Zeros and Passivity of Arnoldi-Reduced-Order Models for Interconnect Networks

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Abstract

CAD tools and research in the area of reduced-order modeling of large linear interconnect networks have evolved from merely finding a Padé approximation for the given network transfer function to finding an approximate transfer function that preserves such circuit-theoretic properties of the network as stability, passivity, and RLC synthesizability. In particular, preserving passivity guarantees that the reduced-order models will be well-behaved when embedded back in the circuit where the interconnect network originated. While stability can be ascertained by studying the poles *of the reduced-order transfer function, passivity depends on both the poles and* zeros *of the network driving-point impedance. In this paper, we present a novel method for studying the zeros of reduced-order transfer functions and show how it yields conclusions about passivity and synthesizability. Moreover, in order to obtain a guaranteed-passive reduced-order model for multiport RC networks, a new algorithm based on the Arnoldi iteration is presented. This algorithm is as computationally efficient as the one used to generate guaranteed-stable reduced-order models [1].*

1 Introduction

The use of reduced-order models to represent large, linear interconnect networks has become a standard component in computer-aided design methodologies for high-density VLSI circuits. Model-order reduction is now used as a model compaction step following the extraction of the large RC netlists needed to perform timing and noise analysis. It is also used to model the RLC circuits representing power buses, clock trees, and long off-chip interconnects where important design issues such as ΔI noise, clock skew, and EMI effects must, respectively, be addressed.

The basic theoretical concept for producing reduced-order models for large linear circuits has been Padé approximation or its variations [2, 3, 4, 5, 6, 7]. While the Padé approximation is mathematically simple to describe and relatively easy to compute, it does not always generate models that satisfy the fundamental circuit-theoretic properties of interconnect networks. Arguably, the most glaring shortcoming of Padé approximation algorithms is that the reduced-order models they generate often fail to preserve the stability of the interconnect circuit. While some of the stability problems are purely numerical [7], some others are inherent in the Padé approximation concept itself [1]. A model-order reduction algorithm based on the Arnoldi iteration was recently shown to generate provably stable reduced-order transfer functions for RLC interconnect networks [1].

It is well known [8] that multiport RLC networks are passive, in the sense that they are energy dissipators. Passivity and stability differ in the following fundamental way: while the connection of two stable networks is not necessarily stable, any multiport connection of passive networks is guaranteed to be passive.

This closure property is of paramount importance from a practical point of view for the following reason. The reduced order model is supposed to replace the original interconnect in the global netlist. It will have the same drivers and loads as the original model. The output impedances of the drivers and the input impedances of the loads are represented with passive elements. If the reduced-order model is just stable but not passive, there is no guarantee that the network composed of output impedances, reduced-order

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model, and input impedances is stable. In the absence of such a guarantee the simulation and analysis of the circuit might become problematic. The passivity of the reduced-order model eliminates such a concern.

In this paper, well-known results about the poles and *zeros* of positive real transfer functions are used to show that the stability-preserving model-order reduction algorithm presented in [1] can fail to preserve passivity. Moreover, using knowledge about the *zeros* of driving point impedances of one-port RC networks, it can be shown that the above mentioned algorithm can fail to generate a reduced-order model that is RC synthesizable!

In order to remedy some of these shortcomings, a new model-order reduction algorithm for multiport RC networks is introduced based on a block version of the stability-preserving Arnoldi iteration described in [1]. It differs from [9] in that preliminary port-preserving congruence transformations are not needed. The algorithm is applied directly to the matrices provided by the modified nodal analysis (MNA) of the circuit equations. The passivity of the resulting reduced-order model is proved rigorously using the positive real characterization of passive networks.

2 Stable Arnoldi Algorithm

In this section we review the stability-preserving Arnoldi algorithm introduced in [1]. The main motivation behind this algorithm was to construct reduced-order models for RLC circuits that are guaranteed to be stable.

