$\mathbb{Z}_2$ Topological Term, the Global Anomaly, and the Two-Dimensional Symplectic Symmetry Class of Anderson Localization

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We discuss, for a two-dimensional Dirac Hamiltonian with a random scalar potential, the presence of a $\mathbb{Z}_2$ topological term in the nonlinear sigma model encoding the physics of Anderson localization in the symplectic symmetry class. The $\mathbb{Z}_2$ topological term realizes the sign of the Pfaffian of a family of Dirac operators. We compute the corresponding global anomaly, i.e., the change in the sign of the Pfaffian by studying a spectral flow numerically. This $\mathbb{Z}_2$ topological effect can be relevant to graphene when the impurity potential is long ranged and, also, to the two-dimensional boundaries of a three-dimensional lattice model of $\mathbb{Z}_2$ topological insulators in the symplectic symmetry class.

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There is strong theoretical and numerical supporting evidence for the premise that metal-insulator transitions in the problem of Anderson localization with short-range correlated disorder can be classified in terms of a few fundamental properties of microscopic Hamiltonians. Whenever the motion of the relevant (fermionic) quasiparticles is diffusive, a nonlinear sigma model (NLσM) can be derived for each symmetry class. A NLσM encodes fluctuations of Nambu-Goldstone bosons (diffuson and Cooperons) that are defined on a curved manifold (target manifold) determined by the symmetries of a microscopic disordered Hamiltonian.

When the fluctuations are small, the dynamics of the Nambu-Goldstone bosons are solely determined by the local data of the target manifold such as the metric and curvature. However, the global topology of the target manifold does have important effects. For example, a weak magnetic field relative to the disorder strength leads to localization of all states in two-dimensional space. On the other hand, a strong magnetic field leads to the integer quantum Hall plateau transition. This difference is captured by the absence or presence, respectively, of a topological term in the NLσM corresponding to the two-dimensional unitary symmetry class [1]. Similarly, a random single-particle Hamiltonian that preserves time-reversal symmetry but breaks spin-rotation symmetry defines the symplectic symmetry class and can be associated to a NLσM. In two-dimensional space, Fendley [2] made the observation that such a NLσM admits a topological term. The configuration space of the relevant NLσM has two topologically distinct sectors. Configurations from different topological sectors can thus be given different Boltzmann weights in the presence of the topological term, thereby realizing the outcome of a metal-insulator transition belonging to a universality class different from the one without the topological term [2].

We show in this Letter that a topological term, encoding a global (i.e., nonperturbative) anomaly, is realized by the Pfaffian of Majorana fermions that originate from the problem of Anderson localization defined by the two-dimensional two-component Dirac Hamiltonian

$$\mathcal{H} = \boldsymbol{\sigma}\cdot\boldsymbol{p} + \sigma_0 V(\boldsymbol{r}), \qquad \boldsymbol{p} := \frac{\partial}{i\partial\boldsymbol{r}}, \tag{1}$$

subject to a white-noise and Gaussian correlated random scalar potential $V(\boldsymbol{r})$, $\overline{V(\boldsymbol{r})} = 0$, $\overline{V(\boldsymbol{r})V(\boldsymbol{r}')} = g\delta(\boldsymbol{r}-\boldsymbol{r}')$. Here, $\boldsymbol{r}\in\mathbb{R}^2$, $g\geq 0$ measures the disorder strength, and $\sigma_{x,y,z}$ denote the standard Pauli matrices with σ_0 the corresponding unit matrix. In the recent independent derivation by Ostrovsky *et al.* [3], the topological term is not presented in the form of the sign of a Pfaffian which, however, is essential for it to be interpreted as a global anomaly. The fabrication of graphene [4] has triggered a renewed theoretical interest in the properties of the random Dirac Hamiltonian (1). We will argue that the random Dirac Hamiltonian (1) can be realized at the two-dimensional boundaries and in the low-energy limit of three-dimensional lattice models.

As observed by Ludwig *et al.* [5] the symmetry $i\sigma_y\mathcal{H}^*(-i\sigma_y) = \mathcal{H}$, which we shall abusively call time-reversal symmetry (TRS) as it might not necessarily realize the TRS of the underlying microscopic model, puts the random Dirac Hamiltonian (1) in the two-dimensional symplectic symmetry class. The disorder in the Dirac Hamiltonian (1) is thus expected to yield weak antilocalization corrections to the mean conductance [6].

