poor), then ($181 \leq PM_{2.5} \leq 225$)	*Z*= <poor, *Unlikely*> *Z*= <very poor, *May be*>
Inverse Fact 4: If AQI is (poor or very poor), then ($226 \leq PM_{2.5} \leq 270$)	*Z*= <poor, *Unlikely*> *Z*= <very poor, *Unlikely*>
Inverse Fact 5: If AQI is (poor or very poor), then ($143 \leq PM_{10} \leq 190$)	*Z*= <poor, *Most Likely*> *Z*= <very poor, *Unlikely*>
Inverse Fact 6: If AQI is (poor or very poor), then ($191 \leq PM_{10} \leq 238$)	*Z*= <poor, *Less Likely*> *Z*= <very poor, *Likely*>
Inverse Fact 7: If AQI is (poor or very poor), then ($239 \leq PM_{10} \leq 286$)	*Z*= <poor, *Unlikely*> *Z*= <very poor, *May be*>
Inverse Fact 8: If AQI is (poor or very poor), then ($287 \leq PM_{10} \leq 334$)	*Z*= <poor, *Unlikely*> *Z*= <very poor, *May be*>

Table 5 Breakpoints for AQI scale 0–500 (units: μg/m^3 unless mentioned otherwise) (CPCB, 2014)

AQI category (range)	PM_{10} 24-h	$PM_{2.5}$ 24-h	NO_2 24-h	O_3 8-h	CO 8-h (mg/m^3)	SO_2 24-h	NH_3 24-h	Pb 24-h
Good (0–50)	0–50	0–30	0–40	0–50	0–1.0	0–40	0–200	0–0.5
Satisfactory (51–100)	51–100	31–60	41–80	51–100	1.1–2.0	41–80	201–400	0.6–1.0
Moderate (101–200)	101–250	61–90	81–180	101–168	2.1–10	81–380	401–800	1.1–2.0
Poor (201–300)	251–350	91–120	181–280	169–208	10.1–17	381–800	801–1200	2.1–3.0
Very poor (301–400)	351–430	121–250	281–400	209–748*	17.1–34	801–1600	1201–1800	3.1–3.5
Severe (401–500)	430+	250+	400+	748+*	34+	1600+	1800+	3.5+

*One hourly monitoring (for mathematical calculation only)

Table 5 as indicated by the CPCB. The assessment results of our present study reveal that the value of $PM_{2.5}$ when lies within the range of $91 \leq PM_{2.5} \leq 135$, the maximum probability of AQI is to be "poor" (Z-based information of AQI is <poor, *Certainly*>) during the Diwali periods over Kolkata (Table 3). From Table 5, AQI is "poor" when the value of $PM_{2.5}$ lies within the range of (91–120), which matches our observations. Similarly, from Table 5, when AQI is "poor," the breakpoint concentrations of PM_{10} lie within the range of (251–350) which contradicts the results obtained in our study. For example, for both the ranges of ($239\text{-} \leq PM_{10} \leq 286$) and ($287 \leq PM_{10} \leq 334$), the Z-measure of the AQI is <poor, *Unlikely*>. Meanwhile, when PM_{10} lies within the range of $143 \leq PM_{10} \leq 190$, Z-based information of AQI is <poor, *Most Likely*>. Likewise, during Diwali over Kolkata, when $PM_{2.5}$ lies within the range of $136 \leq PM_{2.5} \leq 180$, then the maximum probability of occurrences of "very poor" AQI is maximum (Z-based information of AQI is <very poor, *Most Likely*>) which falls within the range of (121–250) as mentioned in Table 5, i.e., when AQI is "very poor," the breakpoint concentrations of $PM_{2.5}$ lie within the range of (121–250). Also, from our study, when $PM_{2.5}$ lies within the range of $181 \leq PM_{2.5} \leq 225$, Z-based measure of AQI is <very poor, *May be*>. Similarly, from Table 5, when AQI is "very poor," the breakpoint concentrations of PM_{10} lie within the range of (351–430) μg/m^3. But maximum concentration of PM_{10} obtained during the Diwali period of Kolkata is 334 μg/m^3 which is lower than the aforesaid range of (351–430) μg/m^3. Thus, there is a wide gap between the assessment result of determining particulate matter concentrations (i.e., PM_{10}) and the index value in the case of both "poor" and "very poor" AQI (Table 5). Thus, concentrations of pollutants can differ from the index value as indicated by the CPCB regarding location and situation.