To simplify the presentation, we deal with single-input, single-output, linear, time-invariant circuit models of the form

$$\begin{aligned} \mathcal{L}\ \dot{\boldsymbol{x}} &= -\mathcal{G}\ \boldsymbol{x} + \boldsymbol{r}\ u \\ y &= \boldsymbol{l}^T \boldsymbol{x} + du \end{aligned} \tag{1}$$

where, $\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{l}, \in R^n$ are, respectively, the state, input, and output vectors, and $\mathcal{L}, \mathcal{G} \in R^{n\times n}$ are, respectively, the dynamic and static matrices obtained using modified nodal analysis (MNA). [1] The scalar d accounts for the direct gain from the input to the output. Applying q steps of the stable Arnoldi process (Algorithm 1) to the system defined in (1) results in the following reduced-order model of order q

$$\begin{aligned} \boldsymbol{H}_q\ \dot{\tilde{\boldsymbol{x}}} &= \tilde{\boldsymbol{x}} + \tilde{\boldsymbol{e}}_1\ u \\ y &= \tilde{\boldsymbol{l}}^T \tilde{\boldsymbol{x}} + du \end{aligned} \tag{2}$$

where $\tilde{\boldsymbol{x}} \in R^q$, $\tilde{\boldsymbol{e}}_1 \equiv (1, 0, \ldots, 0)^T \in R^q$, and

$$\boldsymbol{H}_q = -\boldsymbol{U}_q^T \mathcal{L} \mathcal{G}^{-1} \mathcal{L} \boldsymbol{U}_q \quad \tilde{\boldsymbol{l}} = -h_{00} \boldsymbol{U}_q^T \boldsymbol{l}. \tag{3}$$

The column vectors of the matrix $\boldsymbol{U}_q$ are $\mathcal{L}$-orthonormal, i.e., $\boldsymbol{U}_q^T \mathcal{L} \boldsymbol{U}_q = \boldsymbol{I}$.

[1] These matrices will be assumed nonsingular, which would exclude networks with resistive meshes, capacitive loops, or inductive cutsets.

Algorithm **1 (Modified $\mathcal{L}$-orthogonal Arnoldi)**

```
arnoldi(input L, G, r, q; output
  U_q, u_{q+1}, H_q, h_{q+1,q})
{
  Initialize:
    Solve :  G u_0 = -r
    z_0 = L u_0
    h_00 = sqrt(u_0^T z_0)
    z_1 = z_0/h_00
    u_1 = u_0/h_00
  for (j = 1; j <= q; j++) {
    Solve G w = -z_j
    for (i = 1; i <= j; i++) {
      h_{i,j} = w^T z_i
      w = w - h_{i,j} u_i
    }
    z_{j+1} = L w
    h_{j+1,j} = sqrt(w^T z_{j+1})
    if (h_{j+1,j} != 0) {
      z_{j+1} = z_{j+1}/h_{j+1,j}
      u_{j+1} = w/h_{j+1,j}
    }
  }
  U_q = [u_1 ... u_q]
  H_q = (h_{i,j}),  i, j = 1, ..., q
```

The reduced-order transfer function is thus given by

$$G_q^A(s) = d + \tilde{\boldsymbol{l}}^T (\boldsymbol{I} - s\boldsymbol{H}_q)^{-1} \tilde{e}_1. \tag{4}$$

When the original system describes a driving-point impedance, the transfer function is symmetric, i.e, $\boldsymbol{l} = \boldsymbol{r}$. The impact of this symmetry on the transfer function of the reduced-order model (4) is described by the following [10]:

Proposition 1 *The input vector $\tilde{e}_1$ and the output vector $\tilde{\boldsymbol{l}}$ of the driving-point impedance reduced-order model satisfy*

$$\tilde{\boldsymbol{l}} = h_{00}^2 \boldsymbol{U}_q^T \mathcal{G} \boldsymbol{U}_q \tilde{\boldsymbol{e}}_1. \tag{5}$$

Recall that the choice of the output vector of the reduced-order model is imposed by the requirement of moment matching. An Arnoldi-based reduced-order model of order q matches the first q moments of the original model.

Comparing the driving-point impedance

$$Z(s) = d + \boldsymbol{l}^T (\mathcal{G} + s\mathcal{L})^{-1} \boldsymbol{r} \tag{6}$$

with the reduced-order model

$$Z_q^A(s) = d + h_{00}^2 \tilde{e}_1^T \boldsymbol{U}_q^T \mathcal{G}^T \boldsymbol{U}_q (I - s\boldsymbol{H}_q)^{-1} \tilde{e}_1 \tag{7}$$

we can clearly see that the choice of the Arnoldi output vector (5) makes the transfer function of the reduced order model asymmetric.

