# Zero-Phase Odd-Harmonic Repetitive Controller for a Single-Phase PWM Inverter

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***Abstract*—In this paper, a zero-phase odd-harmonic repetitive control scheme is proposed for pulse-width modulation inverters. The proposed repetitive controller combines an odd-harmonic periodic generator with a noncasual zero-phase compensation filter. It occupies less data memory than a conventional repetitive controller does. Moreover, it offers faster convergence of the tracking error, and yields very low total harmonics distortion (THD) and low tracking error. Analysis and design of the proposed system are presented. Experimental results with the proposed repetitive controller are presented to validate the approach. The phenomena of even-harmonic residues in the proposed control system is discussed and experimentally demonstrated.**

***Index Terms*—Even harmonic, odd harmonic, pulse-width modulation (PWM) inverter, repetitive control, zero phase compensation.**

## I. Introduction

HIGH performance constant-voltage-constant-frequency (CVCF) pulse-width modulated (PWM) inverters should accurately regulate the output ac voltage/current to the reference sinusoidal input with low total harmonics distortion (THD) and fast dynamic response. Nonlinear loads such as the rectifier loads that cause periodic distortion are major sources of THD. According to the internal model principle [1], repetitive control (RC) [2] provides an effective solution for tracking periodic reference signals, or eliminating periodic disturbances. In [3]–[6], RC found its promising usage in CVCF PWM converters for waveform compensation. In a conventional RC system, any reference signal with a fundamental period $N$ can be exactly tracked by including a periodic signal generator $1/(z^N - 1)$ in the closed-loop system. Such a periodic signal generator needs at least $N$ memory cells. A usual repetitive controller introduces infinite gain at both even and odd harmonic frequencies. However, ac references and disturbances contain dominant odd-harmonic frequencies and minor even-harmonic frequencies in the CVCF PWM inverters. In order to improve the convergence performance of RC systems, a new odd-harmonic periodic signal generator [6] is proposed, which only needs $N/2$ memory cells and updates one memory cell each sampling interval sequentially. In such cases, even-harmonic errors can not be reduced. On the other hand, although RC mathematically assures that the tracking errors asymptotically converge to zero when repetition goes to infinity, poor tracking accuracy and transient may be caused by poor design of the filter of repetitive controller in practical applications.

In this paper, a discrete-time zero-phase odd-harmonic RC scheme is proposed for a CVCF PWM inverter to achieve high tracking accuracy (low THD and low tracking error) and good dynamic response (fast monotonic convergence) in the presence of linear and nonlinear loads, and parameter uncertainties. The analysis and design of the proposed controller are discussed. Experimental results are presented to validate the proposed approach. Moreover, the drawback of the proposed controller is discussed.

## II. Zero-Phase Odd-Harmonic Repetitive Control

### A. Odd-Harmonic Periodic Signal Generator

In the discrete time domain, a conventional periodic signal generator can be written as

$$G_r(z) = \frac{z^{-N}}{1 - z^{-N}} = \frac{1}{z^N - 1} \tag{1}$$

where $N = T_s/T$, $T_s$ and $T$ being the signal period and the sampling time, respectively. The generator in (1) can eliminate the harmonics that are below the Nyquist frequency $\omega(= \pi/T)$ by introducing infinite gain at both even and odd harmonic frequencies [2].

However, for systems such as the CVCF PWM converters, the references and disturbances mainly contain odd-harmonic frequencies. Therefore a new odd-harmonic periodic signal generator is proposed [6]. A discrete time odd-harmonic periodic signal generator has the following transfer function:

$$G_r(z) = -\frac{1}{z^{\frac{N}{2}} + 1} \tag{2}$$

where the period $N$ of the signal is even. The generator in (2) has its poles at

$$z = e^{j(2k+1)\frac{2\pi}{NT}} \quad k = 0, 1, \ldots, \frac{N}{2} - 1. \tag{3}$$

Equation (3) means that (2) has infinite gain at frequency $\omega_k = (2k+1)2\pi/(NT)$ of all odd harmonics. Furthermore, if the odd-harmonic signal generator (2) is incorporated in a system, it will achieve perfect asymptotic tracking or disturbance rejection for this class of periodic signals. As compared to the traditional periodic signal generator in (1), all even harmonic poles $\omega_k = 4k\pi/(NT)$ are removed.

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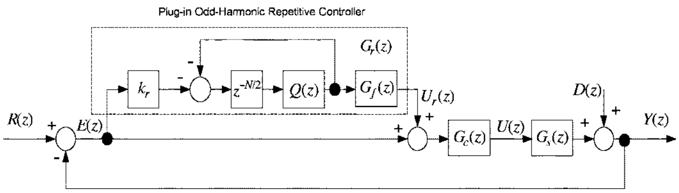



Fig. 1. Plug-in discrete-time odd-harmonic RC system.

### B. Odd-Harmonic Repetitive Control

Fig. 1 shows the proposed discrete-time odd-harmonic repetitive control system under consideration, where $R(z)$ is the reference input, $Y(z)$ is the output, $E(z) = R(z) - Y(z)$ is the tracking error, $D(z)$ is the disturbance, $G_c(z)$ is the conventional controller, $G_s(z)$ is the plant, $G_r(z)$ is a feedforward plug-in odd-harmonic repetitive controller, $k_r$ is the repetitive control gain, $U_r(z)$ is the output of the repetitive controller, $G_f(z)$ is a filter to obtain a stable overall closed-loop system, and $Q(z)$ is a low pass filter to enhance the robustness of the overall system.

As shown in Fig. 1, the proposed plug-in odd-harmonic repetitive control law can be expressed as

$$U_r(z) = -Q(z)\left(z^{-\frac{N}{2}}U_r(z) + k_r z^{-\frac{N}{2}}G_f(z)E(z)\right). \quad (4)$$

From Fig. 1, the transfer functions from $R(z)$ and $D(z)$ to $Y(z)$ in the overall closed-loop control system can be derived as

$$\frac{Y(z)}{R(z)} = \frac{(1+G_r(z))\,G_c(z)G_s(z)}{1+(1+G_r(z))\,G_c(z)G_s(z)} = \frac{\left(1+z^{-\frac{N}{2}}Q(z)\,(1-k_rG_f(z))\right)H(z)}{1+z^{-\frac{N}{2}}Q(z)\,(1-k_rG_f(z)H(z))} \quad (5)$$

$$\frac{Y(z)}{D(z)} = \frac{1+z^{-\frac{N}{2}}Q(z)}{1+G_c(z)G_s(z)} \times \frac{1}{1+z^{-\frac{N}{2}}Q(z)\,(1-k_rG_f(z)H(z))} \quad (6)$$

where $H(z) = G_c(z)G_s(z)/(1+G_c(z)G_s(z))$.

