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ZEROS OF AN ENTIRE FUNCTION CONNECTED BY A LOADED FIRST-ORDER DIFFERENTIAL OPERATOR ON A SEGMENT

In the paper, we consider the problem on eigenvalues of a loaded differential operator of the first order with a periodic boundary condition on the interval [–1; 1], that is, equation contains a load at the point (–1) and the function of bounded variation Φ(t), with the condition $\Phi(-1)=\Phi(1)=1$ *. A characteristic determinant of spectral problem is constructed for the considered loaded differentiation operator, which is an entire analytical function on the spectral parameter. On the basis of the characteristic determinant formula, conclusions are proved about the asymptotic behavior of the spectrum and eigenfunctions of the loaded spectral problem for the differentiation operator, the characteristic determinant of which is an entire analytic function of the spectral parameter λ. A theorem on the location of eigenvalues on the complex plane λ is formulated, where the regular growth of an entire analytic function is indicated. A theorem is proved on the asymptotics of the zeros of an entire function, that is, the eigenvalues of the original considered spectral problem for a loaded differential operator of differentiation, and the asymptotic properties of an entire function with distribution of roots are studied.*

***Keywords**: loaded differential operator, perturbed, characteristic determinant, zeros of entire functions, asymptotics, eigenvalues, spectrum, eigenfunctions, basis.*

Introduction. It is well known that in the case of non-self-adjoint ordinary differential operators, the basicity of root function systems, in addition to boundary conditions, can also be influenced by the values of the coefficients of the differential operator. At the same time, the basic properties of the root functions can change even with an arbitrarily small change in the values of the coefficients. This result was first noted in the work of V.A.Ilyin [1]. V.A.Ilyin's ideas were developed by A.S.Makin [2] in the case of a non-self-adjoint perturbation of a self-adjoint periodic problem. The operator in [2] was changed due to the perturbation of one of the boundary conditions. In [3, 4, 5], another variant of the perturbation of the self-adjoint problem was considered, namely, the spectral problem for a loaded second-order differential operator with periodic boundary conditions, which the second term on the left side of the equation contains the value of the desired function at zero. Such a problem is a non-self-adjoint perturbation of a self-adjoint periodic problem. In contrast to [2], in [3, 4, 5], the perturbation occurs due to a change in the equation.

The issues of the basicity of the root functions of loaded differential operators were studied in the works of I.S.Lomov [6, 7]. He extended the method of spectral expansions by V.A.Ilyin [1] to the case of loaded differential operators. Another method investigated the issues of unconditional basicity of functional differential equations in [8].

Problem statements. In the function space consider the eigenvalue problem of the loaded differentiation operator

$$L_1 y = y'(t) + \lambda y(-1)\Phi(t) = \lambda y(t), \quad -1 \le t \le 1, \tag{2.1}$$

with a boundary condition

$$y(-1)=y(1), \tag{2.2}$$

where $\Phi(t)$ is a function of bounded variation and $\Phi(-1)=\Phi(1)=1$, λ is a complex number, a spectral parameter.

It is required to find those complex values of λ for which the operator equation (2.1) has nonzero solutions.

Construction of a characteristic determinant of a spectral problem (2.1) - (2.2)

Considering $y(-1)$ to be some independent constant, we make sure that the general solution of the equation (2.1) for $\lambda \neq 0$ is representable as

$$y(t)=C\cdot e^{\lambda t}-\lambda e^{\lambda}\cdot y(-1)\cdot\int_{-1}^{t}\Phi(\xi)e^{\lambda\xi}d\xi. \tag{3.1}$$

Hence, assuming first $t=-1$, and then satisfying (3.1) the boundary condition (2.2), we obtain a system of two equations, which in vector-matrix form is representable as:

$$\begin{bmatrix} e^{\lambda}-e^{-\lambda} & \lambda e^{\lambda}\cdot\int_{-1}^{1}\Phi(\xi)e^{\lambda\xi}d\xi \\ e^{-\lambda} & -1 \end{bmatrix}\begin{bmatrix} C \\ y(-1)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}. \tag{3.2}$$

By simple calculations, we obtain that the characteristic determinant $\Delta_1(\lambda)$ of the spectral problem (2.1 - (2.2) is represented as

$$\Delta_1(\lambda)=e^{-\lambda}-e^{\lambda}-\lambda\cdot\int_{-1}^{1}e^{\lambda t}\cdot\Phi(t)dt. \tag{3.3}$$

Thus, it is proved.