It is important to note that *even* in the RC case, the above Arnoldi algorithm does *not* reduce to the SyPVL symmetric Lanczos algorithm [11]. Nor does it reduce to the Cholesky-Lanczos algorithm presented in [12].

The computational cost of Algorithm 1 is that of executing one sparse LU factorization for $\mathcal{G}$, $q+1$ matrix-vector products for computing the z_j vectors, and $q+1$ back substitutions for computing u_0 and the w vectors. It has therefore about the same computational cost as PVL, assuming one back substitution to be roughly equivalent to one matrix-vector product.

For multiport networks, a block version of the above algorithm can be readily obtained. The coefficients $h_{j+1,j}$ will have to be replaced by the unique, symmetric, positive-definite *matrix* square roots of the *matrices* $\boldsymbol{W}\boldsymbol{Z}_{j+1}$ which are symmetric positive-definite by construction. The net result of the algorithm is a *block* Hessenberg matrix in which each block is of size $p \times p$ where p is the number of ports. The numerical examples given in Section 5 use the block implementation of Algorithm 1.

3 Passivity in a Nutshell

Passive networks are networks whose net electrical energy balance is nonpositive, i.e., the energy that is dissipated by the network is at least equal to the energy supplied by the sources. The network is strictly passive when the dissipated energy is strictly larger than the supplied energy. The fundamental theorem relating passivity to the linear network mathematical description, be it in the impedance or admittance form, is the following [8]:

Theorem 2 *A one port-network is passive if and only if its driving-point impedance (admittance), denoted by $F(\sigma + j\omega)$, is positive real, i.e,*

$$(pr1) \qquad \forall \sigma \in R, F(\sigma) \in R$$
$$(pr2) \qquad \forall \sigma \geq 0, Re\{F(\sigma + j\omega)\} \geq 0$$

Using standard arguments from complex function theory, it is not difficult to prove that [8]:

Proposition 3 *The poles and zeros of a positive real function are all in the left-half plane.*

The above proposition means that a one-port network can be stable without being passive. It also means that a passive one-port network is stable whether it is driven, at its port, by an independent voltage source or an independent current source.

Arnoldi zeros
-65.8236
-1.2073
0.0033 + 0.4190i
0.0033 - 0.4190i
-0.0590 + 0.2745i
-0.0590 - 0.2745i

Table 1: The first six zeros of the Arnoldi reduced-order model of a one-port RLC circuit. Note the generation of zeros with positive real parts. The reduced-order model cannot be a passive one-port network.

There are two fundamental properties satisfied by passive networks but not by stable networks. They are the closure property and the RLC realizability property [8]:

Proposition 4 *The connection of any two passive multiport networks is passive.*

Proposition 5 *A linear, time-invariant, multiport network is passive if and only if it can be synthesized exactly using positive R, C, L, and M elements.*

Note that if the interconnect reduced-order model is to be incorporated into the original circuit for simulation or timing analysis, passivity is a more relevant property than stability. For instance, it is important to ensure that adding (passive) capacitive and resistive loads at the output of the reduced-order model cannot make the total circuit (reduced-order model plus loads) unstable. This can be ensured if it can be guaranteed that the reduced-order model is passive whenever the original model is.

4 Zeros of the Arnoldi Model

The passivity of a one-port network implies that both the poles and zeros of the driving-point transfer function (impedance or admittance) are in the left-half plane (Proposition 3). It follows that one way of disproving passivity is to show that the driving-point transfer function has zeros in the right half plane.

Let
$$Z(s) = d + \boldsymbol{l}^T(\mathcal{G} + s\mathcal{L})^{-1}\boldsymbol{r} \tag{8}$$
be the transfer impedance of some one-port network. Since at infinite frequency the one-port network behaves as a two-terminal resistor, $d > 0$. The poles of $Z(s)$ are the roots of the polynomial $q(s) = det[\mathcal{G} + s\mathcal{L}]$.