From (5) and (6), the overall closed-loop system is stable if the following conditions hold:

1) the roots of $1 + G_c(z)G_s(z) = 0$ are located inside the unit circle;

2)

$$\|Q(z)\,(1-k_rG_f(z)H(z))\| < 1 \quad \forall z = e^{j\omega}, 0 < \omega < \frac{\pi}{T}. \quad (7)$$

It should be pointed out that the above stability conditions for an odd-harmonic RC system are the same as those for conventional RCs [2]. It means that, if $Q(z)$, $G_f(z)$ and $H(z)$ are identical in both an odd-harmonic RC system and a conventional RC system, the stability range of RC gain $k_r$ are identical too. An odd-harmonic RC regulates all memory cells once every $N/2$ sampling intervals; while a conventional RC renews all cells in its memory once every $N$ sampling intervals. Therefore, if $Q(z)$, $G_f(z)$, $H(z)$ and $k_r$ are identical for both two RC systems, the tracking error convergence rate of an odd-harmonic RC is about two times as fast as that of a conventional RC.

The error transfer function of the overall system is

$$G_e(z) = \frac{E(z)}{R(z)-D(z)} = \frac{1+z^{-\frac{N}{2}}Q(z)}{1+G_c(z)G_s(z)} \times \frac{1}{1+z^{-\frac{N}{2}}Q(z)\,(1-k_rG_f(z)H(z))}. \quad (8)$$

Thus, if the overall closed-loop system is asymptotically stable, $Q(z) = 1$, and the angular frequency $\omega$ of the reference input $R(t)$ and disturbance $D(t)$ approaches to $\omega_m = (2m+1)2\pi/(NT), m = 0, 1, 2, \ldots, N/2-1$, then $z^{-N/2} \to 1$, thus

$$\lim_{\omega \to \omega_m} \|e(j\omega)\| = 0. \quad (9)$$

According to (9), if the frequencies of odd-harmonic references and/or disturbances are less than half of the sampling frequency (Nyquist frequency), steady-state zero tracking error can be ensured by using the odd-harmonic RC controller $G_r(z)$. Theoretically, for CVCF PWM inverters, the odd-harmonic repetitive controller (4) with $Q(z) = 1$ is a zero tracking error control law, if there are no even-harmonics disturbances. If the low pass filter $Q(z)$ is introduced to improve the stability, high frequency periodic disturbances will not perfectly cancelled. The tracking accuracy will be reduced. Hence, there is a trade-off between tracking accuracy and system robustness [5].

### C. Zero-Phase Compensation

From (4) and (7), we observe that the performance of the RC controller $G_r(z)$ is determined by the design of $G_f(z)$, $k_r$ and $Q(z)$. Within its stability range determined by (7), larger $k_r$ leads to smaller damping ratio with faster transients. In many cases, it is very difficult for all frequencies up to Nyquist frequency to satisfy the inequality (7) if $Q(z) = 1$. To enhance the robustness, a low pass FIR filter $Q(z)$ [5] with $\|Q(z)\| \leq 1$ and zero phase shift, e.g., $Q(z) = \alpha_1 z + \alpha_0 + \alpha_1 z^{-1}$ 1 with $2\alpha_1 + \alpha_0 =$ and $\alpha_0 > 0$, $\alpha_1 \geq 0$, can be introduced to cut out the frequencies which are not able to satisfy (7), and relax the stability range for $k_r$. However, some high frequency periodic disturbances cannot be eliminated exactly due to $Q(z)$. Hence, $Q(z)$ will bring a trade-off between tracking accuracy and the system robustness. A well-designed $G_f(z)$ could make all frequencies up to Nyquist frequency to satisfy (7) with $Q(z) = 1$, then yields high tracking accuracy with good transients.

Suppose $H(z)$ has frequency characteristics $H(j\omega) = N_h(\omega)\exp(j\theta_h(\omega))$ with $N_h(\omega)$ and $\theta_h(\omega)$ being its magnitude and phase; and $G_f(z)$ has frequency characteristics $G_f(j\omega) = N_f(\omega)\exp(j\theta_f(\omega))$ with $N_f(\omega)$ and $\theta_f(\omega)$ being its magnitude and phase. Using these characteristics, (7) leads to [7]

$$0 < k_r < \frac{2\cos(\theta_h + \theta_f)}{N_h(\omega)N_f(\omega)}. \quad (10)$$

Since $k_r$, $N_h(\omega)$ and $N_f(\omega)$ are positive, (10) necessarily yields

$$-90^\circ < \theta_h(\omega) + \theta_f(\omega) < 90^\circ. \quad (11)$$

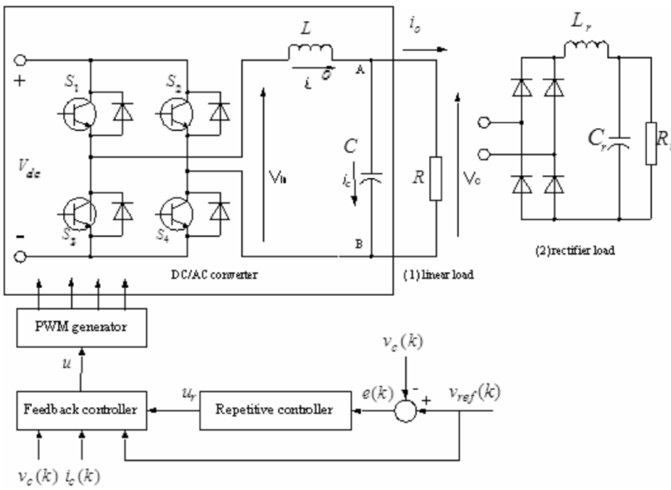



Fig. 2. RC controlled single-phase PWM inverter.