Lemma 3.1 *The characteristic determinant of the spectral problem for the loaded differentiation operator (2.1), which is an integral analytical function of the variable* $[-1,1]$ *with a boundary condition (2.2), is represented as (3.3), which is an integral analytical function of the variable* $\lambda=x+iy$, $Re\lambda=x$, $Im\lambda=y$, $i=\sqrt{-1}$, *where* $\Phi(t)$ *is a function of bounded variation and* $\Phi(-1)=\Phi(1)=1$.

The main result. In the case when $\Phi(t)=0$, it turns out $\Delta_0(\lambda)=e^{-\lambda}-e^{\lambda}$ is the characteristic determinant of the "undisturbed" spectral problem

$$L_0y=y'(t)=\lambda y(t), \quad -1\le t\le 1, \quad y(-1)=y(1). \tag{4.1}$$

The numbers $\lambda_n^0=in\pi$, $n=\pm1,\pm2,\pm3,\dots$ are eigenvalues, while $\forall C>0$, $y_{n0}^0=C\cdot e^{in\pi t}$ are eigenfunctions of the "undisturbed" operator L_0, which forms a complete orthonormal system and a Riesz basis in the space $L_2(-1,1)$.

In the case when $\lambda=0$ we have $y(t)=C\neq 0$, i.e. $\lambda_0=0$ is the eigenvalue of the loaded differentiation operator L_1.

In the case of $\Phi(t)$ is a function of bounded variation and $\Phi(-1)=\Phi(1)=1$, equating the characteristic determinant $\Delta_1(\lambda)$ to zero, we investigate the distribution of zeros of the whole analytic function, which adequately determines the eigenvalues of the loaded differentiation operator L_1.

The research of zeros of integer functions having an integral representation is devoted to the works [9, 10, 11, 12].

The connection of zeros of exponential integer functions with spectral problems is reflected in the works [13,14,15]. Eigenvalue problems for some classes of differential operators on a segment are reduced to a similar problem. In particular, the problem under consideration (2.1) is (2.2) of this article.

The questions of the location of the zeros of the whole function: on one ray, on a straight line, on several rays, at an angle, or arbitrarily in the complex plane have been studied in numerous works [16, 17].

There are the following

Theorem 4.1 *If* $\Phi(t)$ *is a function of bounded variation and* $\Phi(-1)=\Phi(1)=1$ *, then all zeros of the entire function* $\Delta_1(\lambda)$ *, i.e. all eigenvalues of the loaded first-order differential operator* L_1 *belong to the strip* $|\mathrm{Re}\,\lambda|=|x|<k$ *, for some k, where* $\lambda=x+iy$.

Proof. There is a well-known theorem [18] that any function with bounded variation has a finite derivative almost everywhere. By virtue of this theorem, the expression $\int_{-1}^{1} e^{\lambda t}\cdot\Phi(t)\,dt$ can be integrated by parts. Then the function $\Delta_1(\lambda)$ will take the following form:

$$\Delta_1(\lambda)=\frac{2\cdot\left(e^{\lambda}-e^{-\lambda}\right)}{\lambda}-\frac{1}{\lambda}\cdot\int_{-1}^{1-\delta} e^{\lambda t}\cdot d\Phi(t)-\frac{1}{\lambda}\cdot\int_{1-\delta}^{1} e^{\lambda t}\cdot d\Phi(t).$$