The link between single-particle random Hamiltonians and NLσM comes about when setting up a generating function for the mean value taken by the product of the Green's functions. Using the standard machinery of the replica trick [7], the disorder-averaged product of the retarded and advanced Green's functions of Eq. (1) can be obtained from the Grassmann path integral $\bar{Z} := \int\mathcal{D}[\bar{\psi},\psi]\exp(-\int d^2r\mathcal{L})$, where

$$\mathcal{L} = \bar{\psi}_a(\boldsymbol{\sigma}\cdot\boldsymbol{p}\delta_{ab} + i\eta\Lambda_{ab})\psi_b - \frac{g}{2}\bar{\psi}_a\psi_a\bar{\psi}_b\psi_b, \tag{2}$$

and with the Majorana condition $\bar{\psi}_a := \psi_a^T i\sigma_y$, on the Grassmann integration variables ($a = 1, \ldots, 4N_r$). Summation over repeated indices with $a, b = 1, \ldots, 4N_r$ is implied. For any $a = 1, \ldots, 4N_r$, the Pauli matrices act on the two components of the spinor ψ_a. The $4N_r \times 4N_r$ diagonal matrix $\Lambda = \mathrm{diag}(\mathbb{1}_{2N_r}, -\mathbb{1}_{2N_r})$ distinguishes two sectors coming from the retarded and advanced Green's functions. It is multiplied by the infinitesimal positive number η. The replica limit $N_r \to 0$ is understood if the effective partition function is purely fermionic, while the choice $N_r = 1$ is appropriate when (2) is regarded as the fermionic sector of a supersymmetric representation of the product of the retarded and advanced Green's functions. The 4-fermion interaction in (2) originates from the integration over the static random field $V(\boldsymbol{r})$.

When $\eta = 0$, the replicated action is invariant under a $\mathrm{O}(4N_r)$ transformation acting on the (time-reversal) $\otimes$ (retarded or advanced) $\otimes$ (replica) indices. The small imaginary part of the energy η lifts the degeneracy between the retarded and advanced sectors, thereby reducing the $\mathrm{O}(4N_r)$ symmetry down to $\mathrm{O}(2N_r) \times \mathrm{O}(2N_r)$. The Nambu-Goldstone modes associated with this symmetry breaking are described by a bosonic matrix field $Q(\boldsymbol{r})$ living in the coset space $G/H \equiv \mathrm{O}(4N_r)/\mathrm{O}(2N_r) \times \mathrm{O}(2N_r)$. They emerge after performing the Hubbard-Stratonovich decoupling of the interaction by a $\mathrm{O}(4N_r)$ matrix field and then freezing its massive modes (i.e., all modes other than the Nambu-Goldstone modes). Their interactions are governed by the effective local field theory

$$\begin{aligned} \overline{Z}_{\mathrm{eff}} &= \int \mathcal{D}[\bar{\psi}, \psi]\mathcal{D}[Q] e^{-\int d^2 r [\mathcal{L}_f + (\Delta^2/4g)\mathrm{Tr}(QQ^T)]}, \\ \mathcal{L}_f &:= \bar{\psi} D[Q]\psi = \psi^T \tilde{D}[Q]\psi. \end{aligned} \tag{3a}$$

Here, we have introduced the Majorana kernel

$$\begin{aligned} (\tilde{D}[Q])_{ab} &:= i\sigma_y(\boldsymbol{\sigma}\cdot\boldsymbol{p}\delta_{ab} - i\Delta Q_{ab} + i\eta\Lambda_{ab}) \\ &=: i\sigma_y (D[Q])_{ab}, \qquad a, b = 1, \ldots, 4N_r. \end{aligned} \tag{3b}$$

It is skew symmetric, $(i\sigma_y D[Q])^T = -i\sigma_y D[Q]$, in the background of $Q \in G/H$, i.e.,

$$Q^2 = \mathbb{1}_{4N_r}, \qquad Q = Q^T, \qquad \mathrm{Tr}Q = 0. \tag{3c}$$

The real number Δ, which represents the imaginary part of the self-energy caused by disorder, is determined by solving the self-consistent equation $\pi/g = \ln[1 + (a_0\Delta)^{-2}]$, where a_0 is the short-distance cutoff.