If $\theta_h(\omega) = -\theta_f(\omega)$ (i.e., "zero phase"), for all frequencies up to Nyquist frequency, (7) will be satisfied with $k_r \in (0, 2/N_h(\omega)N_f(\omega))$. Since a CVCF PWM inverter is a minimal phase system with phase lag $\theta_h(\omega) > 0$, a phase lead compensation filter $G_f(z)$ with $\theta_f(\omega) < 0$ is needed. To obtain exact phase lead compensation, the poles-zeros cancellation [8] is the most direct approach, i.e., $G_f(z) = 1/H(z)$. (10) will lead to

$$0 < k_r < 2. \tag{12}$$

In practice, since $H(z)$ is only approximately known, our design effort is to cancel the phase lag as close as possible. Since the periodic signal generator in the RC controller brings a delay $z^{-N/2}$ or $z^{-N}$, the noncasual phase lead filter $G_f(z)$ is realizable. Due to uncertainties and disturbances, $Q(z)$ will still be needed in the repetitive controllers in practice.

## III. Zero-Phase Odd-Harmonic Repetitive Controlled PWM Inverter

### A. Modeling of the System

Fig. 2 shows the setup of the odd-harmonic repetitive controlled inverter. The dynamics of the inverter can be described as

$$\begin{pmatrix} \dot{v}_c \\ \ddot{v}_c \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{L_n C_n} & -\frac{1}{C_n R_n} \end{pmatrix} \begin{pmatrix} v_c \\ \dot{v}_c \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{L_n C_n} \end{pmatrix} v_{\text{in}}$$
$$v_{\text{in}} = \begin{cases} -v_{dc}, & \text{if } S_2 \ \& \ S_3 \text{ are on; } S_1 \ \& \ S_4 \text{ are off} \\ +v_{dc} & \text{if } S_1 \ \& \ S_4 \text{ are on; } S_2 \ \& \ S_3 \text{ are off} \end{cases} \tag{13}$$

where $v_c$ is the output voltage; $i_o$ is the output current; $v_{dc}$ is the dc bus voltage; the control input $v_{\text{in}}$ is a PWM pulse; $L_n$, $C_n$, and $R_n$ are the nominal component values of the inductor, the capacitor and the load, respectively.

For a linear system $\dot{x} = Ax + Bu$, its sampled-data equation can be expressed as $x(k+1) = e^{AT}x(k) + \int e^{A(T-\tau)}Bu(\tau)d\tau$. Therefore, the sampled-data form for (13) can be approximately expressed as

$$\begin{pmatrix} v_c(k+1) \\ \dot{v}_c(k+1) \end{pmatrix} = \begin{pmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{pmatrix} \begin{pmatrix} v_c(k) \\ \dot{v}_c(k) \end{pmatrix} + \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} u(k) \tag{14}$$

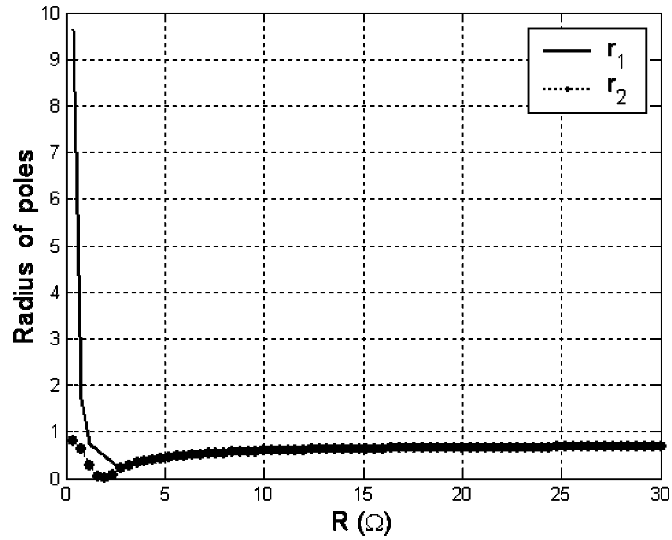



Fig. 3. Radius of poles of $H(z)$.

where the coefficients $\varphi_{11} = 1 - (T^2/2L_nC_n)$, $\varphi_{21} = -(T/L_nC_n) + (T^2/2L_nC_n^2R_n)$, $\varphi_{12} = T - (T^2/2C_nR_n)$, $\varphi_{22} = 1 - (T/C_nR_n) - (T^2/2L_nC_n) + (T^2/2C_n^2R_n^2)$, $g_1 = (T^2/2L_nC_n)$, $g_2 = (T/L_nC_n) - (T^2/2L_nC_n^2R_n)$, and the average active input $u(k) = v_{\text{in}}(k) \approx (2\Delta T(k)/T - 1)v_{dcn}$ with nominal dc bus voltage $v_{dcn}$.

The output equation can be expressed as

$$y(k) = v_c(k). \tag{15}$$

### B. State Feedback Controller

A state feedback controller is chosen as

$$u = -(k_1 v_c + k_2 \dot{v}_c) + h v_{\text{cref}}. \tag{16}$$

With such a feedback controller, the state equation of the closed-loop system can be expressed as

$$\begin{pmatrix} v_c(k+1) \\ \dot{v}_c(k+1) \end{pmatrix} = \begin{pmatrix} \varphi_{11} + g_1 k_1 & \varphi_{12} + g_1 k_2 \\ \varphi_{21} + g_2 k_2 & \varphi_{22} + g_2 k_2 \end{pmatrix} \begin{pmatrix} v_c(k) \\ \dot{v}_c(k) \end{pmatrix} + \begin{pmatrix} g_1 h \\ g_2 h \end{pmatrix} v_{\text{cref}}(k). \tag{17}$$

The poles of the closed-loop system (17) can be arbitrarily assigned by adjusting feedback control gain $k_1$ and $k_2$. When all the poles are placed at the origin, a deadbeat controller is obtained. The transfer function from $y_{\text{ref}}$ to $y$ for the closed-loop system can be written as

$$H(z) = \frac{m_1 z + m_2}{z^2 + p_1 z + p_2} \tag{18}$$

where $p_1 = -\varphi_{11} - g_1 k_1 - \varphi_{22} - g_2 k_2$; $p_2 = (\varphi_{11} + g_1 k_1)(\varphi_{22} + g_2 k_2) - (\varphi_{12} + g_1 k_2)(\varphi_{21} + g_2 k_1)$; $m_1 = g_1 h$; $m_2 = (\varphi_{12} + g_1 k_2) g_2 h - (\varphi_{22} + g_2 k_2) g_1 h$.