Next, we rely on the well-known Rouche theorem [19], and on the basis of this theorem we introduce the function $f(\lambda)=\dfrac{2\cdot\left(e^{\lambda}-e^{-\lambda}\right)}{\lambda}=-\dfrac{2}{\lambda}\Delta_0(\lambda)$, and also the difference $g(\lambda)=\Delta_1(\lambda)-f(\lambda)$. Let us show that the function $\Delta_1(\lambda)$ is outside the strip ($|\mathrm{Re}\,\lambda|<k$, for some *k*) has no zeros. To do this, we estimate the function $\Delta_1(\lambda)$ from below

$$|\Delta_1(\lambda)|\geq\frac{2}{|\lambda|}e^{|\lambda|}-\frac{2}{|\lambda|}e^{-|\lambda|}-\frac{1}{|\lambda|}\cdot\int_{-1}^{1-\delta} e^{|\lambda|t}\cdot d|\Phi(t)|-\frac{1}{|\lambda|}\cdot\int_{1-\delta}^{1} e^{|\lambda|t}\cdot d|\Phi(t)| \qquad (4.2)$$

Therefore, the function $f(\lambda)$ is estimated from below, while the remaining terms of the function $g(\lambda)$ are estimated from above

$$\begin{aligned} |f(\lambda)|&\geq\frac{2}{|\lambda|}e^{x}-\frac{2}{|\lambda|}e^{-x},\\ |g(\lambda)|&\leq\frac{1}{|\lambda|}\left(\int_{-1}^{1-\delta} e^{xt}\cdot d|\Phi(t)|+\int_{1-\delta}^{1} e^{xt}\cdot d|\Phi(t)|\right). \end{aligned} \qquad (4.3)$$

Let us separately consider the estimate of each integral from (4.3). To do this, we use the boundedness of variations of the function $\Phi(t)$.

Then the first integral from (4.3) is evaluated as follows:

$$\int_{-1}^{1-\delta} e^{xt}\cdot d\left|\Phi(t)\right| \le e^{x(1-\delta)}\int_{-1}^{1-\delta}\left|d\Phi(t)\right| \le e^{x(1-\delta)}\int_{-1}^{1}\left|d\Phi(t)\right| = e^{x(1-\delta)}H,$$

Where $H=\int_{-1}^{1}\left|d\Phi(t)\right|$ is a constant value. Consider the estimate of the second integral

$$\int_{1-\delta}^{1} e^{xt}\cdot\left|d\Phi(t)\right| \le e^{x}\cdot\int_{1-\delta}^{1}\left|d\Phi(t)\right| \le e^{x}\cdot\varepsilon(\delta), \tag{4.5}$$

where $\varepsilon(\delta)\to 0$ for $\delta\to 0$.

So, taking into account (4.2) - (4.5), come to an estimate

$$\left|\Delta_1(\lambda)\right| \ge \frac{2}{|\lambda|}\cdot\left(e^{x}-e^{-x}\right)-\frac{1}{|\lambda|}\cdot e^{x(1-\delta)}\cdot H-\frac{1}{|\lambda|}\cdot e^{x}\cdot\varepsilon(\delta),$$

$$|\lambda|\cdot\left|\Delta_1(\lambda)\right| = \left|\Delta(\lambda)\right|.$$

$\left|\operatorname{Re}\Delta(\lambda)\right| \ge \left|e^{x}-e^{-x}\cdot O(1)\right| > \frac{e^{x}}{2}$ for $\operatorname{Re}\lambda = x \ge k,$ i.e. $\Delta_1(\lambda)$ has no zeros for these x values. Similar reasoning for negative *x*, which completes the proof of Theorem 4.1.

Theorem 4.2 *Let $\Phi(t)$ be a function of bounded variation and $\Phi(-1)=\Phi(1)=1$. Then the zeros of the entire analytic function $\Delta_1(\lambda)$, that is, the eigenvalues of the loaded first-order differential operator L_1, form a countable set and have the asymptotic $\lambda_n^{(1)} = i\pi n + \underline{\underline{O}}(1)$ as $n\to\infty$.*

Proof. Calculating the zeros of the function $f(\lambda)$ gives $\lambda_n^0 = i\pi n$, $n=\pm1,\pm2,\dots$, which are the same as the zeros of the function $\Delta_0(\lambda)$, otherwise they coincide with the eigenvalues of the operator L_0, that is, the "unperturbed" spectral problem (4.1).