For smooth spatial variations of the Q field starting from Λ, the spectrum of $\tilde{D}[Q]$ has a gap of order Δ at the band center. We can then ignore η and integrate the Majorana fermions out. An effective action for the Nambu-Goldstone modes follows. This integration, when properly regularized, *defines* the Pfaffian $\int \mathcal{D}[\bar{\psi}, \psi]\exp(-\int d^2 r \mathcal{L}_f) \equiv \mathrm{Pf}(\tilde{D}[Q])$. A possible regularization that preserves TRS starts with wrapping momentum space around the two-torus T^2. Momentum is then quantized, $k_\mu = 2\pi n_\mu/L$, where $\mu = 1, 2$, with the level spacing $2\pi/L$ controlled by the long-distance cutoff L. Momentum is made countably finite, $n_\mu = -(N-1)/2, -(N-1)/2 + 1, \ldots, (N-1)/2$, where $\mu = 1, 2$ with the help of the short-distance cutoff a_0. The ratio $N = L/a_0$ of the ultraviolet to infrared cutoff is chosen as an odd integer for convenience. To complete the definition of the regularized Pfaffian we note that its square is the determinant $\det\tilde{D}[Q]$. This determinant can be rewritten

$$\det\tilde{D}[Q] = \det(i\sigma_y D'[Q]) = \det D'[Q] \tag{4a}$$

with the Dirac kernel ($a, b = 1, \ldots, 4N_r$)

$$(D'[Q])_{ab} = \boldsymbol{\sigma}\cdot\boldsymbol{p}\delta_{ab} + \sigma_z\Delta Q_{ab}. \tag{4b}$$

The real-valued spectrum of $D'[Q]$ comes in pairs of nonvanishing eigenvalues $\{-\lambda'_i, +\lambda'_i\}$ labeled by the index i running over some countably finite set. For some reference configuration Q, we *define*

$$\mathrm{Pf}\,\tilde{D}[Q] := \prod_i \tilde{\lambda}_i, \tag{5}$$

where $\tilde{\lambda}_i \in \mathbb{R}$ represents either one of $-\lambda'_i$ or $+\lambda'_i$ for each i. It follows that $\arg \mathrm{Pf}\tilde{D}[Q] \in \{0, \pi\}$. The sign of the Pfaffian is protected by the spectral gap $\propto \Delta$ under any infinitesimal change of Q. This protection does not extend to all configurations. Indeed, the second homotopy group of the target manifold is nontrivial,

$$\pi_2(G/H) = \begin{cases} \mathbb{Z}_2, & \text{for all } N_r > 1, \\ \mathbb{Z}\times\mathbb{Z}, & \text{for } N_r = 1, \end{cases} \tag{6}$$

and, as we are going to show explicitly, there is a phase difference of π between the two Pfaffians (5) when evaluated at two Q's belonging to distinct $\mathbb{Z}_2$ topological sectors. This ambiguity in fixing the sign of the Pfaffian over all configurations, starting from some reference one, is reminiscent of the SU(2) global anomaly in four spacetime dimensions that follows from $\pi_4[\mathrm{SU}(2)] = \mathbb{Z}_2$ [8].

We turn to the construction of two families of Q-fields that differ in the sign of the Pfaffian (5). To this end, we first compactify two-dimensional Euclidean space $\boldsymbol{r} \in \mathbb{R}^2$ by wrapping it once around the two-sphere S^2. On the two-sphere parametrized by the polar $-\pi/2 \le \theta \le +\pi/2$ and azimuthal $0 \le \phi < 2\pi$ angles, we define, following Weinberg *et al.* [9], the family of fields

$$\begin{aligned} Q_k(\theta, \phi) &= \begin{pmatrix} \mathbb{1}_{2N_r-2} & 0 & 0 \\ 0 & q_k(\theta,\phi) & 0 \\ 0 & 0 & -\mathbb{1}_{2N_r-2} \end{pmatrix}, \\ q_k(\theta, \phi) &= \begin{pmatrix} \cos\theta\mathbb{1}_2 & \sin\theta R_k(\phi) \\ \sin\theta R_k^T(\phi) & -\cos\theta\mathbb{1}_2 \end{pmatrix}, \\ R_k(\phi) &= \begin{pmatrix} \cos k\phi & \sin k\phi \\ -\sin k\phi & \cos k\phi \end{pmatrix}, \end{aligned} \tag{7}$$

labeled by the index $k \in \mathbb{Z}$. One verifies that the Chern integer (Pruisken term) $\mathrm{Ch}[Q_k] := (16\pi i)^{-1} \times \int_{S^2} d^2 r \epsilon_{\mu\nu} \mathrm{Tr}[Q_k \partial_\mu Q_k \partial_\nu Q_k]$ vanishes for any $k \in \mathbb{Z}$. This is expected as TRS holds. On the other hand, we are going to argue that

$$\mathrm{sgn}\,\mathrm{Pf}(\tilde{D}[Q_k]) = -\mathrm{sgn}\,\mathrm{Pf}(\tilde{D}[Q_{k+1}]), \qquad k \in \mathbb{Z}. \quad (8)$$