### C. Plug-In Zero-Phase Odd-Harmonic RC Controller

In practice, there are uncertainties in model parameters, such as $\Delta v = v_{dc} - v_{dcn}$, $\Delta L = L - L_n$, $\Delta C = C - C_n$, load disturbance $\Delta R = R - R_n$, and even un-modeled uncertainties (such as computation delay, inaccurate high frequency characteristics of the plant, and so on). Hence, zero tracking error can't be achieved by state feedback controller. To overcome the

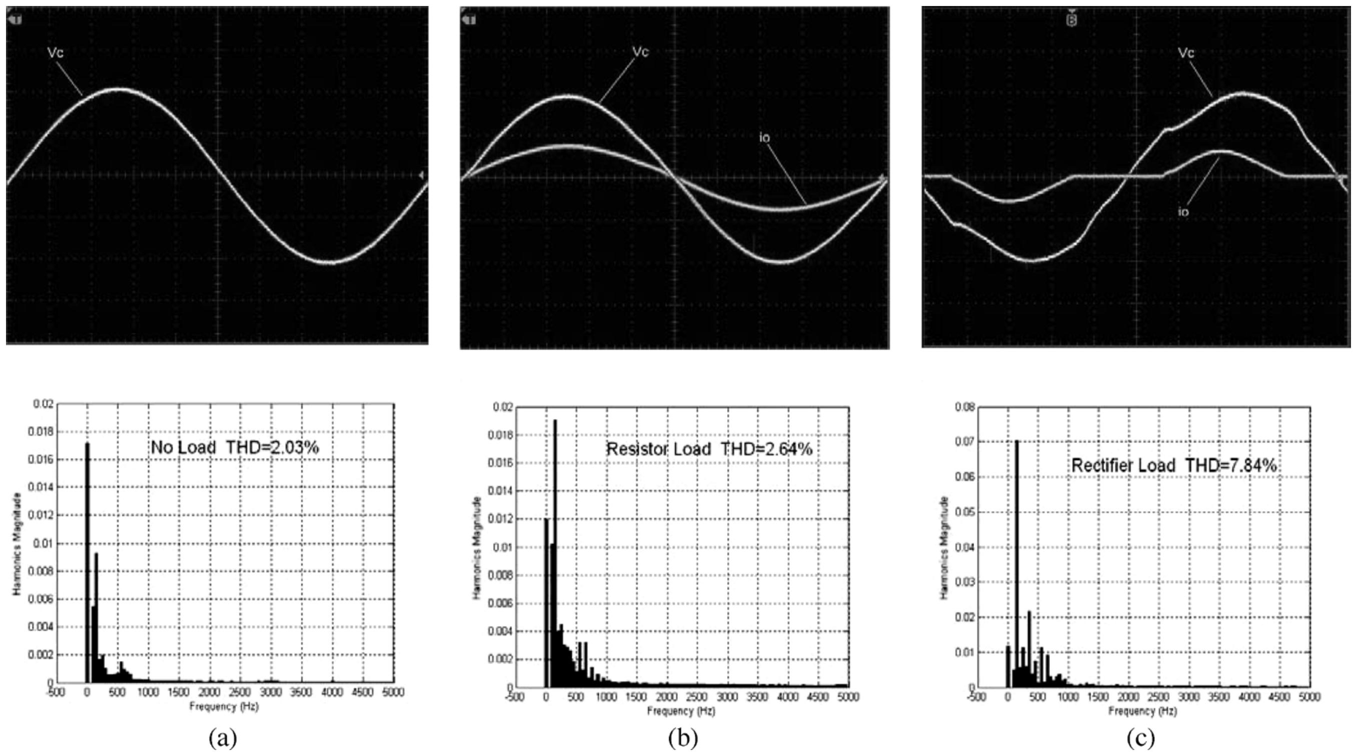



Fig. 4. State feedback controlled steady-state response under different loads: (a) voltage $V_c$ (24 V/div) and its harmonics spectrum under no load, (b) voltage $V_c$ (24 V/div), $i_o$ (2.85 A/div), and harmonics spectrum of $V_c$ under resistor load, and (c) voltage $V_c$ (24 V/div), $i_o$ (7.12 A/div), and harmonics spectrum of $V_c$ under rectifier load.