Consider a square *T* with side 2ε centered at the point λ_n^0 on the complex plane λ. Let us choose the minimal $\varepsilon > 0$ so that the conditions of the Rouche theorem [19] are satisfied for the function $f(\lambda)$, $g(\lambda)$ on the sides of the square *T*, why do we compare the majorant of the function $g(\lambda)$ with the minorant of the function $f(\lambda)$

$$\max_T\left|g(\lambda)\right| < \min_T\left|f(\lambda)\right|.$$

The function $f(\lambda)$ let's evaluate from below

$$\left|f(\lambda)\right| = \frac{2}{|\lambda|}\cdot\left(e^{\lambda}-e^{-\lambda}\right) \ge \min_T\frac{2}{|\lambda|}\cdot\left|e^{\lambda}-e^{-\lambda}\right| = \frac{2}{\lambda_0^{*}}\cdot\left|e^{\lambda^{*}}-e^{-\lambda^{*}}\right|,$$

where $\lambda^{*}\in T$. The last equality follows from the fact that $\left|f(\lambda)\right|$ is a continuous function, and the square *T* is compact.

Let us estimate the function $g(\lambda)$ on the sides of the square T. The imaginary axis divides the square T into two equal parts. Let us estimate from above the function $g(\lambda)$ on the right half of the square

$$|g(\lambda)| \le \frac{1}{|\lambda|}\cdot\int_{-1}^{1-\delta} e^{xt}\cdot d|\Phi(t)| + \frac{1}{|\lambda|}\cdot\int_{1-\delta}^{1} e^{xt}\cdot d|\Phi(t)|.$$

Let us consider the estimate for each term separately. Let us estimate the first term. We choose $\frac{2}{n}\ge\delta\ge\frac{1}{n}$, then, taking into account that $x\ge 0$, $-1\le t\le 1-\delta$, we get the inequalities

$$\frac{1}{|\lambda|}\cdot\int_{-1}^{1-\delta} e^{xt}\cdot d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{x(1-\delta)}\int_{-1}^{1-\delta}\cdot d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{x\left(1-\frac{1}{n}\right)}\int_{-1}^{1-\frac{1}{n}} d|\Phi(t)| = \frac{1}{|\lambda|}\cdot C_1\cdot e^{x\left(1-\frac{1}{n}\right)},$$

where $C_1=\int_{-1}^{1} d|\Phi(t)|$ is a constant value.

Let's evaluate the second term. Since $\max(xt)=x$, we have

$$\frac{1}{|\lambda|}\cdot\int_{1-\delta}^{1} e^{xt}\cdot d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{x}\int_{1-\delta}^{1} d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{x}\int_{1-\frac{2}{n}}^{1} d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{x}\cdot\mu\left(\frac{1}{n}\right),\quad n>>1.$$

Obviously,

$$\mu\left(\frac{1}{n}\right)\to 0,$$

when $n\to\infty$. Now the function $g(\lambda)$ let's estimate from above on the left half of the square

$$|g(\lambda)| \le \frac{1}{|\lambda|}\cdot\int_{-1}^{-1+\delta} e^{xt}\cdot d|\Phi(t)| + \frac{1}{|\lambda|}\cdot\int_{-1+\delta}^{1} e^{xt}\cdot d|\Phi(t)|.$$

The estimate for the first term has the following form:

$$\frac{1}{|\lambda|}\cdot\int_{-1}^{-1+\delta} e^{xt}\cdot d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{-x}\int_{-1}^{-1+\delta} d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{-x}\int_{-1}^{-1+\frac{2}{n}} d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{-x}\cdot\mu\left(\frac{1}{n}\right),\quad n>>1.$$