Conclusions

Despite not being as filthy as Delhi, Kolkata is becoming as India's second most polluted metropolis (WHO, 2018). Like other metro cities, Kolkata has recognized several issues that contribute to air pollution. The air quality of the highly polluted mega city Kolkata has deteriorated as a result of the rampant cracker bursting on Diwali, despite the restrictions. Furthermore, the temperature inversion causes a blanket of smog to grow over the research area. As a result, air quality as measured by the AQI has deteriorated significantly.

In the present study, rough set theoretic approach is used for analyzing air quality data to obtain prediction during the post-monsoon months of October, November, and December. The findings of the present study led to the conclusion that the certainty and coverage values, as obtained for different rough set theoretic decision rules under various conditions, play the most crucial role in prediction. Combining the impact of certainty and coverage factors of a decision rule provides an information measure (AQI) of the prediction.

The method of using Z-numbers to provide a linguistic description of such prediction of two different categories in terms of AQI is unique. With the use of such linguistic summarization, Z-numbers describe the predictability of the results as obtained from the rough set-based condition-decision support system. This measure is also treated as a quantitative index for determining the threshold ranges of $PM_{2.5}$ and PM_{10} that cause the degradation of air quality in the aforementioned months during Diwali leading to AQI "poor" and "very poor." Thus, the public can track the status of their local, regional, and national air quality without knowing the details of the monitoring data on which they are based. Moreover, a more sophisticated technique has been developed to convey the health risk associated with the ambient concentrations.

The interpretation of the decision is done using the concept of explainable artificial intelligence (XAI). We have applied flow graph as an explanation method to represent the rough set-based condition-decision support system for the prediction of air quality. This determines all possible $PM_{2.5}$ and PM_{10} ranges with different strength, certainty, and coverage values for a given prediction by indicating the corresponding multiple probable pathways.

Traditional computational approaches do not appear to be flexible or capable enough to address complicated real-world environmental challenges successfully. On the other hand, uncertainty management, temporal reasoning, spatial reasoning, and evaluation are among the major concerns behind designing IEDSS. The primary purpose of the study is to demonstrate how to combine the notion of rough set-based condition-decision support system with decision rules, inverse-decision rules with flow graph for decision-making, and Z-numbers for quantification of semantic information towards the measurement, prediction, and explanation. Combination of these techniques can therefore be viewed as a hybrid system in soft computing paradigm leading to an advanced IEDSS for air pollution monitoring and prediction. The work is significant not only in the area of air pollution but also in the domain of soft computing and machine intelligence.

Our study has certain concerns because of some assumptions made, and these may lead to several scopes for future research in the areas we have highlighted. (1) We have split the ranges of $PM_{2.5}$ and PM_{10} that make the air quality in Kolkata "very poor" and "poor" in four intervals. One may consider splitting these ranges at different intervals. Then the results may vary, though the way of representation will be the same. (2) When splitting the dataset, it is assumed that there is no initial overlapping.

Our study demonstrates that for assessing "poor" and "very poor" AQI during the Diwali period in Kolkata, satisfactory results are obtained for different ranges of $PM_{2.5}$. However, a significant discrepancy is found between the observed concentration range of PM_{10} and the breakpoint concentrations for predicting AQI "poor" and "very poor." Moreover, the observed concentration ranges of PM_{10} during the Diwali period, signifying "poor" and "very poor" AQI, are lower than the breakpoint concentrations. Thus, concentrations of pollutants may vary depending on the location and situation. It may be mentioned that the AQI is utilized by national and local environmental organizations to provide real-time air quality information for a particular location. WHO (2006) recommends that when formulating policy targets, such as AQI, governments should consider their own local circumstances carefully before implementing the guidelines directly as legally based standards. Focusing on the aforementioned, it is suggested that the regulatory and enforcement agencies should review the present air quality monitoring requirements. Furthermore, it is essential to develop a more comprehensive monitoring system, regulations, and appropriate implementation to obtain the most effective and efficient method of improving air quality. Additionally, there are several secondary pollutants that are formed in the lower atmosphere as a result of chemical reactions of primary pollutants. While paying more attention to the combined effects of several pollutants, low level exposure, and quick public reporting, there is still a lot of progress to be made. This study suggests the formulation of an improved AQI.

Finally, we propose that the future work on this topic be expanded to include other pollutants, e.g., nitrogen dioxide (NO_2) and ozone (O_3), to measure their impact on the deterioration of air quality. The novel concept of Z-number-based AQI, as developed here, can be implemented to characterize other tasks in pollution analytics.

Author contribution DD formulated the research problem, wrote the programs, and made the first draft. SKP is the mentor and principal investigator who gave the guidance in machine learning aspects of the paper and provided corrections of the overall manuscript for better organization and understanding.

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Data availability The corresponding author will provide the data on request.

Declarations

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