The zeros of $Z(s)$ are the poles of the admittance $Y(s) = 1/Z(s)$. In terms of the matrices and vectors in (8), the admittance is given by [10]

$$Y(s) = \frac{1}{d} - \frac{\boldsymbol{l}^T}{d}\left(\mathcal{G} + \frac{\boldsymbol{l}\boldsymbol{r}^T}{d} + s\mathcal{L}\right)^{-1}\frac{\boldsymbol{r}}{d} \tag{9}$$

The following comments are a direct result of (9):

Arnoldi zeros
-46.4754
-1.0819 + 0.0972i
-1.0819 + 0.0972i
-0.5103
-0.0857
-0.0179

Table 2: The six zeros of the Arnoldi reduced-order model of a one-port RC circuit. Note the generation of zeros with imaginary parts. The reduced-order model is therefore not synthesizable as a one-port RC network.

1. The zeros of $Z(s)$ are the roots of the polynomial
$$p(s) = det\left[\boldsymbol{\mathcal{G}} + \frac{\boldsymbol{l}\boldsymbol{r}^T}{d} + s\boldsymbol{\mathcal{L}}\right].$$
The above polynomial shows clearly the effect of the input vector, the output vector, and the resistor at infinite frequency on the zeros of $Z(s)$.

2. The admittance function behaves as if it had a matrix $\boldsymbol{\mathcal{G}}' \equiv \boldsymbol{\mathcal{G}} + \frac{\boldsymbol{l}\boldsymbol{r}^T}{d}$, where $\boldsymbol{\mathcal{G}}'$ is a rank-one perturbation of $\boldsymbol{\mathcal{G}}$.

3. Note that the matrix $\boldsymbol{\mathcal{L}}$ is always symmetric, positive-definite. Thus the poles and zeros of $Z(s)$ are also given by the eigenvalues of the matrices
$$\begin{aligned}\boldsymbol{K}_1 &\equiv -\boldsymbol{\mathcal{L}}^{1/2}\boldsymbol{\mathcal{G}}\boldsymbol{\mathcal{L}}^{1/2}\\ \boldsymbol{K}_2 &\equiv -\boldsymbol{\mathcal{L}}^{1/2}\left(\boldsymbol{\mathcal{G}} + \frac{\boldsymbol{l}\boldsymbol{r}^T}{d}\right)\boldsymbol{\mathcal{L}}^{1/2},\end{aligned}$$
respectively, the latter matrix being a rank one perturbation of the former.

4. Since $\boldsymbol{\mathcal{L}}^{1/2}$ is symmetric, $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are obtained by congruence transformation from $-\boldsymbol{\mathcal{G}}$ and $-(\boldsymbol{\mathcal{G}} + \frac{\boldsymbol{l}\boldsymbol{r}^T}{d})$, respectively, which preserves the signs of the real parts of the matrix eigenvalues ([13], Theorem 2.4.10).

It follows then that the locations of the zeros of $Z(s)$ depend on the effect of the rank-one perturbation of $\boldsymbol{\mathcal{G}}$ on the eigenvalues of $\boldsymbol{\mathcal{G}}$.

Application to reduced-order models: Applying (9) to the transfer function $Z_q^A(s)$ (7), we conclude that the zeros of the reduced order model of a (symmetric) driving point impedance $Z(s)$ are obtained as the roots of the polynomial
$$p_q^A(s) = det\left[\boldsymbol{I}_q + h_{00}^2\frac{\tilde{e}_1\tilde{e}_1^T}{d}\boldsymbol{U}_q^T\boldsymbol{\mathcal{G}}^T\boldsymbol{U}_q - s\boldsymbol{H}_q\right].$$

At this stage we shall treat the RLC case and RC case separately.

1. *The RLC case*: The matrix $\boldsymbol{H}_q$ has all its eigenvalues in the (closed) left half plane. However, due to the asymmetric rank-one perturbation of the identity matrix, some of the roots of $p_q^A(s)$ might move to the right-half plane. This is indeed the case of the the example shown in Table 1 where the first six zeros of the Arnoldi reduced-order model of an RLC circuit are shown. The RLC circuit is a randomly generated RLC network with 225 capacitors, 225 resistors, and 196 inductors. The order of the reduced-order model is $q = 14$. Note that two of the zeros have positive real parts. By Proposition 3, the reduced-order model cannot be that of a one-port *passive* RLC network.