Here, for $\frac{1}{n}\to 0$, the value $\mu\left(\frac{1}{n}\right)\to 0$. The second term is estimated as follows:

$$\frac{1}{\lambda}\cdot\int_{-1+\delta}^{1} e^{xt}d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{x(-1+\delta)}\int_{-1+\delta}^{1} d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{x\left(-1+\frac{1}{n}\right)}\int_{-1+\frac{1}{n}}^{1} d|\Phi(t)| \le \frac{1}{|\lambda|}\cdot e^{x\left(-1+\frac{1}{n}\right)}\cdot C_1,$$

where $C_1 = \int_{-1}^{1} d\left|\Phi(t)\right|$ is a constant value.

Taking these inequalities into account, we choose $\varepsilon > 0$ so that the following inequality holds on the sides of the square *T*:

$$\frac{2}{\left|\lambda_0^*\right|} \cdot \left|e^{\lambda^*} - e^{-\lambda^*}\right| > \frac{1}{|\lambda|} \cdot e^{|x|} \cdot C \cdot \left[\mu\left(\frac{1}{n}\right) + e^{-\frac{1}{n}|x|}\right].$$

To the left side of the inequality, we apply Lagrange's theorem on finite increments [20], then it is sufficient to fulfill the inequality

$$\frac{2}{\left|\lambda_0^*\right|} \cdot H \cdot \varepsilon > \frac{1}{|\lambda|} \cdot e^{|x|} \cdot C\left[\mu\left(\frac{1}{n}\right) + e^{-\frac{1}{n}|x|}\right],$$

where *H* is a constant value the derivative $\left(e^{\lambda} - e^{-\lambda}\right)$ from below on the sides of the square *T*; e^x - is limited because $-\varepsilon < \operatorname{Re}\lambda < \varepsilon$ implies $e^{-\varepsilon} < e^x < e^{\varepsilon}$. Since the difference $\lambda - \lambda_n^0$, $\lambda^* - \lambda_n^0$ is bounded, it follows that for $n \to \infty$ relation $\left|\frac{\lambda}{\lambda^*}\right|$ is limited. Since $e^{-\frac{1}{n}\cdot x} > \mu\left(\frac{1}{n}\right)$, therefore, we get $\varepsilon \approx \underline{\underline{O}}(1)$. Theorem 2 is proved.

Remark 4.1 One of the features of the problem under consideration is that the conjugate to (2.1) - (2.2) is the spectral problem in $W_2^1(-1,1)$ for the differentiation operator on a segment with a linear occurrence of the spectral parameter in the boundary condition with an integral perturbation

$$\begin{aligned} &L_1^* v = v'(t) = \bar{\lambda} v(t), \quad -1 \le t \le 1, \\ &v(-1) - v(1) = \bar{\lambda} \int_{-1}^{1} v(t)\Phi(t)\,dt, \end{aligned} \tag{4.6}$$

where $\Phi(t)$ is a function of bounded variation and $\Phi(-1) = \Phi(1) = 1$, $\bar{\lambda}$ is a complex number, a spectral parameter.

Remark 4.2 According to the result of Theorem 2, the system of eigenfunctions of spectral problems (2.1) - (2.2) and (4.6) has an asymptotic representation: $y_n^{(1)} = v_n^{(1)} \approx Ce^{in\pi t}e^{\varepsilon t}$ for $n \to \infty$, $\varepsilon \approx \underline{\underline{O}}(1), \forall C > 0$. In this case, such a system is not orthonormal, but forms a Riesz basis in $L_2(-1,1)$. Так как $\exists \alpha > 0, \beta > 0$: $e^{\varepsilon t}$ does not tend to zero, nor does it tend to infinity, $\alpha \le e^{\varepsilon t} \le \beta$, i.e. there is a bounded reversible transformation, so the system $y_n^{(1)} = v_n^{(1)}$ forms a Riesz basis in $L_2(-1,1)$.