Any element $Q \in G/H$ is also an element of the larger coset space $\mathrm{U}(4N_r)/\mathrm{U}(2N_r) \times \mathrm{U}(2N_r)$. From this point of view, the Chern integer is the signature of the second homotopy group $\pi_2[\mathrm{U}(4N_r)/\mathrm{U}(2N_r) \times \mathrm{U}(2N_r)] = \mathbb{Z}$. As the Chern integer is also the phase of the fermion determinant of the Dirac kernel $D'[U]$ (through the chiral anomaly) in the background of $U \in \mathrm{U}(4N_r)/\mathrm{U}(2N_r) \times \mathrm{U}(2N_r)$, the fact that the Chern integer vanishes is a consequence of the positivity of the determinant (4a). While the Chern integer is blind to the second homotopy group (6), the Pfaffian is sensitive to it.

We give a "numerical proof" of Eq. (8) in Fig. 1 by showing 4 evolutions of eigenvalues of the Dirac kernel (4b) evaluated at $Q_t := (1-t)Q_i + tQ_f$ as a function of $0 \le t \le 1$. Here, the initial, Q_i, and final, Q_f, field configurations belong to G/H [8]. Although Q_t is not a member of G/H for $0 < t < 1$, it remains real-valued, symmetric, and traceless. Consequently, the spectrum of $D'[Q_t]$ is symmetric about the band center. Configurations Q_i and Q_f have Pfaffians of opposite signs whenever an odd number of level crossing occurs at the band center

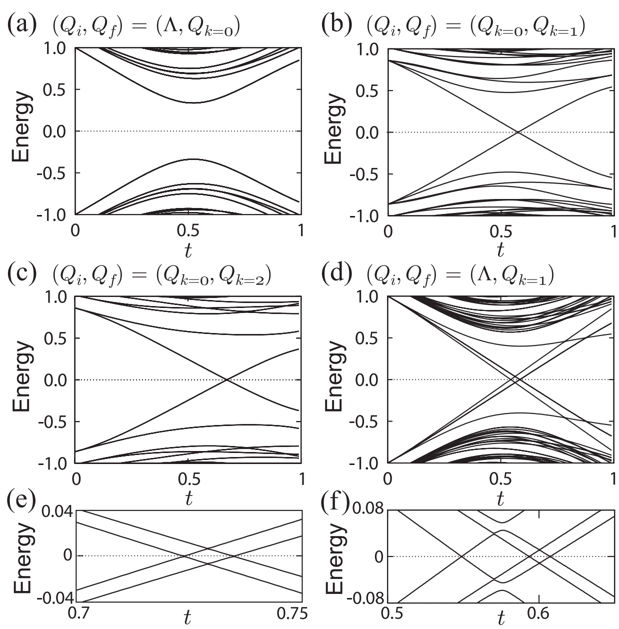

FIG. 1. The energy eigenvalue spectrum in the vicinity of the band center for the Dirac kernel $D'[Q_t]$, Eq. (4b), is computed numerically as a function of the parameter $0 \le t \le 1$ for $(a_0\Delta)^{-1} = 1$ and $N = 11$. The field Q_t interpolates between Q_i when $t = 0$ and Q_f when $t = 1$. (e) and (f): Same as in (c) and (d), respectively, except for the addition of a small perturbation so as to lift any accidental or quasi degeneracies.