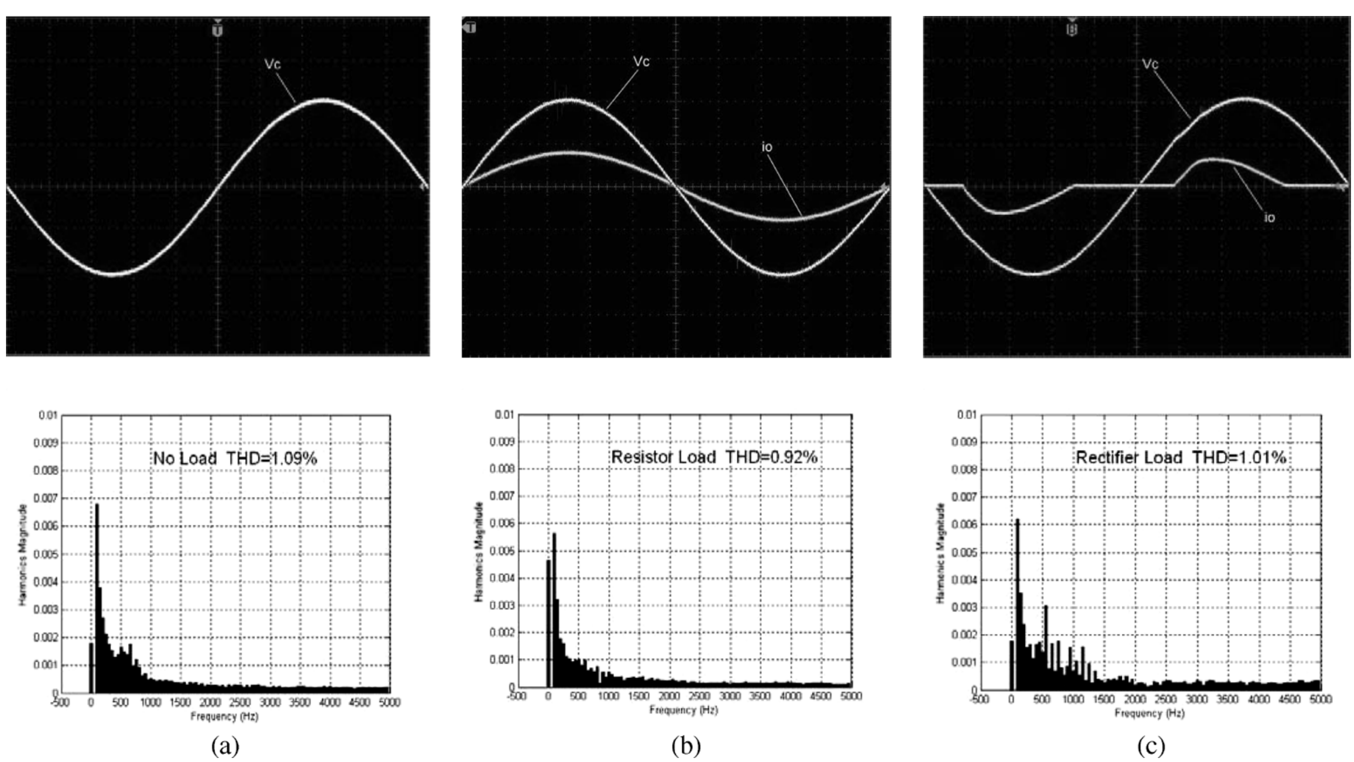
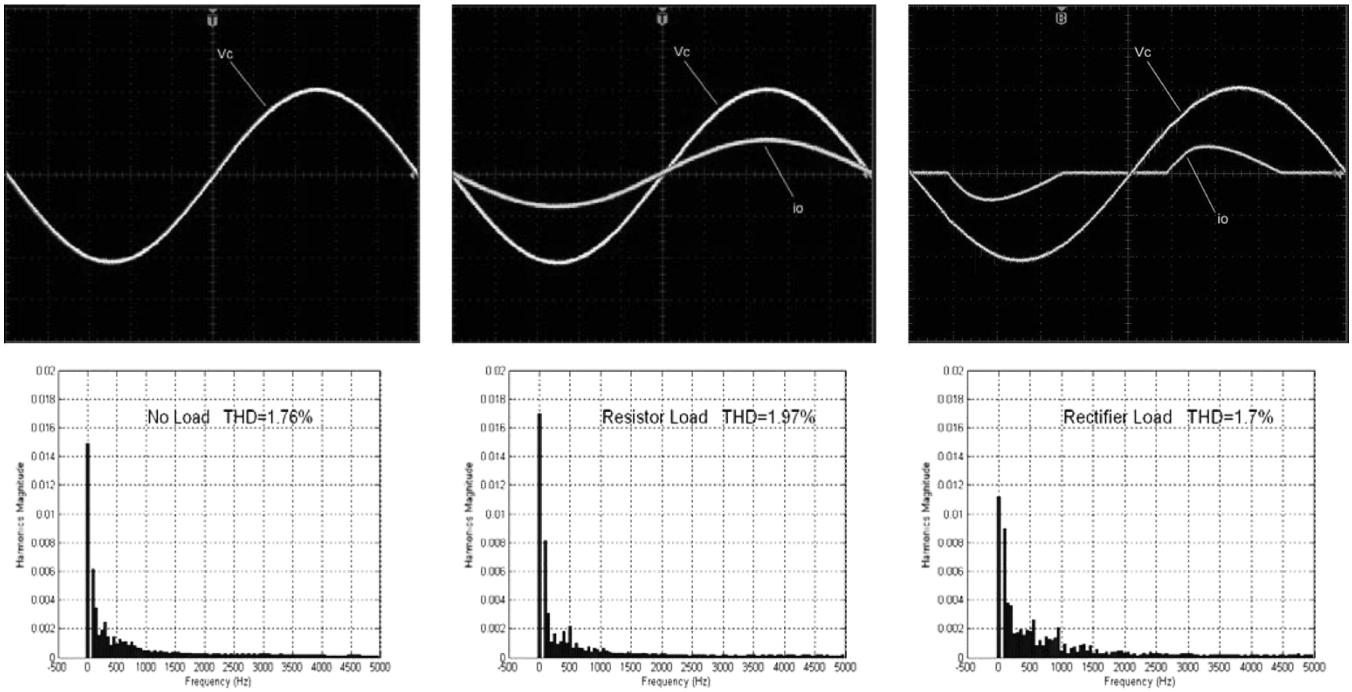