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REFERENCES

1 Ilin V.A. On the connection between the types of boundary conditions and the properties of basis property and equiconvergence with a trigonometric series of expansions in root functions of a non-self-adjoint differential operator / Differential Equations. 1994. – V.30, – No.9, – p.1516-1529.

2 Makin A.S. On Nonlocal Perturbation of a Periodic Eigenvalue Problem / Differential Equations. 2006. – V.42, – No.4, – p.560-562.

3 Imanbaev N.S., Sadybekov M.A. Ob ustoychivosti svoystva bazisnosti odnogo tipa zadach na sobstvennyye znacheniya pri nelokalnom vozmushchenii krayevogo usloviya [On the stability of the basis property of one type of eigenvalue problems under a nonlocal perturbation of the boundary condition] / Ufimskiy matematicheskiy zhurnal [Ufa Mathematical Journal]. 2011. – V.3, – No.2, – p.28-33 [in Russian].

4 Imanbaev N.S., Sadybekov M.A. (2016). Stability of basis property of a periodic problem with nonlocal perturbation of boundary conditions / AIP Conf. Proc. 2016. – Vol.1759, article №020080, DOI: https://doi.org/10.1063/1.4959694

5 Sadybekov M.A., Imanbaev N.S. Characteristic determinant of a boundary value problem, which does not have the basis property / Eurasian Mathematical Journal. 2017. – No.2, – p.40-46

6 Lomov I.S. Basis Property of Root Vectors of Loaded Differential Operators / Differential Equations. 1991. – V27, – No.1, – p.80-94.

7 Lomov I.S. The unconditional basis property theorem for root vectors of loaded second-order differential operators / Differential Equations. 1991. – V.27, – No.9, – p.1550-1563.

8 Gomilko A.M., Radziyevskiy G.V. Basic properties of eigenfunctions of a regular boundary value problem for a vector functional differential equation / Differential Equations. 1991. – V.27, – No.3, – p.384-396.

9 Cartwright M.L. The zeros of certain integral functions / The Quarterly Journal of Mathematics. 1930. – V.1, – No.1, – p.38-59. DOI:https://doi.org/10.1093/gmath/os-1.1.38

10 Hald O.H. Discontinuous inverse eigenvalue problems / Communications on Pure and Applied Mathematics. 1984. – V.37, – No.5, – p.539--577. DOI:https://doi.org/10/1002/cpa.3160370502

11 Imanbaev N.S., Kanguzhin, B.E., Kalimbetov B.T. On zeros of the characteristic determinant of the spectral problem for a third-order differential operator on a segment with nonlocal boundary conditions / Advances in Difference Equations. 2013. – No.110. DOI: https://doi.org//10/1186/1687-1847-2013-110

12 Titchmarsh, E.C. The zeros of certain integral functions / Proc. of the London Mathematical Society. 1926. – V25, – No.1, – p.283-302. DOI:https://doi.org/10.1112/plms/s2-25.1.283

13 Imanbaev N.S. Distribution of eigenvalues of a third-order differential operator with strongly regular boundary conditions / AIP Conf. Proc. 2018. – V1997, – No. 020027. DOI: https://doi.org/10/1063/1.5049021

14 Sadovnichii V.A., Lyubishkin V.A., Belabbasi Yu. On regularized sums of root of an entire function of a certain class / Sov. Math. Dokl. 1980. – No.22, – p.613-616.

15 Sedletskii A.M. (1993). On the zeros of the Fourier transform of finite measure / Mathematical Notes. 1993. – V.53, – No.1, – p.77-84. DOI:https://doi.org/10.1007/BF01208527

16 Bellman R., Kuk K. Differentsialno-raznostnyye uravneniya [Differential-difference equations]. M.: Mir, 1967 [in Russian].

17 Leontyev A.F. Tselyye funktsiy i ryady eksponent [Entire functions and series of exponents]. M.: Nauka, 1983. [in Russian].