("spectral flow") during the t evolution of the Dirac kernel $D'[Q_t]$. This is accompanied by the closing of the spectral gap of $D'[Q_t]$ by an odd number of pairs $(-\lambda'_i, +\lambda'_i)$ as t interpolates between 0 and 1. The spectral t evolution is obtained numerically using the regularization of the Dirac kernel $D'[Q_t]$ in momentum space as described above Eq. (4) [10]. We show with Fig. 1(a) that Λ (the uniform configuration) and $Q_{k=0}$ belong to the same $\mathbb{Z}_2$ topological sector as the spectral gap never closes under the evolution of the spectrum: $\mathrm{Pf}(\tilde{D}[\Lambda])$ and $\mathrm{Pf}(\tilde{D}[Q_{k=0}])$ share the same sign. We show with Fig. 1(b) that $Q_{k=0}$ and $Q_{k=1}$ belong to different $\mathbb{Z}_2$ topological sectors as level crossing at the band center takes place for a single pair of levels: $\mathrm{Pf}(\tilde{D}[Q_{k=0}])$ and $\mathrm{Pf}(\tilde{D}[Q_{k=1}])$ differ by their sign. We show with Figs. 1(c) and 1(e) that $Q_{k=0}$ and $Q_{k=2}$ belong to the same $\mathbb{Z}_2$ topological sector as level crossing at the band center takes place for 2 pairs of levels [11]: $\mathrm{Pf}(\tilde{D}[Q_{k=0}])$ and $\mathrm{Pf}(\tilde{D}[Q_{k=2}])$ share the same sign. Finally, we show with Figs. 1(d) and 1(f) that Λ and $Q_{k=1}$ belong to different topological sectors as level crossing at the band center takes place for 3 pairs of levels [11]: $\mathrm{Pf}(\tilde{D}[\Lambda])$ and $\mathrm{Pf}(\tilde{D}[Q_{k=1}])$ have opposite signs.

From now on, we shall assign the topological quantum number 0 (1) to all configurations belonging to the same homotopy class as Q_k with k even (odd). If so, the effective action (3a) can be approximated by [12]

$$Z_{\mathrm{NL}\sigma\mathrm{M}}^{\mathrm{topolo}} = \int \mathcal{D}[Q](-1)^{n[Q]} e^{-S[Q]}, \qquad (9)$$

where $S[Q]$ is the usual local action for the NLσM on G/H and $n[Q] = 0, 1$ is the $\mathbb{Z}_2$ topological quantum number of Q. The topological term has its origin in the Pfaffian arising from Majorana fermions, i.e., the global anomaly. When the bare NLσM coupling constant is small (when the bare conductivity is large), the effect of $n[Q] = 0, 1$ on the renormalization group flow is small. Weak antilocalization, as it occurs in the absence of the topological term, is then expected. The effect of $n[Q] = 0, 1$ is more important in the strong coupling regime (when the bare conductivity is small) [6]. Numerical simulations [13,14] of the random Dirac Hamiltonian (1) suggest that the topological term makes the two-dimensional symplectic stable metallic fixed point the only fixed point.

As is well known from lattice gauge theory [15], the fermion doubling problem forbids the emergence of an odd number of massless spinors from two-dimensional tight-binding models with conserved electric charge, TRS, locality, and the translation symmetry of a regular lattice. That is, the NLσM (9) cannot emerge from microscopic electronic models on the square or honeycomb lattices [16]. However, as pointed out by Ando and Suzuura [17], graphene approximately realizes the symplectic symmetry if the potential range of static point impurities is much larger than the lattice spacing.

One way out of the fermion doubling problem is to consider d-dimensional space as the boundary of $(d+1)$-dimensional space [18,19] and to study the physics of localization at this boundary. In this context, the effect of disorder on the two-dimensional boundary states of three-dimensional $\mathbb{Z}_2$ topological insulators [20–22] could yield microscopic realizations of the two-dimensional symplectic class with a $\mathbb{Z}_2$ topological term. The situation is here similar to that in a quasi-one-dimensional tight-binding model belonging to the symplectic symmetry class. When the number of Kramers doublets propagating in the wire is even, all states are exponentially localized [23]. When it is odd, one Kramers doublet remains extended [24,25]. The former case is always realized in a quasi-one-dimensional tight-binding model [23], whereas the latter case can be realized at the edges of two-dimensional $\mathbb{Z}_2$ topological insulators [26].