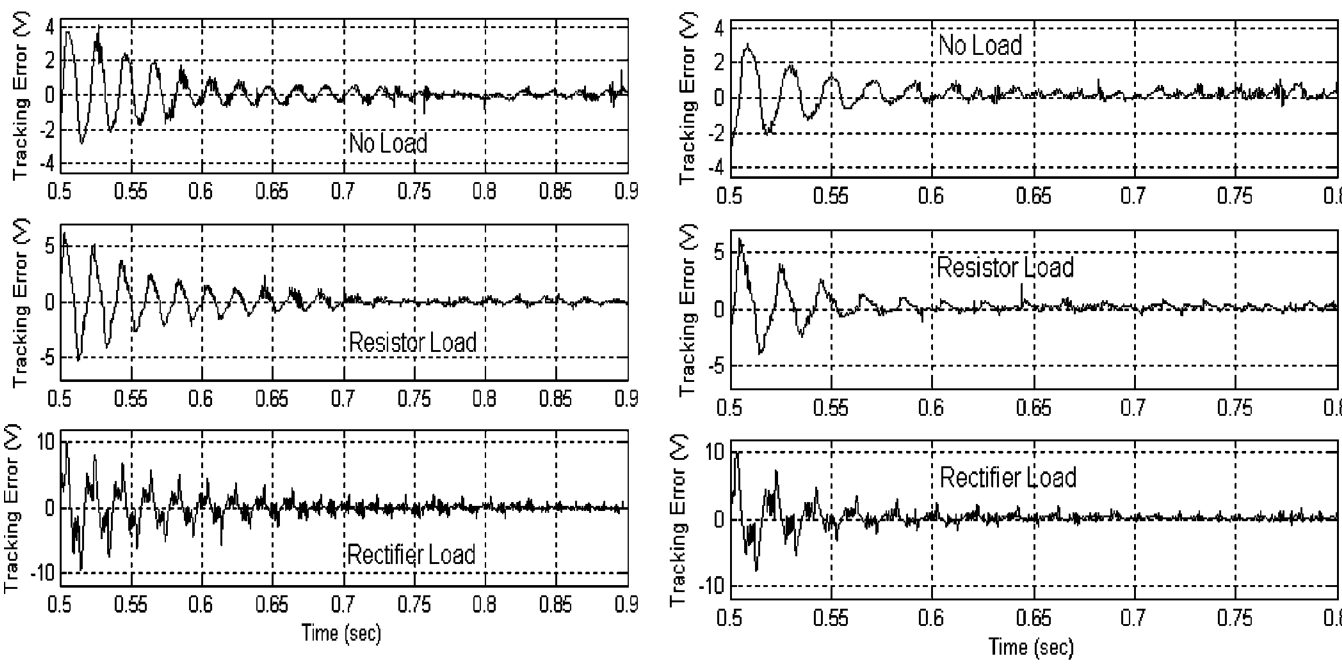

Fig. 5. Conventional RC controlled steady-state response under different loads: (a) voltage $V_c$ (24 V/div) and its harmonics spectrum under no load, (b) voltage $V_c$ (24 V/div), $i_o$ (2.85 A/div), and harmonics spectrum of $V_c$ under resistor load, and (c) voltage $V_c$ (24 V/div), $i_o$ (7.12 A/div), and harmonics spectrum of $V_c$ under rectifier load.

uncertainties and disturbances, an odd-harmonic repetitive controller $G_r(z) = -(k_r z^{-N/2} Q(z)/(1 + z^{-N/2} Q(z))) G_f(z)$ is plugged into the prior state feedback controlled inverter, where $N = f/f_s$ (even); $f_s$ is the frequency of $y_{\text{ref}}$; $f = 1/T$ is the sampling frequency; $G_f(z) = 1/H(z)$ is a phase lead filter; $Q(z) = \alpha_1 z + \alpha_0 + \alpha_1 z^{-1}$ with $2\alpha_1 + \alpha_0 = 1$ and $\alpha_1 \geq 0$,




(a) (b) (c)

Fig. 6. Odd-harmonic RC controlled steady-state response under different loads: (a) voltage $V_c$ (24 V/div) and its harmonics spectrum under no load, (b) voltage $V_c$ (24 V/div), $i_o$ (2.85 A/div), and harmonics spectrum of $V_c$ under resistor load, (c) voltage $V_c$ (24 V/div), $i_o$ (7.12 A/div), and harmonics spectrum of $V_c$ under rectifier load.




(a) (b)

Fig. 7. Tracking error histories with RC controller being plugged into state feedback control loop at time $t \approx 0.5$ s under different loads: (a) conventional RC and (b) odd-harmonic RC.

$\alpha_0 > 0$. If $\alpha_1 = 0$, $\alpha_0 = 1$, then $Q(z) = 1$. Of course, since it is impossible to get accurate $H(z)$ in practice, "zero phase" can only be roughly obtained. Furthermore, in case of compensating the un-modeled pure delays in the plant, a pure lead $z^m$ can be added into the compensation filter as $G_f(z) = z^m/H(z)$, where the value of $m$ is determined by experiments.

## IV. Experimental Verification

To validate the theoretical study, we have setup an experimental system in our laboratory. DSPACE DS1102 and Matlab/Simulink have been used in fast prototyping the experimental platform and collecting experimental data. As shown

TABLE I
TRANSIENT RESPONSE DATA

| Load Type | Conventional RC | | Odd-harmonic RC | |
|---|---|---|---|---|
| | Convergence Time (sec) | Peak Error (V) | Convergence Time (sec) | Peak Error (V) |
| No load | 0.2 ~ 0.25 | 4 → 0.5 | 0.1 ~ 0.13 | 4 → 0.5 |
| Resistor | 0.2 ~ 0.25 | 6 → 0.5 | 0.1 ~ 0.13 | 6 → 0.5 |
| Rectifier | 0.2 ~ 0.25 | 10 → 1 | 0.1 ~ 0.13 | 10 → 1 |