18 Riss F., Sekefalvi-Nad B. Lektsii po funktsionalnomu analizu [Lectures on Functional Analysis]. M.: Mir, 1979. [in Russian].

19 Shabat, B.V. Vvedeniye v kompleksnyy analiz. V 2 chastyakh. Chast 1. Funktsii odnogo peremennogo [Introduction to complex analysis. In 2 parts. Part 1. Functions of one variable]. M: URSS, 2015. [in Russian].
20 Kudryavtsev L.D. Kratkiy kurs matematicheskogo analiza [Short Course in Calculus]. M: Nauka, 1989. [in Russian].

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КЕСІНДІДЕГІ ЖҮКТЕЛГЕН БІРІНШІ РЕТТІ ДИФФЕРЕНЦИАЛДЫҚ ОПЕРАТОРМЕН БАЙЛАНЫСТЫ БҮТІН ФУНКЦИЯНЫҢ НӨЛДЕРІ

Бұл мақкалада [-1; 1] кесіндісіндегі периодтық шартпен берілген бірінші ретті дифференциалдық операторлың меншікті мәндерін зерттеуге қойылған есеп қарастырылады. Қарастырылып отырған жүктелген дифференциалдау операторының спектралдық есебінің сипаттамалық анықтауышы құрылып, оның спектралдық параметрден тәуелді болатын бүтін аналитикалық функция болатындығы көрсетілген. Сипаттамалық анықтауыштың негізінде жүктелген (толқытылған) теңдеудің құрамында $\Phi(-1)=\Phi(1)=1$ *шартын қанағаттандыратын өзгеруі шенелген функциясы бар дифференциалдау операторының спектрінің асимптотикасы мен меншікті функциялары туралы қорытындылар жасалған. Меншікті мәндердің комплексті жазықтықтағы орналасуы туралы теорема дәлелденіп, бүтін аналитикалық функцияның регулярлы өсімі көрсетілген. Бүтін аналитикалық функцияның асимптотикалық қасиеттері мен түбірлерінің таралымы зерттелген.*

***Түйін сөздер**: жүктелген дифференциалдық оператор, толқытылу, сипаттамалық анықтауыш, бүтін функцияның нөлдері, асимптотика, меншікті мәндер, спектр, меншікті функциялар, базис.*

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НУЛИ ЦЕЛОЙ ФУНКЦИИ, СВЯЗАННОЙ НАГРУЖЕННЫМ ДИФФЕРЕНЦИАЛЬНЫМ ОПЕРАТОРОМ ПЕРВОГО ПОРЯДКА НА ОТРЕЗКЕ

В статье рассматривается задача на собственные значения нагруженного дифференциального оператора первого порядка с периодическим краевым условием на отрезке [– 1; 1], то есть уравнение содержит нагрузку в точке (- 1) и функции $\Phi(t)$ *– ограниченной вариации, с условием* $\Phi(-1)=\Phi(1)=1$. *Построен характеристический определитель спектральной задачи для рассматриваемого нагруженного оператора дифференцирования, который является целой аналитической функцией от спектрального параметра. На основе формулы характеристического определителя доказаны выводы об асимптотике спектра и собственных функций нагруженной («возмущенной») спектральной задачи для оператора дифференцирования, характеристический определитель которого является целой аналитической функцией от спектрального параметра* *λ. Сформулирована теорема о расположении собственных значений на комплексной плоскости λ, где указан регулярный рост целой аналитической функции. Исследуются мероморфные функции вполне регулярного роста в верхней полуплоскости относительно функции роста и асимптотические свойства целых функций с заданным законом распределения корней. Доказывается теорема об асимптотике нулей целой функции, то есть собственных значений исходной рассматриваемой спектральной задачи для нагруженного дифференциального оператора дифференцирования, при этом изучаются асимптотические свойства целой функции с распределением корней.*

Ключевые слова: *нагруженный дифференциальный оператор, возмущенный, характеристический определитель, нули целой функции, асимптотика, собственные значения, спектр, собственные функций, базис.*