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[1] A. M. M. Pruisken, Nucl. Phys. **B235**, 277 (1984).
[2] P. Fendley, Phys. Rev. B **63**, 104429 (2001).
[3] P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, Phys. Rev. Lett. **98**, 256801 (2007).
[4] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, Y. Zhang, S. V. Dubonos, I. V. Grigorieva, and A. A. Firsov, Science **306**, 666 (2004).
[5] A. W. W. Ludwig, M. P. A. Fisher, R. Shankar, and G. Grinstein, Phys. Rev. B **50**, 7526 (1994).
[6] It was also shown by T. Ando and T. Nakanishi, J. Phys. Soc. Jpn. **67**, 1704 (1998), and by T. Ando, T. Nakanishi, and R. Saito, J. Phys. Soc. Jpn. **67**, 2857 (1998) that the Berry phase, π, that is accumulated around the Dirac cone in the momentum space leads to the absence of back scattering from the random scalar potential. We believe that this effect is related to the topological term (9).
[7] K. B. Efetov, A. I. Larkin, and D. E. Khemlnitskii, Sov. Phys. JETP **52**, 568 (1980).
[8] E. Witten, Phys. Lett. B **117**, 324 (1982).
[9] E. J. Weinberg, D. London, and J. L. Rosner, Nucl. Phys. **B236**, 90 (1984).
[10] Without loss of generality, we choose $N_r = 1$ in which case Q_k in Eq. (9) reduces to the 4×4 matrix q_k. We choose N sufficiently large to avoid accidental level crossing. Although, $a_0\Delta$ is to be determined self-consistently, we treat Δ as a free parameter. The topologically nontrivial Q-field configurations on the torus can be constructed in the same way as on the sphere. We still call these configurations Q_k with $k \in \mathbb{Z}$. One should also bear in mind that numerics cannot rigorously distinguish level crossing from avoided crossing.
[11] These levels can be accidentally or quasi degenerate for the choice (7) [for example, $Q_{k=0,2}$ have an accidental symmetry, $Q_{k=0,2}(\mathbf{r}) = Q_{k=0,2}(-\mathbf{r})$ on the torus]. Adding a small perturbation to $Q_{i,f}$, as is done in Figs. 1(e) and 1(f), lifts this (quasi) degeneracy.
[12] Observe first that the 4×4 block-matrix q_k in Eq. (9) is independent of N_r. The global anomaly is thus expected to survive the replica limit $N_r \to 0$. Second, the derivation of the NLσM that requires a diffusive regime can be achieved with the help of N_f flavors for the spinors. If so, the global anomaly is only possible when N_f is odd. Third, the Dirac kinetic energy in Eq. (1) yields nothing but the $\widehat{\mathrm{so}}(4N_r)_1$ Wess-Zumino-Witten (WZW) critical point. After disorder averaging, the disorder strength g drives the system to the G/H NLσM.
[13] J. H. Bardarson, J. Tworzydlo, P. W. Brouwer, and C. W. J. Beenakker, arXiv:0705.0886 [Phys. Rev. Lett. (to be published)].
[14] K. Nomura, M. Koshino, and S. Ryu, arXiv:0705.1607 [Phys. Rev. Lett. (to be published)]..
[15] H. B. Nielsen and M. Ninomiya, Nucl. Phys. **B193**, 173 (1981).
[16] H. Obuse, A. Furusaki, S. Ryu, and C. Mudry, Phys. Rev. B **76**, 075301 (2007); A. M. Essin and J. E. Moore, arXiv:0705.0172v1.
[17] T. Ando and H. Suzuura, J. Phys. Soc. Jpn. **71**, 2753 (2002); H. Suzuura and T. Ando, Phys. Rev. Lett. **89**, 266603 (2002).
[18] C. G. Callan and J. A. Harvey, Nucl. Phys. **B250**, 427 (1985).
[19] E. Fradkin, E. Dagotto, and D. Boyanovsky, Phys. Rev. Lett. **57**, 2967 (1986).
[20] L. Fu, C. L. Kane, and E. J. Mele, Phys. Rev. Lett. **98**, 106803 (2007); L. Fu and C. L. Kane, Phys. Rev. B **76**, 045302 (2007).
[21] J. E. Moore and L. Balents, Phys. Rev. B **75**, 121306(R) (2007).
[22] R. Roy, arXiv:cond-mat/0607531.
[23] P. W. Brouwer and K. Frahm, Phys. Rev. B **53**, 1490 (1996).
[24] Y. Takane, J. Phys. Soc. Jpn. **73**, 9 (2004); **73**, 1430 (2004); **73**, 2366 (2004).
[25] Delocalization in the quasi-one-dimensional symplectic class was discovered by M. R. Zirnbauer, Phys. Rev. Lett. **69**, 1584 (1992); A. D. Mirlin, A. Müller-Groeling, and M. R. Zirnbauer, Ann. Phys. (N.Y.) **236**, 325 (1994), although the distinction between even and odd number of channels was not appreciated then.
[26] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 146802 (2005).