2. *The RC case:* The matrix $\boldsymbol{H}_q$ is a symmetric, tridiagonal, negative-definite matrix, with all its eigenvalues on the negative real axis [1]. However, the asymmetric rank-one perturbation might cause some zeros to be complex conjugates. This is illustrated in Table 2 which lists the 6 zeros of the 6th-order Arnoldi reduced-order model of a random one-port RC circuit with 100 resistors and 100 capacitors. It is a classical result of linear circuit theory ([8], p. 66) that the poles and zeros of a one-port RC network are alternating negative reals. Since the reduced-order model of this one-port RC network has *complex* zeros, it cannot be synthesized as a one-port RC circuit.

The appearance of complex conjugate zeros in the RC case is due to the fact that although the system matrix is symmetric, the rank-one perturbation is not. The reader should also note that, unlike SyPVL [11] and Cholesky-Lanczos [12], the stable Arnoldi algorithm 1 destroys the symmetry of driving-point impedances, which could result in the generation of complex conjugate zeros.

5 Passive Arnoldi Algorithm

In this section, a passivity-preserving reduced-order model for multiport RC circuits is derived from the stability-preserving Arnoldi iteration reviewed in Section 2.

The results described in Section 4 illustrate the crucial role played by the input and output vectors in controlling the passivity behavior of the reduced-order model.

Consider the multiport driving-point impedance
$$\boldsymbol{Z}(s) = \boldsymbol{D} + \boldsymbol{E}^T(\boldsymbol{\mathcal{G}} + s\boldsymbol{\mathcal{L}})^{-1}\boldsymbol{E} \qquad (10)$$
where $\boldsymbol{D}$ is a symmetric, positive-definite matrix since the multiport network behaves purely resistively at infinite frequency. The port-node incidence matrix $\boldsymbol{E}$ is assumed to be full rank with p columns ($p < n$). In the RC case, the matrix $\boldsymbol{\mathcal{G}}$ is symmetric positive definite.

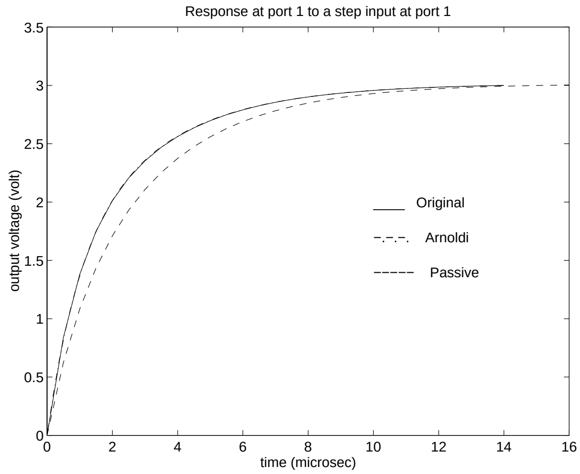

Figure 1: Rise time: Response of port 1 to a step input at port 1.

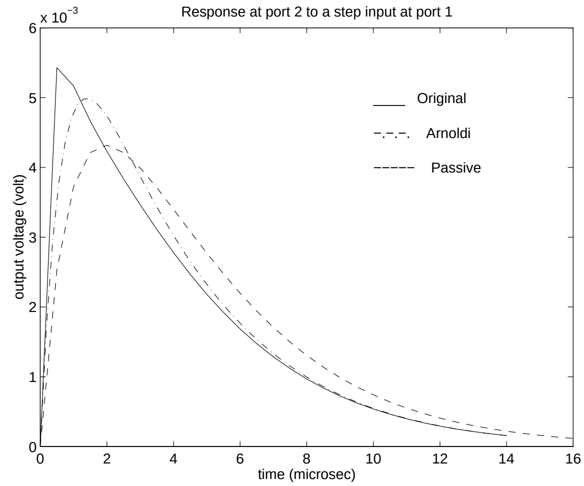

Figure 2: Coupling noise: Response of port 2 to a step input at port 1.