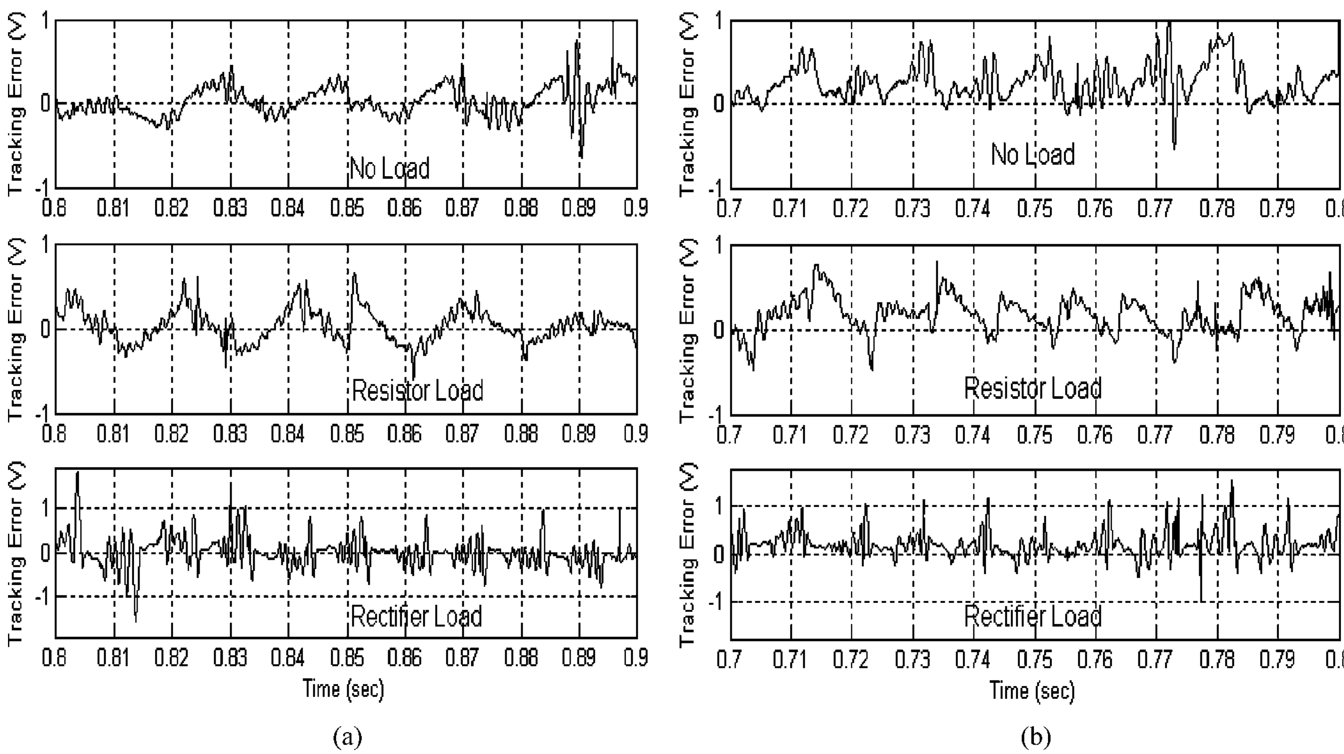



Fig. 8. RC controlled steady-state tracking error under different loads. (a) Conventional RC and (b) odd-harmonic RC.

in Fig. 2, the inverter parameters are setup as follows: $L_n = 20$ mH; $L = 30$ mH; $C_n = 45$ $\mu$F; $C = 50$ $\mu$F; $R_n = 15$ $\Omega$; $v_{dc} = 70$ V; $v_{dcn} = 80$ V; $y_{\rm ref} = v_{\rm cref}$ is $(f_s =)$ 50-Hz 50-V (peak) sinusoidal voltage; $f = 1/T = 10$ kHz; $N = f/f_s = 200$; resistive load $R = 22\,\Omega$; uncontrolled rectifier $L_r = 1$ mH, $C_r = 500\,\mu$F, $R_r = 22\,\Omega$; $k_1 = 1.8 \times 10^{-2}$, $k_2 = 1.67 \times 10^{-6}$, $h = 1.8 \times 10^{-2}$.

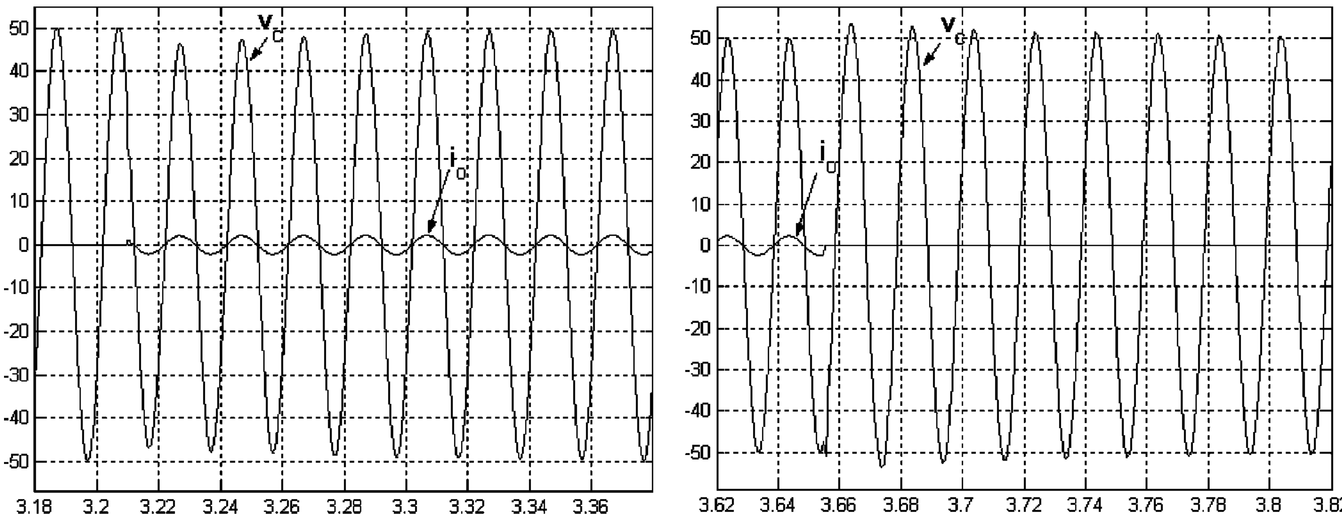
Fig. 3 shows that, if load $R > 1.1\ \Omega$ all the two poles of $H(z)$ of (18) are located inside the unit circle. Hence, for the resistance load $R \in (1.1, \infty)\Omega$, the state feedback controlled inverter $H(z)$ is stable. To compensate the phase lag of $H(z)$ and unmodeled delay in the range of $(1.1, \infty)\Omega$, the filter $G_f(z)$ is chosen as $G_f(z) = z^3/(H(z)|_{R=30\ \Omega})$. In our experiments, repetitive control gain $k_r = 0.8 \in (0,2)$; $Q(z) = (z + 2 + z^{-1})/4$.

## A. Steady-State Response

Fig. 4 show the steady-state responses of the output voltage $v_c(t)$ and the load current $i_o(t)$ of the state feedback controlled inverter with different loads. The results indicate that, state feedback control offers low THD (up to 2.64%) output voltage under linear load (resistor and no load), but yields worse THD (7.84%) output voltage under nonlinear rectifier load. Note that, the fundamental frequency components (50 Hz, with normalized unit amplitude) are removed in the output voltage harmonics spectrum.