Running the block Arnoldi algorithm for q iterations results in the block tridiagonal system matrix $\boldsymbol{H}_q$ of order $pq \times pq$. Instead of defining the input and output matrices as in (3), we let $\tilde{\boldsymbol{E}}$ be a $p \times pq$ matrix such that the reduced-order driving-point matrix impedance is given by

$$\boldsymbol{Z}_q^A(s) = \boldsymbol{D} + \tilde{\boldsymbol{E}}^T(\boldsymbol{I}_q - s\boldsymbol{H}_q)^{-1}\tilde{\boldsymbol{E}} \qquad (11)$$

Moreover, we constrain $\tilde{\boldsymbol{E}}$ to be of the form $\tilde{\boldsymbol{E}}^T = [\boldsymbol{F}|\boldsymbol{O}]$ where $\boldsymbol{F}$ is a *symmetric* $p \times p$ matrix and $\boldsymbol{O}$ is a block of zeros. To compute $\boldsymbol{F}$, we impose the DC constraint $\boldsymbol{Z}_q^A(0) = \boldsymbol{Z}(0)$, which yields

$$\boldsymbol{F}^2 = \boldsymbol{E}^T\mathcal{G}^{-1}\boldsymbol{E}.$$

In other words, the matrix $\boldsymbol{F}$ is the unique, symmetric, positive-definite square root of the symmetric positive-definite matrix $\boldsymbol{E}^T\mathcal{G}\boldsymbol{E}$.

Note that in the RLC case, such a definition of the input/output matrix is not possible as $\mathcal{G}$ is not symmetric, positive-definite.

We now claim the following:

Proposition 6 *The reduced-order model (11) defined with*

$$\tilde{\boldsymbol{E}}^T = [\boldsymbol{F}|\boldsymbol{O}] \quad \boldsymbol{F} = \left[\boldsymbol{E}^T\mathcal{G}^{-1}\boldsymbol{E}\right]^{1/2}$$

is passive.

Proof. See [10]. □

In order to illustrate the time-domain behavior of this passivity-preserving algorithm, we consider the case of a large two-port RC network. The network is built out of two random, fully connected one-port RC networks that are capacitively coupled. The first network has 20100 capacitors while the second has 5050 capacitors. The network was reduced using the block Arnoldi algorithm to a two-port network with just four states. Figure 1 shows the step response of port 1 to an input at port 1. Three curves are compared: the response of the original network, the response of the stability-preserving block Arnoldi, and the response of the passivity-preserving block Arnoldi. Figure 2 shows the effect of capacitive coupling and how it is approximated by the reduced-order model. Although the reduced-order two-port network has only four poles, the direct and crosstalk step responses are quite satisfactory. The difference between the two block-Arnoldi models, the stable and the passive, is due to the fact that the input/output matrix of the passive network contains no information about the capacitance matrix $\boldsymbol{C}$. In the case of stable-Arnoldi the $\boldsymbol{C}$ matrix is used to compute the matrix $\boldsymbol{H}_{00}$, the block matrix equivalent of h_{00} in Algorithm 1. This results in a better match, at higher frequencies, between the stable-Arnoldi step response and the original-system step response than between the latter and the passive-Arnoldi step response.

The computational efficiency of the passive Arnoldi algorithm is very close to that of the stable Arnoldi algorithm, the latter being comparable to block PVL [7]. The additional computation in passive Arnoldi is that of the matrix $\boldsymbol{F}$ as the matrix square root of $\boldsymbol{E}^T\mathcal{G}\boldsymbol{E}$. This is equivalent to solving a symmetric eigenvalue problem whose order is p, the number of ports, which is typically much smaller than the number of network nodes.

6 Conclusions

This paper investigated two circuit-theoretic properties pertaining to reduced-order models generated by the stability-preserving Arnoldi algorithm of [1], namely passivity and synthesizability. Once it has been understood how the input and output vectors that result from the Arnoldi algorithm affect the zeros of the reduced-order transfer function, it is relatively straightforward to understand the mechanism by which the algorithm fails to produce passive reduced-order models for RLC networks or RC-synthesizable reduced-order models for RC networks. We have also shown how to modify the output matrix of the Arnoldi algorithm so that the passivity of RC circuit reduced-order models is provably guaranteed.

After this paper had been submitted, Reference [14] was communicated to us, in which the authors use a "vanilla" version of the block Arnoldi algorithm to construct provably passive reduced-order models of RLC interconnects. We note that their algorithm, which was developed for single-point expansions, can be readily extended to deal with the case of multipoint expansions [5]. The resulting multipoint algorithm yields reduced-order models for RLC inteconnects that are provably passive and guaranteed accurate for any finite number of frequency points [10].

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