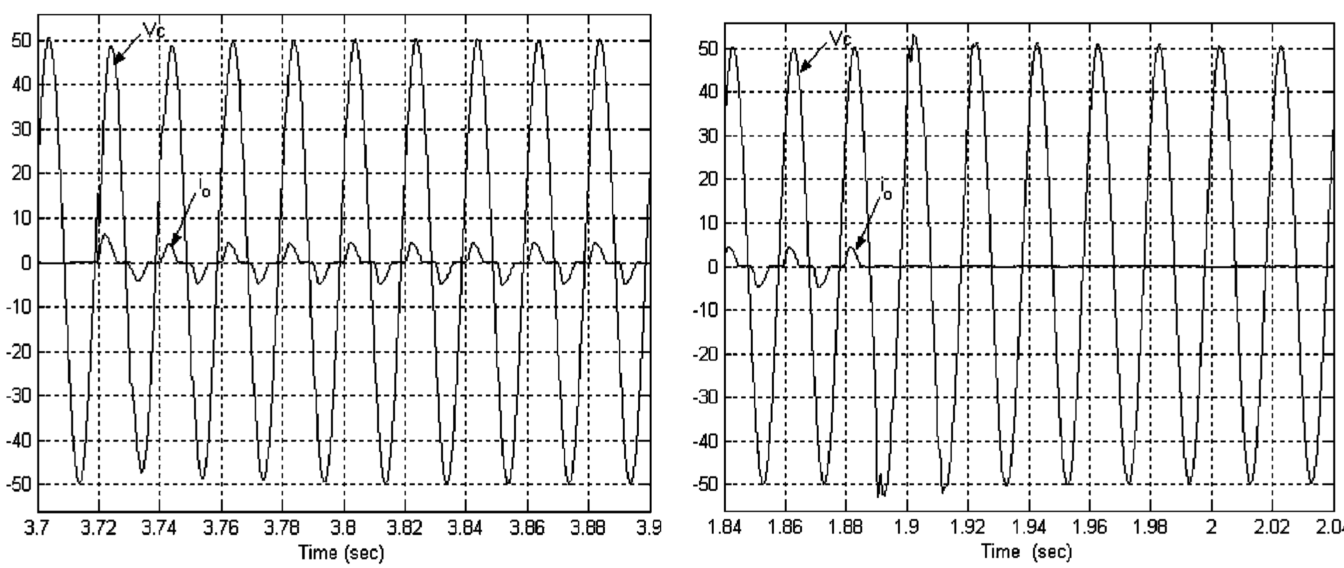
Fig. 5 show the steady-state responses of the output voltage $v_c(t)$ and the load current $i_o(t)$ of the conventional RC controlled inverter under different loads. The results indicate that conventional RC control offers very low THD ($\approx$1%) output voltage under both linear load (resistor and no load) and nonlinear rectifier load.

Fig. 6 show the steady-state responses of the output voltage $v_c(t)$ and the load current $i_o(t)$ of the odd-harmonic RC controlled inverter under different loads. The results indicate that odd-harmonic RC control also offers very low THD (<2%) output voltage under both linear load (resistor and no load) and nonlinear rectifier load. Therefore, in terms of eliminating THD, both odd-harmonic RC and conventional RC are efficient control schemes.




(a) (b)

Fig. 9. Odd-harmonic RC controlled response under resistor load change: (a) load change $R = \infty \rightarrow 22\ \Omega$ and (b) load change $R = 22 \rightarrow \infty\ \Omega$.




(a) (b)

Fig. 10. Odd-harmonic RC controlled response under rectifier load change: (a) load change $R = \infty\ \Omega \rightarrow$ rectifier load and (b) load change rectifier load $\rightarrow R = \infty\ \Omega$.

## *B. Transient Response*

Fig. 7 shows the transient tracking error $e = v_{\mathrm{cref}}(t) - v_{\mathrm{c}}(t)$ with conventional RC controller and odd-harmonic RC controller being respectively plugged into state feedback controlled inverter with different loads at $t \approx 0.5$ s. The transient response data of Fig. 7 are listed in Table I. Table I shows that, under identical conditions, the tracking error convergence rate of an odd-harmonic RC is about two times as fast as that of a conventional RC. It should be pointed out that, as shown in Fig. 7(b), there are minor dc voltage bias residues (even-harmonic component) in the odd-harmonic controlled steady-state tracking errors.

Fig. 8 shows the zoomed-in steady-state tracking errors under different loads. In Fig. 8(a), there are no obvious even-harmonic voltage residues in conventional RC controlled steady-state tracking errors; whereas, in Fig. 8(b), for odd-harmonic RC controller, there are obvious even-harmonic voltage residues (especially dc and second harmonics) in the tracking errors, which include about 0.2∼0.3 V dc voltage bias under no load and resistor load, and about 0.5 V dc voltage bias under rectifier load, respectively.

## *C. Sudden Step Load Change*

Figs. 9 and 10 show the odd harmonic repetitive controller operates with sudden step load changes. It is clear from the diagrams that, odd-harmonic RC controlled output voltages do not varied too much (3∼4 V), and recover from the sudden step load changes (between no load and resistor load, and between no load and rectifier load) within about 4∼5 cycles (i.e., 80∼100 ms).

In summary, the results shown in Figs. 4–10 indicate that, under different loads (both linear loads and nonlinear loads) and parameter uncertainties, an odd-harmonic RC offers significantly faster convergence rate than a conventional RC does.

The odd-harmonic RC controlled inverter provides very low THD output voltage, and is robust to sudden step load changes. However, odd-harmonic RC is not immune from even-harmonic disturbances. dc voltage bias residues in the odd-harmonic RC controlled converter may have a bad impact upon magnetic components, such as transformers and inductors. In case of high power applications, much attention should be paid to the phenomena of dc voltage bias residues in odd-harmonic RC controlled converters.

## V. Conclusion

In this paper, a zero-phase odd-harmonic repetitive control scheme is proposed for the CVCF PWM inverters. The data memory occupied by the odd-harmonic periodic signal generator is only half of the conventional repetitive controller. It simplifies the implementation and offers faster convergence of the tracking error. The well-designed phase lead compensation filter $G_f(z)$ helps repetitive controller to achieve high tracking accuracy by good phase lead compensation. Experimental results show that the proposed approach effectively eliminated the dominant odd-harmonic distortion under different loads (linear and nonlinear loads). The proposed odd-harmonic RC is robust to sudden step load changes too. The drawback of an odd-harmonic RC system is that even-harmonic residues may occur in the tracking errors. The phenomena of even-harmonic residues is pointed out and experimentally demonstrated.

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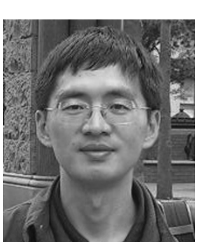

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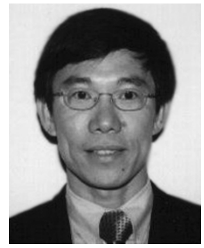

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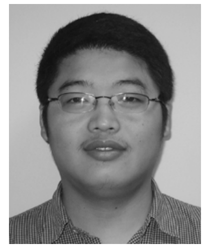

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